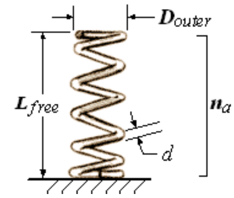
Chapter 13: Springs

with several figures from:
MACHINE DESIGN - An Integrated Approach, 2ed by Robert L. Norton, Prentice-Hall 2000

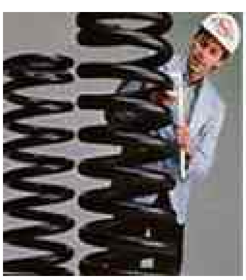
Outline

- Spring Functions & Types
- Helical Springs
  - Compression
  - Extension
  - Torsional



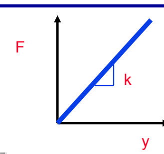
The Function(s) of Springs

Most fundamentally: to STORE ENERGY



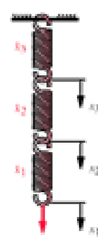
Many springs can also: ***push***
pull
twist

Some Review



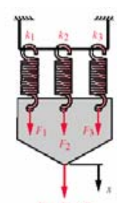
linear springs: k=F/y

nonlinear springs: $k = \frac{dF}{dy}$



Series

$$\frac{1}{k_{total}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

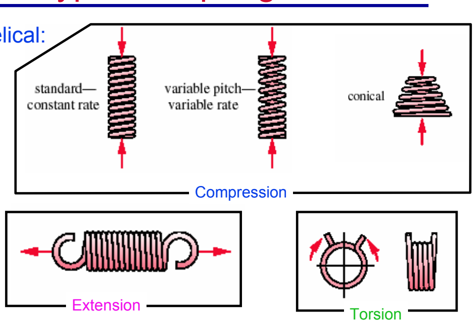


Parallel

$k_{total}=k_1+k_2+k_3$

Types of Springs

Helical:

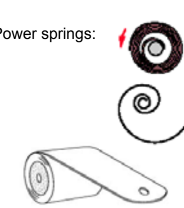


More Springs

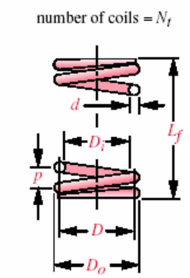
Washer Springs:

Beams:

Power springs:



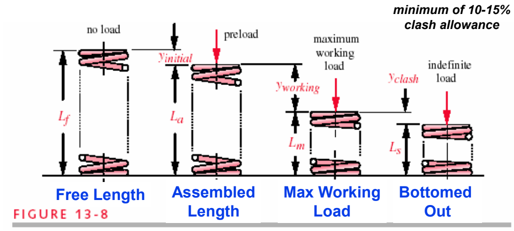
Helical Compression Springs



d	diameter of wire
D	mean coil diameter
L_f	free length
p	pitch
N_t	Total coils

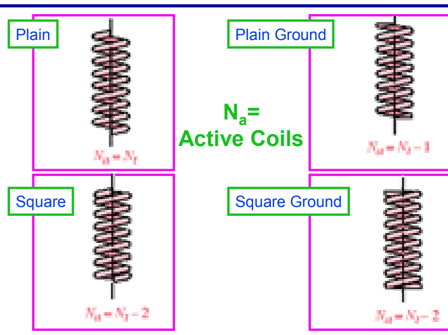
may also need: D_o and D_i

Length Terminology

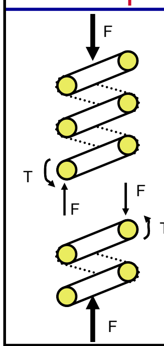


L_f L_a L_m L_s

End Conditions



Stresses in Helical Springs



Spring Index *C=D/d*

Typically: $4 \le C \le 12$

$$\tau_{max} = K_s \frac{8FD}{\pi d^3}, \quad where\ K_s = \frac{2C+1}{2C}$$

Curvature Stress

Inner part of spring is a stress concentration

(see Chapter 4)

K_w includes both the direct shear factor and the stress concentration factor

$$\tau_{max} = K_w \frac{8FD}{\pi d^3}, \quad where\ K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

- under static loading, local yielding eliminates stress concentration, so use K_s
- under dynamic loading, failure happens below S_y: use K_s for mean, K_w for alternating

Spring Deflection

$$y \approx \frac{8FD^3 N_a}{d^4 G}$$

Spring Rate

$$y \approx \frac{8FD^3 N_a}{d^4 G}$$

k=F/y

$$k \approx \frac{d^4 G}{8D^3 N_a}$$

Helical Springs

- Compression
  - Nomenclature
  - Stress
  - Deflection and Spring Constant
  - Static Design
  - Fatigue Design
- Extension
- Torsion

Static Spring Design

- Inherently iterative
  - Some values must be set to calculate stresses, deflections, etc.
- Truly *Design*
  - there is not one "correct" answer
  - must synthesize (a little bit) in addition to analyze

Material Properties

- S_{ut} *ultimate tensile strength*
  - Figure 13-3
  - Table 13-4 with $S_{ut}=Ad^b$
- S_{ys} *torsional yield strength*
  - Table 13-6 – a function of S_{ut} and set

Spring/Material Treatments

- Setting
  - overstress material in same direction as applied load
    - » increase static load capacity 45-65%
    - » increase energy storage by 100%
  - use K_s, not K_w (stress concentration relieved)
- Load Reversal with Springs
- Shot Peening
  - What type of failure would this be most effective against?

What are You Designing?

Given

F, y
k, y

Find

k, F + d, C, D^*, L_f^*, N_a^*, clash allowance (α)**, material**

→ ***design variables***

Such that:

Safety factor is > 1
Spring will not buckle
Spring will fit in hole, over pin, within vertical space

* - often can calculate from given
** - often given/defined

Static Spring Flow Chart

if GIVEN F,y, then find k; If GIVEN k, y, then find F

d, C → D, K_s, K_w → material strengths

material

STRESSES

$N_s = S_{ys}/\tau$

for shut spring if possible
if not, for max working load

N_a, α

DEFLECTION

L_f, y_{shut}, F_{shut}

CHECK

buckling, N_{shut}, D_i, D_o

$N_{shut} = S_{ys}/\tau_{shut}$

ITERATE?

Three things to know:
- effect of d
- shortcut to finding d
- how to check buckling

Static Design: Wire Diameter

$$\tau_{max} = K_s \frac{8FD}{\pi d^3} \qquad y \approx \frac{8FD^3 N_a}{d^4 G}$$

Based on $N_s = S_{sy}/\tau$ and above equation for τ:

$$d = \left\{ \frac{8N_s(C+0.5)[F_{work}(1+\alpha) - F_{initial}(\alpha)]}{\pi K_m A} \right\}^{1/(2+b)}$$

Three things to know:
- effect of d
- shortcut to finding d
- how to check buckling

use Table 13-2 to select standard *d* near calculated *d*

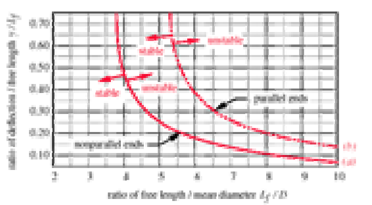
$K_m = S_{ys}/S_{ut}$

*maintain units (in. or mm) for A, b

Buckling

$$S.R. = \frac{L_f}{D}$$

$$y' = \frac{y_{init} + y_{working}}{L_f}$$



Three things to know:
- effect of d
- shortcut to finding d
- how to check buckling

In general; $S.R. = \frac{L_f}{D} < 4$ for safe design

Helical Springs

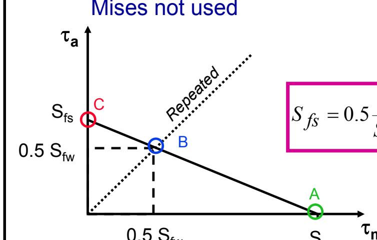
- Compression
  - Nomenclature
  - Stress
  - Deflection and Spring Constant
  - Static Design
  - Fatigue Design
- Torsion

Material Properties

- S_{us} *ultimate shear strength*
  - $S_{us} \approx 0.67\, S_{ut}$
- $S_{fw'}$ *torsional fatigue strength*
  - Table 13-7 -- function of S_{ut}, # of cycles
  - repeated, room temp, 50% reliability, no corrosion
- $S_{ew'}$ *torsional endurance limit*
  - for steel, d < 10mm
  - see page 816 (=45 ksi (310 MPa) if unpeened, =67.5 ksi (465 MPa) if peened)
  - repeated, room temp., 50% reliability, no corrosion

Modified Goodman for Springs

- S_{fw}, S_{ew} are for torsional strengths, so von Mises not used

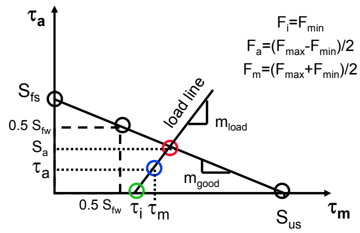


$$S_{fs} = 0.5 \frac{S_{fw} S_{us}}{S_{us} - 0.5(S_{fw})}$$

Fatigue Safety Factor

$F_i = F_{min}$
$F_a = (F_{max} - F_{min})/2$
$F_m = (F_{max} + F_{min})/2$

$$N_{fs} = \frac{S_a}{\tau_a}$$



$\tau_{a,load} = \tau_{a,good}$ at intersection

$$N_{fs} = \frac{S_{fs}(S_{us} - \tau_i)}{S_{fs}(\tau_m - \tau_i) + S_{us}\tau_a}$$

...on page 628

What are you Designing?

Given

$F_{max}, F_{min}, \Delta y$
$k, \Delta y$

Find

k
F
+ d, C, D*, L_f*, N_a*, clash allowance (α)**, material**

design variables

Such that:

Fatigue Safety Factor is > 1
Shut Static Safety Factor is > 1
Spring will not buckle
Spring is well below natural frequency
Spring will fit in hole, over pin, within vertical space

* - often can calculate from Given
** - often given/defined

Fatigue Spring Design Strategy

if GIVEN F,y, then find k; If GIVEN k, y, then find F

d, C → D, K_s, K_w → material strengths

material

STRESSES

$$N_{fs} = \frac{S_{fs}(S_{us} - \tau_i)}{S_{fs}(\tau_m - \tau_i) + S_{us}\tau_a}$$

N_a, α

DEFLECTION

L_f, y_{shut}, F_{shut}

CHECK
buckling, frequency,
N_{shut}, D_i, D_o

$N_{shut} = S_{ys}/\tau_{shut}$

ITERATE?

Two things to know:
- shortcut to finding d
- how to check frequency

Fatigue Design:Wire Diameter

as before, you can iterate to find d, or you can use an equation derived from relationships that we already know:

$$d = \left\{\frac{8CN_{fs}}{0.67\pi A}\left[K_s F_m - \frac{N_{fs}-1}{N_{fs}} K_s F_{min} + \left(1.34\frac{Ad^b}{S_{fw}} - 1\right) K_w F_a\right]\right\}^{1/(2+b)}$$

use Table 13-2 to select standard *d* near calculated *d*

Two things to know:
- shortcut to finding d
- how to check frequency

*maintain units (in. or mm) for A, b

Natural Frequency: Surge

Surge == longitudinal resonance

for fixed/fixed end conditions:

$$f_n = \frac{1}{2}\sqrt{\frac{kg}{W_a}} \quad \text{(Hz)}$$

ideally, f_n will be at least 13x more than $f_{forcing}$…
it should definitely be multiple times bigger

Two things to know:
- shortcut to finding d
- how to check frequency

...see pages 814-815 for more

Review of Design Strategy

ITERATIVE

Find Loading
Select C, d

Find stresses
Determine material properties
Find safety factor

USING d EQUATION

Find Loading
Select C, safety factor

Solve for d, pick standard d
Find stresses
Determine material properties
Check safety factor

Strategy Review Continued

- Find spring constant, N_a, N_t
- Find F_{SHUT} (must find lengths and y's to do this)
  Find static shut shear stress and safety factor
- Check Buckling
- Check Surge
- Check D_i, D_o if pin to fit over, hole to fit in

Consider the Following:

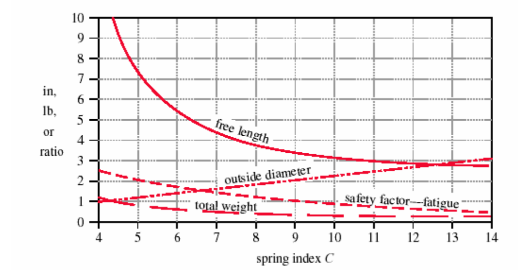
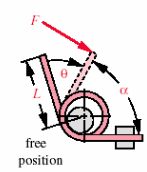


FIGURE 13-18

Helical Springs

- Compression
  - Nomenclature
  - Stress
  - Deflection and Spring Constant
  - Static Design
  - Fatigue Design
- Torsion

Torsion Springs



Deflection & Spring Rate

$$\theta_{rev} = \frac{1}{2\pi}\frac{ML_w}{EI},$$

where,

$L_w = \text{length of wire} = \pi D N_a$

$I = \frac{\pi d^4}{64}$

$$\theta_{rev,roundwire} = 10.2\frac{MDN_a}{d^4E}$$

$$\theta_{rev,roundwire} = 10.8\frac{MDN_a}{d^4E}$$ (if we account for Friction)

$$k = \frac{M}{\theta_{rev}}$$

Stresses

(1) **Static** -Compressive is Max –($K_{bi} > K_{bo}$) – Inside of Coil

$$\sigma_{i_{max}} = K_{b_i}\frac{M_{max}c}{I} = K_{b_i}\frac{32M_{max}}{\pi d^3}$$

$$K_{b_i} = \frac{4C^2 - C - 1}{4C(C-1)}$$

(2) **Fatigue** – (since fatigue is a tensile stress phenomenon) – Outside of Coil

$$\sigma_{o_{max}} = K_{b_o}\frac{32M_{max}}{\pi d^3} \qquad \sigma_{o_{min}} = K_{b_o}\frac{32M_{min}}{\pi d^3}$$

$$K_{b_o} = \frac{4C^2 + C - 1}{4C(C+1)}$$

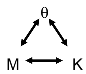
Materials

see Tables 13-13 and 13-14, page 850

follow book on $S_{ewb}=S_{ew}/0.577$… for now

Strategy

Select C, d



- fit over pin (if there is one)
- don’t exceed stresses

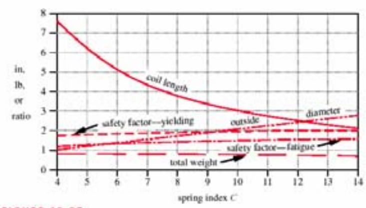


FIGURE 13-27

Helical Springs

- Compression
  - Nomenclature
  - Stress
  - Deflection and Spring Constant
  - Static Design
  - Fatigue Design
- Torsion