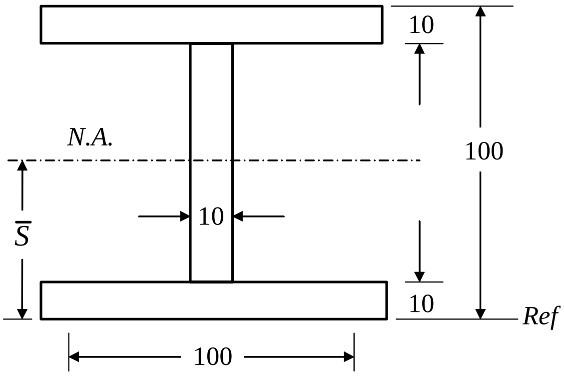
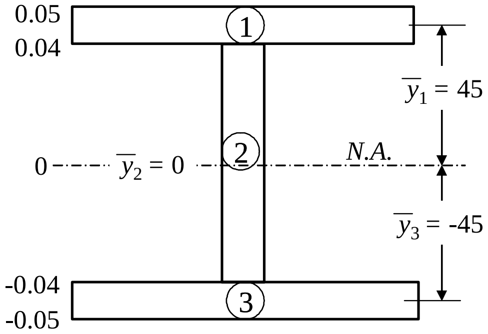
Example 1: Determine shear stress distribution in the following I beam, with a shear force of $V = 25\ k$N applied.





Parallel Axis Theorem to find global I

Step 1: Determine the sectional geometric properties

Neutral Axis Location: $\bar{S} = \dfrac{\sum s_i A_i}{\sum A_i} = \dfrac{95\times100\times10 + 50\times80\times10 + 5\times100\times10}{100\times10 + 80\times10 + 100\times10} = 50mm$

Parallel Axis Theorem: $I = \sum\left(I_{i\,local} + \bar{y}_i^2 A_i\right) = \left(I_{1loc} + \bar{y}_1^2 A_1\right) + \left(I_{2loc} + \bar{y}_2^2 A_2\right) + \left(I_{3loc} + \bar{y}_3^2 A_3\right)$

$$I = \left(\frac{100\times10^3}{12} + (45)^2\times100\times10\right) + \left(\frac{10\times80^3}{12} + 0^2\times10\times80\right) + \left(\frac{100\times10^3}{12} + (-45)^2\times100\times10\right)$$

$I = 4.493\times10^{+6}\ mm^4 = 4.493\times10^{-6}\ m^4$

Step 2: Determine shear stress distribution

Start the integration from the top and work yourself down through all sub-sections of constant thickness, ***ALWAYS integrating about the Neutral Axis***.

The shear stress equation is:

$$\tau_{xy} = \frac{V(x)}{It(y)}\int_{y}^{y_{top}} yt(y)dy = \frac{VQ}{It}$$

We need to express shear stress segment by segment as divided in Step1.

i) For the range between $0.04 \le y \le 0.05$, i.e. *Area* 1, the shear stress is given by:

$$\tau_{xy} = \frac{25\times10^3}{4.493\times10^{-6}\times0.1}\int_{y}^{0.05} 0.1ydy = 2.782\times10^9\left(0.0025 - y^2\right)$$

ii) Range $-0.04 \le y \le 0.04$, i.e. *Area* 2, the shear stress is given by:

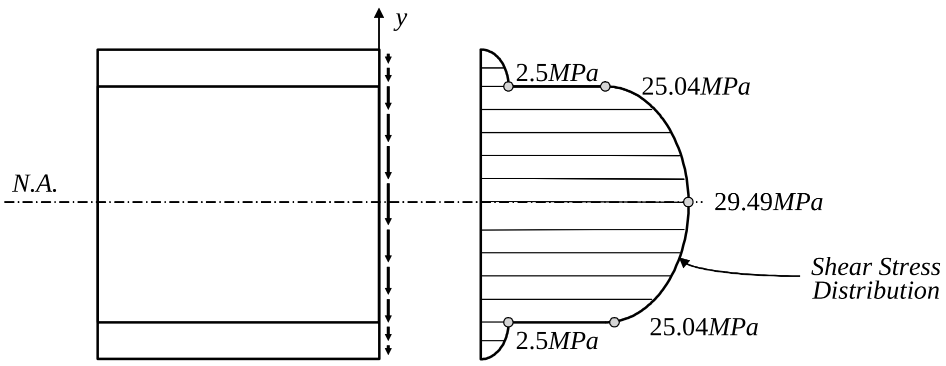
$$\tau_{xy} = \frac{25\times10^3}{4.493\times10^{-6}\times0.01}\left(\int_{0.04}^{0.05} 0.1ydy + \int_{y}^{0.04} 0.01ydy\right)$$

$$= 5.564\times10^{10}\left[0.1\times\left(\frac{0.05^2}{2} - \frac{0.04^2}{2}\right) + 0.01\times\left(\frac{0.04^2}{2} - \frac{y^2}{2}\right)\right] = 2.782\times10^9\left(0.0106 - y^2\right)$$

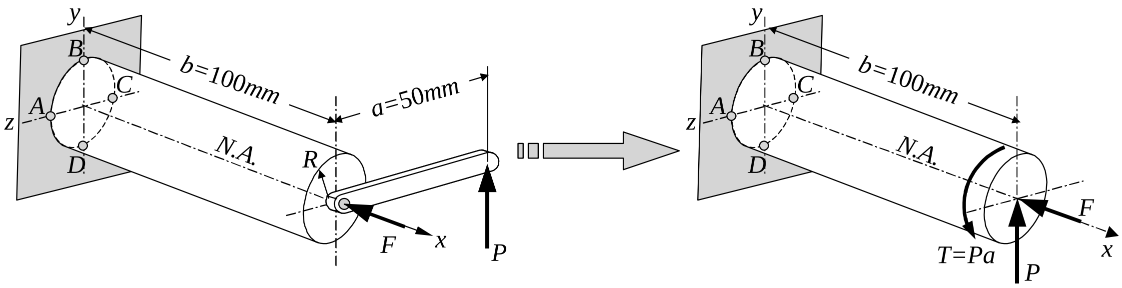
iii) Range $-0.05 \le y \le -0.04$, i.e. *Area* 3, the shear stress is given by:

$$\tau_{xy} = \frac{25\times10^3}{4.493\times10^{-6}\times0.1}\left(\int_{0.04}^{0.05} 0.1ydy + \int_{-0.04}^{0.04} 0.01ydy + \int_{y}^{-0.04} 0.1ydy\right) = 2.782\times10^9\left(0.0025 - y^2\right)$$

Plotting these distributions between their limits, gives the following discontinuous ***parabolic distribution*** of shear stress:


y
N.A.
2.5MPa
25.04MPa
29.49MPa
Shear Stress
Distribution
25.04MPa
2.5MPa

Example 2: Two forces P=18kN and F=15kN are applied to the shaft with a radius of R=20mm as shown. Determine the maximum normal and shear stresses developed in the shaft.



Step 0: Determine the geometrical properties of cross section:

Area of cross section: $A = \pi R^2 = 3.1416 \times 0.02^2 = 1.257 \times 10^{-3} m^2$

Polar moment of inertia: $J = \pi R^4 / 2 = 3.1416 \times 0.02^4 / 2 = 251.3 \times 10^{-9} m^4$

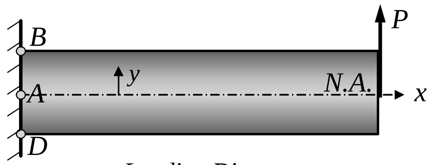
Second moment of area: $I = \pi R^4 / 4 = 3.1416 \times 0.02^4 / 4 = 125.7 \times 10^{-9} m^4$

First moment of semicircle: $Q = A^{'} \bar{y}^{'} = \left(\dfrac{\pi R^2}{2}\right) \times \left(\dfrac{4R}{3\pi}\right) = 5.33 \times 10^{-6} m^3$

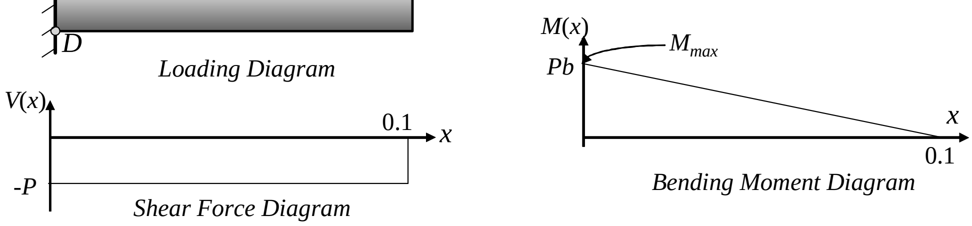
Step 1: Move eccentric force P to the center of the shaft

This causes a uniform torsional moment (Torque) about axis x by $T=Pa$=18000×0.05=900Nm as shown. Centric force P also will produce a varying bending moment $M(x)$ along axis x. Axial force F leads to a constant average compressive normal stress at cross sections along the shaft.

Step 2: Determine the maximum bending moment M_{max} and maximum shear force V_{max}



Loading Diagram



Shear Force Diagram

Bending Moment Diagram

From the shear force and bending moment diagrams, one can identify that the shear force is uniform along the shaft with $V=P$=18000N, and the maximum bending moment occurs at the section ABCD with a magnitude of $M_{max} = Pb$=18000×0.1=1800Nm. So the ***critical*** section is ABCD.

Step 3: Apply the superposition for determining the maximum normal stress

The maximum compressive stress occurs at point B, where both the maximum bending moment M_{max} and axial force F will form a highest combined compressive stress as

$$\sigma_{max} = \sigma_B = \frac{P}{A} - \frac{M_{max} y_{max}}{I} = \frac{-15000}{1.257 \times 10^{-3}} - \frac{1800 \times 0.02}{125.7 \times 10^{-9}} = -11.93 - 286.40 = -298.33 MPa$$

Step 4: Apply the superposition for determining the maximum shear stresses

As shown in table 7.1, the maximum shear stress occurs at point C, where both the transverse shear force $V=P$ and the torsional moment $T=Pa$ give a highest combined shear stress as

The max twist shear stress $\tau^T_{max} = \dfrac{TR}{J} = \dfrac{900 \times 0.02}{251.3 \times 10^{-9}} = 71.63 MPa$ (at outer surface)

The max shear stress in bending $\tau^V_{max} = \dfrac{VQ}{It} = \dfrac{(18000) \times (5.33 \times 10^{-6})}{(125.7 \times 10^{-9}) \times (2 \times 0.02)} = 19.08 MPa$ (at $N.P.$)

The total combined max shear stress: $\tau_{max} = \tau_C = \tau^T_{max} + \tau^V_{max} = 71.36 + 19.08 = 90.44 MPa$