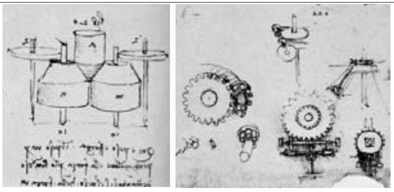
Gears



with several figures from:
MACHINE DESIGN - An Integrated Approach, 2ed by Robert L. Norton, Prentice-Hall 2000
HOW STUFF WORKS – http://www.howstuffworks.com

Types

Spur gears: noisy operation: cement mixer, steel mill drives, corn picker, washing machine

Bevel gears: changes the direction: car differential

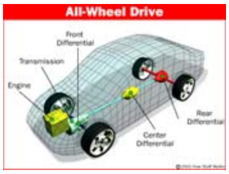
Helical gears: smooth operation: automative tranmission

Worm gears: Enables large gear reductions: conveyors Gear cannot turn worm

Rack and Pinion: Rotary to linear motion: scale, car steering

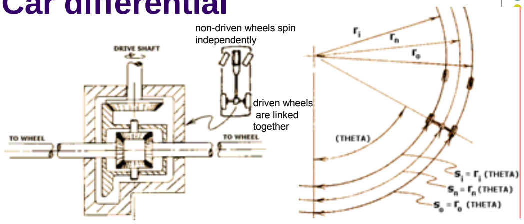
---

Example: Car differential



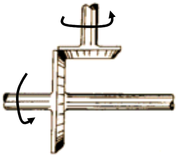
Example: Car differential

Car negotiates a turn

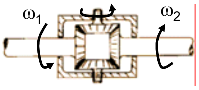


Driven wheels connected to the engine via the driveshaft, however, must both be turned by gearing and this gear train must allow for *differential* movement of the left wheel with respect to the right wheel.
Outer wheel should turn slightly faster than the inner wheel to prevent binding and slippage of the tires
on the road.

Without the "square" set of four gears in the middle of the above diagram which yields to the figure below, both wheels turn at the same angular velocity.
This leads to problems when the car negotiates a turn.

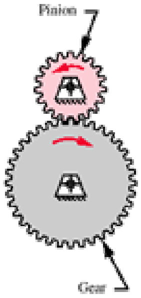


Now imagine the differential "square" alone, as illustrated in the figure on the right.
It should be apparent that turning one wheel results in the opposite wheel turning in the opposite direction at the same rate.
This is how the automobile differential works.
It only comes into play when one wheel needs to rotate *differentially* with respect to its counterpart.
When the car is moving in a straight line, the differential gears do not rotate with respect to their axes.
When the car negotiates a turn, however, the differential allows the two wheels to rotate differentially with respect to each other.



Outline

- Gear Theory
  - Fundamental Law of Gearing
  - Involute profile
- Nomenclature
- Gear Trains
- Loading
- Stresses

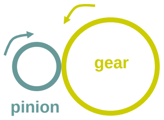


Fundamental Law of Gearing

functionally, a gearset is a device to exchange torque for velocity

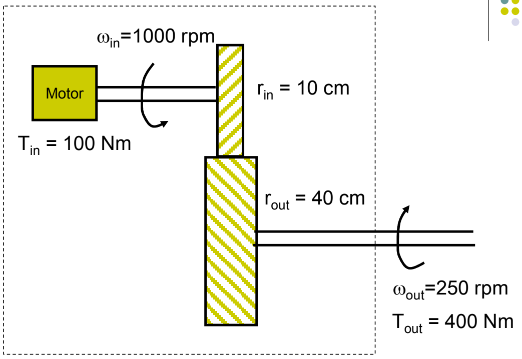
P=Tω

the angular velocity ratio of the gears of a gearset must remain constant throughout the mesh



$$m_V = \frac{r_{in}}{r_{out}} = \frac{\omega_{out}}{\omega_{in}} = \frac{N_{in}}{N_{out}}$$

Example:



The Involute Profile

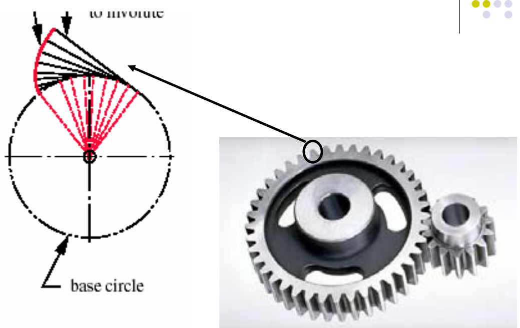
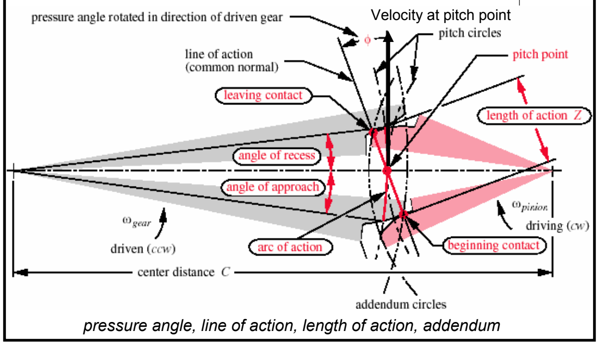


FIGURE 11-3

Profile of the Involute Profile



pressure angle, line of action, length of action, addendum

Nomenclature

$$p_c = \frac{\pi d}{N}$$

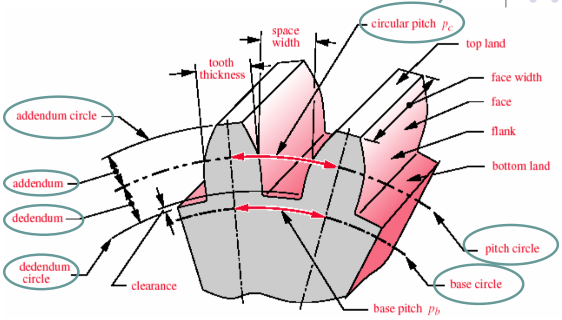


Figure 11-8

Pitches, Etc.

circular pitch (mm, in.) $p_c = \frac{\pi d}{N}$

base pitch (mm, in.) $p_b = p_c \cos\phi$

diametral pitch (teeth/in.) $p_d = \frac{N}{d}$

module (mm/teeth) $m = \frac{d}{N}$

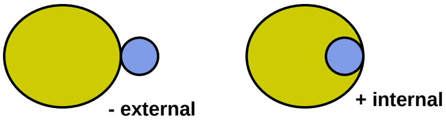
Note: d is pitch diameter

Velocity Ratio

pitches must be equal for mating gears, therefore

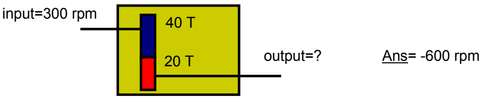
$$m_V = \left|R_{ij}\right| = \pm\frac{d_i}{d_j} = \pm\frac{N_i}{N_j} = \frac{\omega_j}{\omega_i}$$

R_{ij} : *Gear Ratio*
d : *diameter*
N : *number of teeth*
ω : *angular rotation*



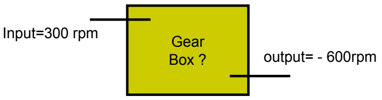
Kinematic Design of Gears:

TYPE A:



Ans= -600 rpm

TYPE B



Ans=A gear and a pinion
Gear ratio=2

Contact Ratio (m_p)

average number of teeth in contact at any one time
=
length of action (Z) divided by the base pitch (P_b), or,

$$m_p = \frac{p_d Z}{\pi\cos\phi}$$

$$Z = \sqrt{(r_p + a_p)^2 - (r_p\cos\phi)^2} + \sqrt{(r_g + a_g)^2 - (r_g\cos\phi)^2} - C\sin\phi$$

where C : distance between the centers
$C = r_g + r_p$
$= (N_g + N_p)*1/p_d*1/2$

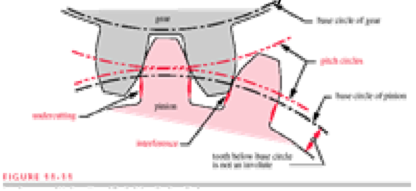
a : addendum

Minimum # of Teeth

Undercutting occurs if dedendum of pinion extends below the base circle and removes material from the root of the gear.

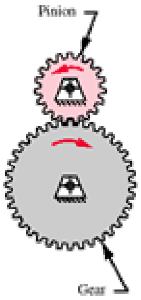
minimum # of full depth teeth on a pinion to avoid undercutting with gear and rack:

$$N_{\min} = \frac{2}{\sin^2\phi}$$

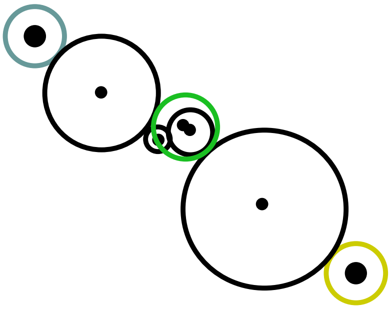


Outline

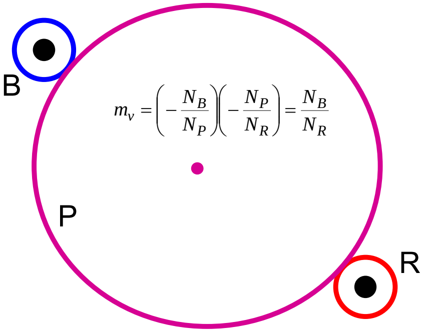
- Gear Theory
  - Fundamental Law of Gearing
  - Involute profile
- Nomenclature
- Gear Trains
- Loading
- Stresses



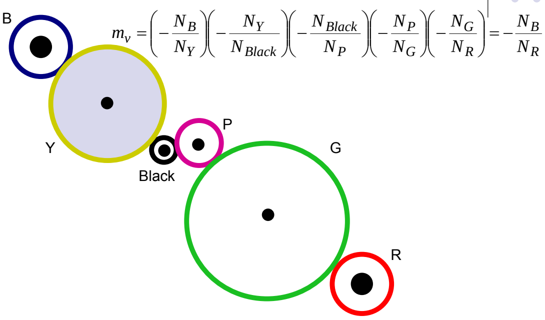
Simple Gear Trains



Simple Gear Train



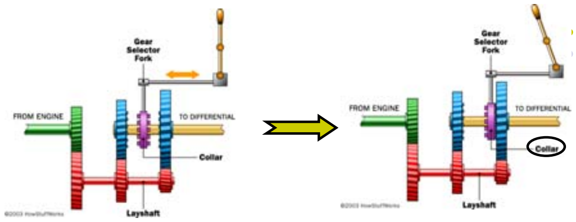
$$m_v = \left(-\frac{N_B}{N_P}\right)\left(-\frac{N_P}{N_R}\right) = \frac{N_B}{N_R}$$



$$m_v = \left(-\frac{N_B}{N_Y}\right)\left(-\frac{N_Y}{N_{Black}}\right)\left(-\frac{N_{Black}}{N_P}\right)\left(-\frac{N_P}{N_G}\right)\left(-\frac{N_G}{N_R}\right) = -\frac{N_B}{N_R}$$

Simple Gear Train

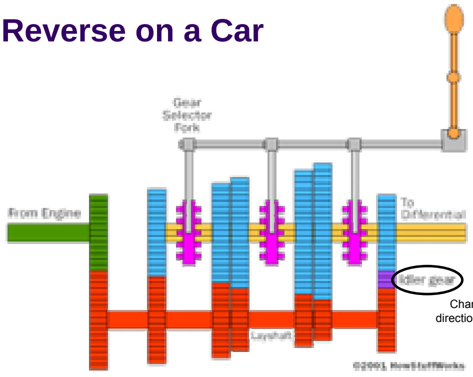
- Fine for transmitting torque between
  - shafts in close proximity
  - when m_v does not need to be too large
- Use third gear (“idler”) only for directional reasons (not for gear reduction)



The green shaft comes from the engine through the **clutch**. The clutch is a device that lets you connect and disconnect the engine and the transmission. When you push in the clutch pedal, the engine and the transmission are disconnected so the engine can run even if the car is standing still.

The purpose of the **collar** is to connect one of the two blue gears to the yellow drive shaft. Teeth on the collar, called **dog teeth**, fit into holes on the sides of the blue gears to engage them. In this picture, the green shaft from the engine turns the layshaft, which turns the blue gear on the right.

Reverse on a Car

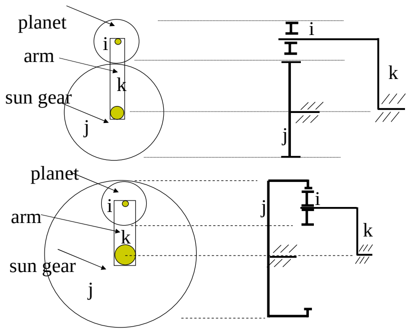


Changes the direction of motion

Actual Manual Transmission

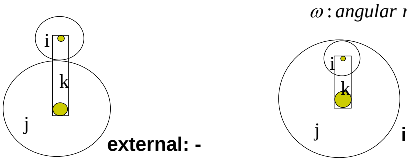
Most of the gears in a manual transmission have helical teeth. The three gears that make up reverse have straight teeth. The large spur gear on the right slides up to put the car in reverse.

Planetary Gear Trains



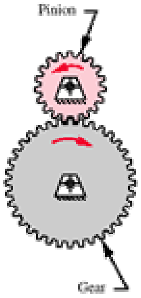
$$m_V = \left|R_{ij}\right| = \pm\frac{d_i}{d_j} = \pm\frac{N_i}{N_j} = \frac{\omega_j - \omega_k}{\omega_i - \omega_k}$$

R_{ij} : Gear Ratio
d : diameter
N : number of teeth
ω : angular rotation



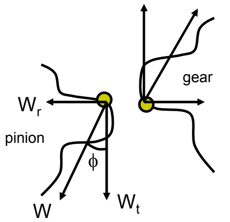
Outline

- Gear Theory
  - Fundamental Law of Gearing
  - Involute profile
- Nomenclature
- Gear Trains
- Loading
- Stresses



Loading of Gears

W_t= tangential (transmitted) load
W_r= radial load
W= total load



$$W_t = \frac{T_p}{r_p} = \frac{2T_p}{d_p}$$

$$W_r = W_t \tan\phi$$

$$W = \frac{W_t}{\cos\phi}$$

MACHINE DESIGN - An Integrated Approach, 2ed by Robert L. Norton, Prentice-Hall 2000

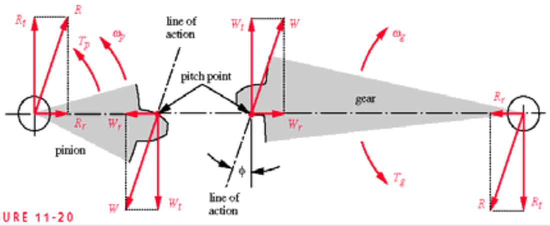
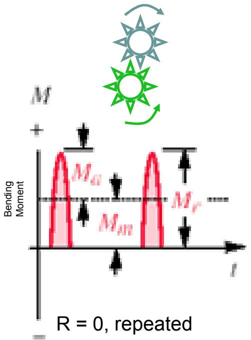


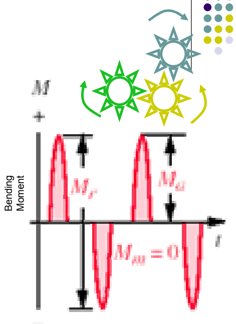
FIGURE 11-20

Forces on the Pinion and Gear in a Gearset (Gears Separated for Illustration—Pitch Points are Actually in Contact)

Gear vs. Idler



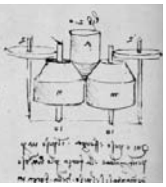
R = 0, repeated
moment on nonidler tooth



R = -1 fully reversed
moment on idler tooth
(more critical)

Outline

- Gear Theory
  - Fundamental Law of Gearing
  - Involute profile
- Nomenclature
- Gear Trains
- Loading
- Stresses



Gear Failure

Fatigue Loading

from bending of teeth
- infinite life possible
- failure can be sudden

Surface Failure

from contact b/n of teeth
- infinite life not possible
- failure is gradual

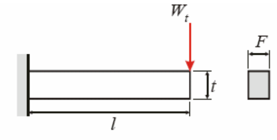
Lewis, 1892, first formulation of gear tooth fatigue failure

$$\sigma_b = \frac{W_t \cdot p_d}{F \cdot Y}$$

F: Face width
Y: Geometry factor
W_t:Tangential force
p_d:diametral pitch

Lewis Equation $$\sigma_b = \frac{W_t \cdot p_d}{F \cdot Y}$$

Cantilever Beam Assumption:



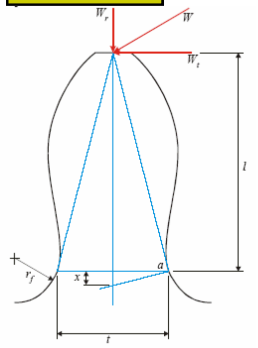
$$\sigma_b = \frac{Mc}{I} = \frac{6W_t l}{Ft^2}$$

where, $M = W_t l$

$c = t/2$

$I = \frac{1}{12}Ft^3$

Lewis Equation $$\sigma_b = \frac{W_t \cdot p_d}{F \cdot Y}$$



Using similar triangles:

$$\frac{t/2}{x} = \frac{l}{t/2} \Rightarrow x = \frac{t^2}{4l}$$

$$\sigma_b = \frac{W_t\ p}{F(\frac{2}{3})xp}$$ p is circular pitch

Define y = 2x/3p

$$\sigma_b = \frac{W_t}{Fyp} \qquad \sigma_b = \frac{W_t\ p_d}{FY}$$

p_d is diametral pitch = π / p
Y is geometry factor = πy

AGMA Gear Stress Formula

many assumptions: see page 714…
…including that the contact ratio $1<m_p<2$

$$\sigma_b = \frac{W_t \cdot p_d}{F \cdot J} \cdot \frac{K_a K_m}{K_v} K_s K_B K_I$$

J K_v K_m K_a K_s K_B K_I

Bending Strength Geometry Factor (J)

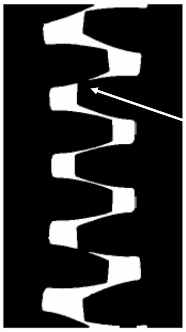
- from tables on pgs. 666-668
- Inputs
  - pinion or gear
  - number of teeth
  - pressure angle
  - long-addendum or full-depth
  - tip loading or HPSTC

J K_v K_m K_a K_s K_B K_I

Tip vs. HPSCT Loading

For high precision gears, with load sharing between the gear teeth, use the HPSCT value.

For low precision gears, there will be more likely be no load sharing so that the load will be carried by a single tooth at its tip.



poor load sharing due to tooth İnaccuracy (only tip is in contact)

Full-Depth vs. Long Addendum

MACHINE DESIGN - An Integrated Approach, 2ed by Robert L. Norton, Prentice-Hall 2000

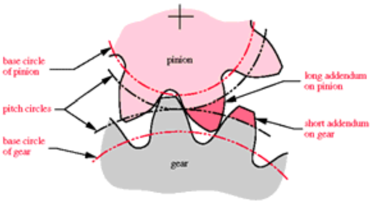


FIGURE 11-12

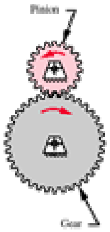
Profile-Shifted Gear Teeth with Long and Short Addenda to Avoid Interference and Undercutting

In order to avoid interference and undercutting on small pinions, the tooth form can be changed from the standard, **full-depth** shape (equal addenda on both pinion and gear) to **longer addendum** on the pinion and a shorter one on the gear (so that pinion teeth becomes stronger).

Finding J for the Example

- pinion is driving gear with 50hp at 1500 rpm
- $N_p=20$, $N_g=40$
- $p_d=4$ /in.
- $\phi=20$ degrees
- $W_t=420$ lb.

$J_p=0.34$, $J_g=0.38$



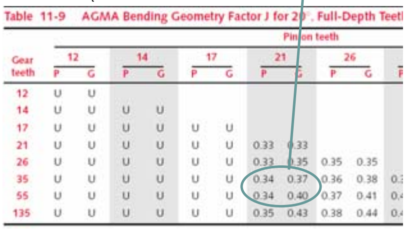


Table 11-9 AGMA Bending Geometry Factor J for 20°, Full-Depth Teet

Gear teeth	12		14		17		21		26		
	P	G	P	G	P	G	P	G	P	G	P
12	U	U									
14	U	U	U	U							
17	U	U	U	U	U	U					
21	U	U	U	U	U	U	0.33	0.33			
26	U	U	U	U	U	U	0.33	0.35	0.35	0.35	
35	U	U	U	U	U	U	0.34	0.37	0.36	0.38	0.3
55	U	U	U	U	U	U	0.34	0.40	0.37	0.41	0.4
135	U	U	U	U	U	U	0.35	0.43	0.38	0.44	0.4

U means undercutting

Dynamic (Velocity) Factor

to account for tooth-tooth impacts and resulting vibration loads

from Figure 11-22 or from Equations 11.16-11.19 (pages 665-668)

Inputs needed:

- Quality Index (Table 11-6)
- Quality Index (Table 11-7)
- Pitch-line Velocity

V_t = (radius)(angular speed in radians)

ft/min	US units
m/s	SI units

pp. 659

Table 11-7 Recommended Gear Quality Numbers for Pitch Line Velocity

Pitch Velocity	Q_v
0–800 fpm	6–8
800–2000 fpm	8–10
2000–4000 fpm	10–12
Over 4000 fpm	12–14

J K_v K_m K_a K_s K_B K_I

Dynamic Factor

(Note: Q_v is the quality index of the lower quality gear in the mesh)

$$B=\frac{(12-Q_v)^{2/3}}{4} \text{ for } 6 \le Q_v \le 11$$

$$A=50+56(1-B)$$

$$K_v=\left(\frac{A}{A+\sqrt{V_t}}\right)^B \quad (US)$$

$V_{t,max}=[A+(Q_v-3)]^2$

$$K_v=\left(\frac{A}{A+\sqrt{200V_t}}\right)^B \quad (SI)$$

$V_{t,max}=[A+(Q_v-3)]^2/200$

J K_v K_m K_a K_s K_B K_I

Determining K_v for the Example

V_t=(1500rev/min)(2.5 in)(1ft/12in)(2π rad/1rev)=1964 ft/min

(well below $V_{t,max}$)

therefore, Q_v=10

Table 11-7 Recommended Gear Quality Numbers for Pitch Line Velocity

Pitch Velocity	Q_v
0–800 fpm	6–8
800–2000 fpm	8–10
2000–4000 fpm	10–12
Over 4000 fpm	12–14

$$B=\frac{(12-10)^{2/3}}{4}=0.397$$

$$A=50+56(1-0.397)=83.8$$

$$K_v=\left(\frac{83.8}{83.8+\sqrt{1964}}\right)^{0.397}=0.845$$

Load Distribution Factor K_m

to account for distribution of load across face

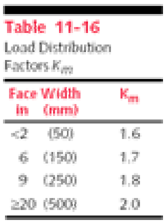


Table 11-16
Load Distribution
Factors K_m

Face Width in	(mm)	K_m
<2	(50)	1.6
6	(150)	1.7
9	(250)	1.8
≥20	(500)	2.0

F: Face width

$8/p_d < F < 16/p_d$
can use $F = 12/p_d$ as a starting point

J K_v K_m K_a K_s K_B K_I

K_m for the Example

Assume $F=12/p_d=3$ in.

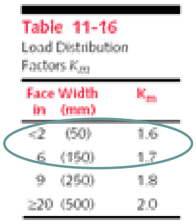


Table 11-16
Load Distribution
Factors K_m

Face Width in	(mm)	K_m
<2	(50)	1.6
6	(150)	1.7
9	(250)	1.8
≥20	(500)	2.0

K_m=1.63

Application Factor, K_a

to account for non-uniform transmitted loads

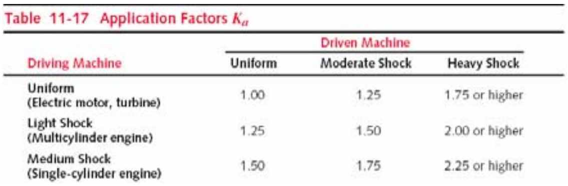


Table 11-17 Application Factors K_a

	Driven Machine		
Driving Machine	Uniform	Moderate Shock	Heavy Shock
Uniform (Electric motor, turbine)	1.00	1.25	1.75 or higher
Light Shock (Multicylinder engine)	1.25	1.50	2.00 or higher
Medium Shock (Single-cylinder engine)	1.50	1.75	2.25 or higher

for example, assume that all is uniform

J K_v K_m K_a K_s K_B K_I

Other Factors

K_s

to account for size

K_s=1 unless teeth are very large
If large, a value of 1.25 to 1.5 would be a conservative assumption

K_B

to account for gear with a rim

K_B=1 for solid gears
(Large-diameter gears are sometimes manufactured with a rim rather than as a solid disk).

K_I

to account for extra loading on idler

K_I=1 for non-idlers, K_I=1.42 for idler gears

J K_v K_m K_a K_s K_B K_I

Back to the Example

$$\sigma_b = \frac{W_t \cdot p_d}{F \cdot J} \cdot \frac{K_a K_m}{K_v} K_s K_B K_I$$

$$\sigma_{b_{gear}} = \frac{(420)(4)}{(3)(0.38)} \cdot \frac{(1)(1.63)}{(0.845)} (1)(1)(1) = 2842\ psi$$

$$\sigma_{b_{pinion}} = \frac{(420)(4)}{(3)(0.34)} \cdot \frac{(1)(1.63)}{(0.845)} (1)(1)(1) = 3177\ psi$$

Compared to What??

σ_b is great, but what do I compare it to?

$$S_{fb} = \frac{K_L}{K_T K_R} S_{fb'}$$

similar concept to S_e, but particularized to gears

$S_{fb'}$ → Table 11-20
Figure 11-25 for steels

Life Factor K_L

to adjust test data from 1E7 to any number of cycles

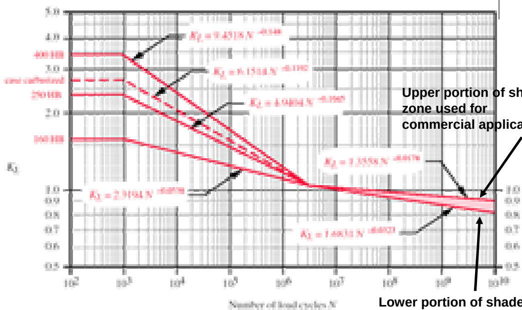


FIGURE 11-24

Upper portion of shaded zone used for commercial applications

Lower portion of shaded zone used for critical service applications

Temperature & Reliability Factors

K_T=1 if T < 250°F

see Equation 11.24*a* if T > 250°F

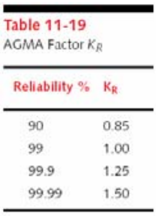


Table 11-19

AGMA Factor K_R

Reliability %	K_R
90	0.85
99	1.00
99.9	1.25
99.99	1.50

Fatigue Bending Strength (example continued)

Class 40 Cast Iron
450 rpm, 8 hours/day, 10 years
T = 80°F
Reliability=99%

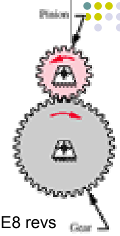


Table 11-20 ⟶ S_{fb}'= 13ksi

N=(450 rev/min)(60 min/hr)(8 hr/dy)(250 dy/yr)(10 yrs)=5.4E8 revs

K_L=1.3558$N^{-0.0178}$=0.948

S_{fb} = (0.948)13ksi = 12.3 ksi

Safety Factor

$$N_b = \frac{S_{fb}}{\sigma_b}$$

$N_{b\,pinion}$ =(12300 psi)/(3177psi)= 3.88

$N_{b\,gear}$ =(12300 psi)/(2842psi)= 4.33

Gear Failure

Fatigue Loading

from bending of teeth
- infinite life possible
- failure can be sudden

Surface Failure

from contact b/n of teeth
- infinite life not possible
- failure is gradual

Buckingham pioneered this work

$$\sigma_c = C_p \sqrt{\frac{W_t}{F \cdot I \cdot d} \boxed{\frac{C_a C_m}{C_v} C_s} \boxed{C_f}}$$

equal to 1

same as for bending

d= pitch diameter of smaller gear

Geometry Factor ("I")

considers radius of curvature of teeth and pressure angle

$$I = \frac{\cos\phi}{\left(\frac{1}{\rho_p} \pm \frac{1}{\rho_g}\right) d_p}$$

ρ: **radius of curvature**

$$\rho_p = \left[\left(r_p + \frac{1 + x_p}{p_d}\right)^2 - \left(r_p \cos\phi\right)^2\right]^{1/2} - \frac{\pi}{p_d}\cos\phi \qquad \rho_g = C \sin\phi \mp \rho_p$$

top sign is for external gears, bottom for internal

(e.g. İf the mating is external, use plus sign for I and minus sign for the calculation of ρ_g)

For example: ρ_p=0.691, ρ_g=1.87, I = 0.095

Elastic Coefficient (C_p)

considers differences in materials

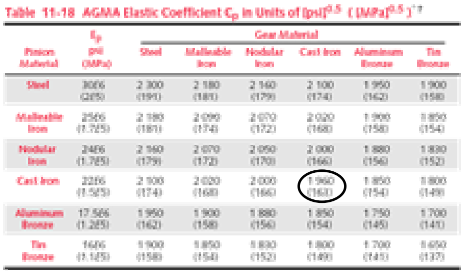


Table 11-18 AGMA Elastic Coefficient C_p in Units of $[\text{psi}]^{0.5}$ ($[\text{MPa}]^{0.5}$)*†

Pinion Material	E_p psi (MPa)	Gear Material					
		Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminum Bronze	Tin Bronze
Steel	30E6 (2E5)	2 300 (191)	2 180 (181)	2 160 (179)	2 100 (174)	1 950 (162)	1 900 (158)
Malleable Iron	25E6 (1.7E5)	2 180 (181)	2 090 (174)	2 070 (172)	2 020 (168)	1 900 (158)	1 850 (154)
Nodular Iron	24E6 (1.7E5)	2 160 (179)	2 070 (172)	2 050 (170)	2 000 (166)	1 880 (156)	1 830 (152)
Cast Iron	22E6 (1.5E5)	2 100 (174)	2 020 (168)	2 000 (166)	1 960 (163)	1 850 (154)	1 800 (149)
Aluminum Bronze	17.5E6 (1.2E5)	1 950 (162)	1 900 (158)	1 880 (156)	1 850 (154)	1 750 (145)	1 700 (141)
Tin Bronze	16E6 (1.1E5)	1 900 (158)	1 850 (154)	1 830 (152)	1 800 (149)	1 700 (141)	1 650 (137)

For example, C_p=1960 psi (163 MPa)

Surface-Fatigue Strengths

σ_c is great, but what do I compare it to?

$$S_{fc} = \frac{C_L C_H}{\boxed{C_T C_R}} \boxed{S_{fc}'}$$

from Table 11-21

same as for bending

Life Factor C_L

to adjust test data from 1E7 to any number of cycles

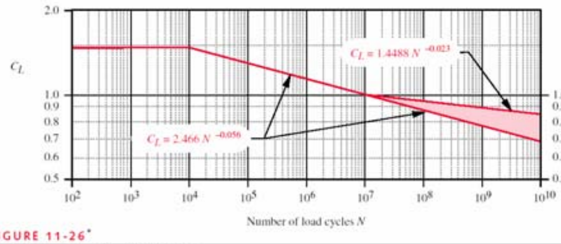


FIGURE 11-26*

AGMA Surface-Fatigue Strength Life Factor C_L

Hardness Ratio C_H

to account for pitting resistance

when pinion is harder than gear, then gear is cold-worked

only apply C_H to the gear-tooth strength, not to the pinion

when HB_p=HB_G, then C_H=1

Otherwise, see Eqs. 11-26a, 11-26b, 11-26c, 11-26d

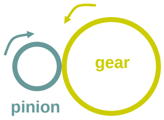
Safety Factor for Loading

stress is based on the square root of loading, therefore:

$$N_c = \left(\frac{S_{fc}}{\sigma_c} \right)^2$$

Gear
Design

Gear Design



Same for Pinion and Gear

- **p_d (p_c), ϕ, F**
- **Power ($T_g\ \omega_g = T_p\ \omega_p$)**
- **W, W_t, W_r**
- **V_t**
- **N_c (Safety Factor for Surface Failure)**

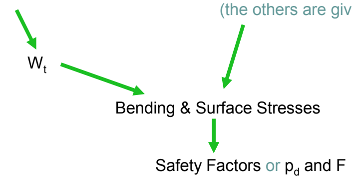
Different for Pinion and Gear

- **d, N**
- **T, ω ,**
- **N_b (Safety Factor for Bending Failure)**

Gear Design Strategy

Given or Set:
gear ratio
Power, ω, T

Properties of Gears to Determine:
d_p, d_g, N_p, N_g
p_d, F -OR- Safety Factors
(the others are given)



W_t

Bending & Surface Stresses

Safety Factors or p_d and F