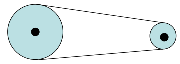
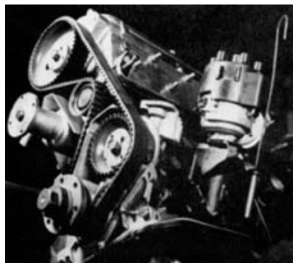
Flexible Elements: Belts



Belts, ropes, chains, and other similar flexible elements are used in conveying systems and in the transmission of power over comparatively long distances. In many cases, their use simplifies the design of a machine and substantially reduce the cost.



Advantages/Disadvantages

Advantages:

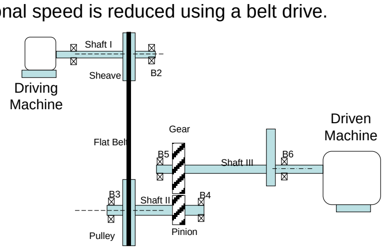
- Small amount of installation work
- Low maintainance
- High reliability
- In some applications, shock and sound absurption
- Transmission of power over long distances

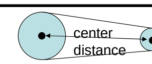
Disadvantages:

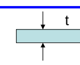
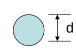
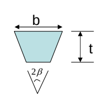
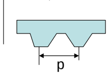
- Limited power transmission. If very large ratios of speed reduction are required in the drive, gear reducers are desirable because they can typically accomplish large reductions in a rather small package.

Flexible Elements: Belts

Motors generally operate too fast and deliver too low torque to be appropriate for the final application. The torque is increased in proportion to the amount that rotational speed is reduced using a belt drive.

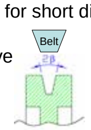




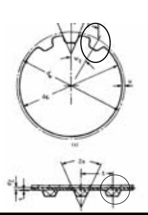
Type	Figure	Joint	Size	Center Distance
Flat		Yes	t = 0.75-5 mm	No upper Limit
Round		Yes	d = 0.3-2 mm	No upper limit
V-belt		None	t = 8-19 mm	Limited
Timing Belt		None	p = 2 mm and up	Limited

Flexible Elements: Belts

- Flat and round belts are typically used for long distances V-belt and timing belts are used for short distances.
- V-belt sits tightly into the groove increasing friction and allowing high torques to be transmitted.



- Timing belts used with sprockets that have mating grooves for the teeth on the belt.



Timing Belt

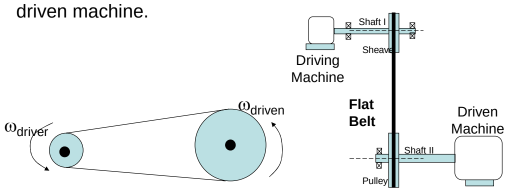
Timing belt is a toothed belt used to coordinate the turnings of crankshafts and camshafts in internal combustion engines. The timing belt needed to synchronize the camshaft to the crankshaft position, so the valves will open and close at the proper time in the relation to the position of the pistons. The camshaft rotates at exactly 1/2 speed of the crankshaft.

Timing belts are engineered to last at least 100,000 kms, with some newer designs lasting the life of the engine. Vehicle owners are urged to replace timing belts according to the car manufacturer's recommendations. If a timing belt fails completely, the entire engine will grind to a halt and the car will become inoperable.

4-cylinder twin-cam engine

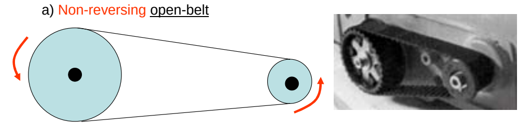
Function

When the belt is used for speed reduction, the smaller sheave is mounted on the high speed shaft, like the shaft of an electric motor. The larger sheave is then put on the driven machine.

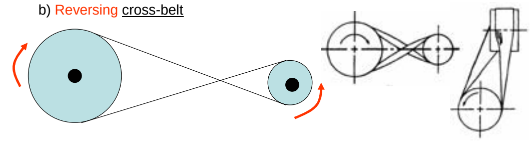


Belt Geometry

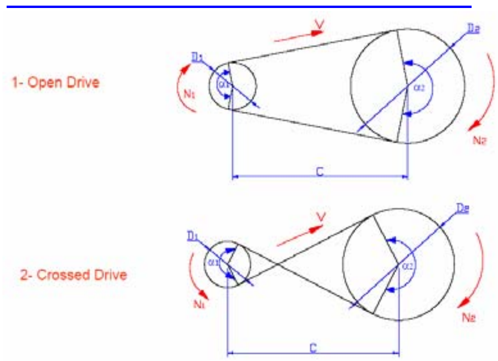
a) Non-reversing open-belt



b) Reversing cross-belt



Belt Geometry



Belt Geometry

D_1: Diameter of small Pulley

D_2: Diameter of Large Pulley

C: Center Distance

α_1: Angle of contact between the belt and the small pulley

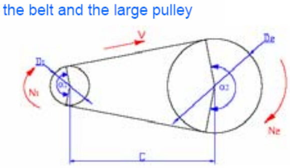
α_2: Angle of contact between the belt and the large pulley

L: Length of belt

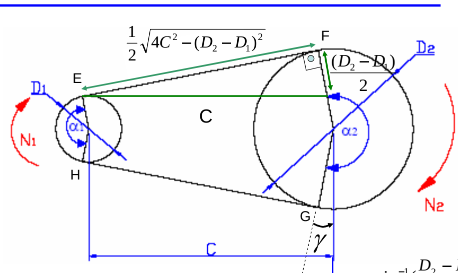
N_1: Speed of small pulley

N_2: Speed of large pulley

V: Velocity of belt



Belt Geometry



L (Length of Belt) = EF + FG + GH + EH

$$L = \sqrt{4C^2-(D_2-D_1)^2} + \frac{D_1\alpha_1 + D_2\alpha_2}{2}$$

$$\gamma = \sin^{-1}\left(\frac{D_2-D_1}{2C}\right)$$

$$\alpha_1 = \pi - 2\gamma$$

$$\alpha_2 = \pi + 2\gamma$$

Belt Geometry

Relations

$$\frac{N_1}{N_2} = \frac{D_2}{D_1} \qquad V = \frac{\pi D_1 N_1}{60} = \frac{\pi D_2 N_2}{60}$$

For open belt drive

$$\alpha_1 = \pi - 2\sin^{-1}\left(\frac{D_2-D_1}{2C}\right) \quad rad$$

$$\alpha_2 = 2\pi - \alpha_1 = \pi + 2\sin^{-1}\left(\frac{D_2-D_1}{2C}\right) \quad rad$$

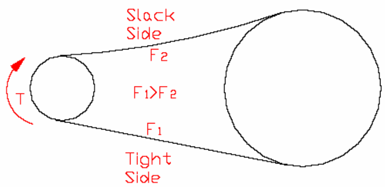
$$L = Belt\ Length = \sqrt{4C^2-(D_2-D_1)^2} + \frac{D_1\alpha_1 + D_2\alpha_2}{2}$$

For crossed belt drive

$$\alpha_1 = \alpha_2 = \pi + 2\sin^{-1}\left(\frac{D_2+D_1}{2C}\right) \quad rad$$

$$L = Belt\ Length = \sqrt{4C^2-(D_2+D_1)^2} + \frac{\alpha(D_1+D_2)}{2}$$

Power and Force Analysis of Flat Belt



$P = (F_1 - F_2)\,V$ ....(1)

P: Transmitted Power

T: Transmitted Torque

F_1: Tension in tight side

F_2: Tension in slack side

Power and Force Analysis of Flat Belt

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\alpha|_{min}} \quad ....(2)$$

F_c: Centrifugal Tension

μ: Coefficient of friction between belt and pulley

$\mu\alpha|_{min}$ is the smaller value between $\mu_1\alpha_1$, $\mu_2\alpha_2$ where μ_1 and μ_2 are the coefficient of friction between belt and small and large pulley consequently. If $\mu_1 = \mu_2 = \mu$ (The two pulleys are made from the same material), $\mu\alpha|_{min} = \mu\,\alpha_1$ for open belt.

Dividing each parameter in the left hand side of Equation (2) by the area of the belt, the following equation results:

$$\frac{\sigma_1 - \sigma_c}{\sigma_2 - \sigma_c} = e^{\mu\alpha|_{min}} \quad ....(3)$$

σ_1, σ_2: Max. and Min. stresses in the belt

σ_c: Centrifugal stress

How to calculate hoop force/stress?

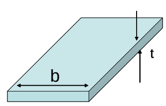
$$F_c = mV^2$$

$$V = V_s = \omega_p r_p = V_p = \omega_s r_s$$

$$m = \frac{Weight}{g}$$

$$Weight = (Weight\ Density) * (Volume)$$

$$Weight = (\rho)(1 * b * t)$$



$$\sigma_c = \frac{F_c}{Area} = \frac{F_c}{bt}$$

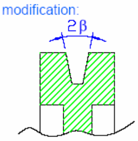
Power and Force Analysis of V-Belt

All the equations of flat belt are applied for v-belt with one modification:

$\mu\alpha|_{min}$ in the equations is replaced by $(\mu\alpha|_{min})/\sin(\beta)$

Where 2β is the groove angle of v-belt pulley

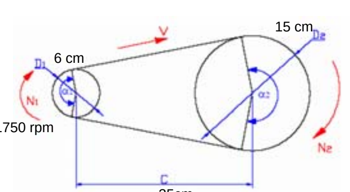
$2\beta = 32°, 34°, 36°, 38°$.



$$\frac{F_1 - F_c}{F_2 - F_c} = e^{[\mu\alpha|_{min} / \sin(\beta)]}$$

Belt Problem:

A flat belt is used to transmit power from a variable diamter sheave to a flat pulley with an effective diameter of 15 cm. The effective diamter of the sheave is 6 cm. Belt cross-sectional area is 2 cm2. The coefficient of friction is 0.2. The allowable tension in the belt is 12 kN/m3. Determine the maximum power that can be transmitted by this arrangement if the speed of the sheave is 1750 rpm.



Solution:

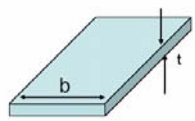
$$D_s = 6\,cm$$
$$D_p = 15\,cm$$
$$C = 35\,cm$$
$$\sigma_{all} = 1.2\,MPa$$
$$A = 2\,cm^2$$
$$\rho = 12\,kN/m^3$$

$$\gamma = \sin^{-1}\left(\frac{D_p - D_s}{2C}\right)$$
$$\alpha_1 = \pi - 2\gamma$$
$$\alpha_2 = \pi + 2\gamma$$

$$\alpha_1 = 2.88\,rad$$
$$\alpha_2 = 3.40\,rad$$

$$F_c = mV^2$$
$$V = V_s = \omega_p r_p = V_p = \omega_s r_s$$
$$\omega_s = 1750 \frac{2\pi}{60}$$
$$V_s = 5.5\, m/s$$

$$m = \frac{Weight}{g} \frac{\rho b t}{9.81} = \frac{12000(2.10^{-4})}{9.81}$$
$$m = 0.245\, kg/m$$



$$F_c = 0.245(5.5)^2 = 7.41 N\, (per\ length)$$
$$\sigma_c = \frac{7.41}{Area} = \frac{7.41}{2.10^{-4}} = 0.037\, MPa$$

The design is based on the smaller angle of contact.

$$\frac{\sigma_{1s} - \sigma_c}{\sigma_{2s} - \sigma_c} = e^{\mu \alpha_1}$$
$$\sigma_{1s} = \sigma_{all} = 1.2\, MPa$$
$$\frac{1.2 - 0.037}{\sigma_{2s} - 0.037} = e^{0.576} \Rightarrow \sigma_{2s} = 0.69\, MPa$$

$$F_1 - F_2 = A(\sigma_{1s} - \sigma_{2s})$$
$$F_1 - F_2 = 2.10^{-4}(1.2 - 0.69) = 102\, N$$

$$T = \frac{D_s}{2}(F_1 - F_2) = 3.06\, Nm$$
$$H = T\omega$$
$$H = 3.06 * 1750 \frac{2\pi}{60} = 560\, Watts$$
or
$$H = (F_1 - F_2)V = 102(5.5) = 560\, Watts$$