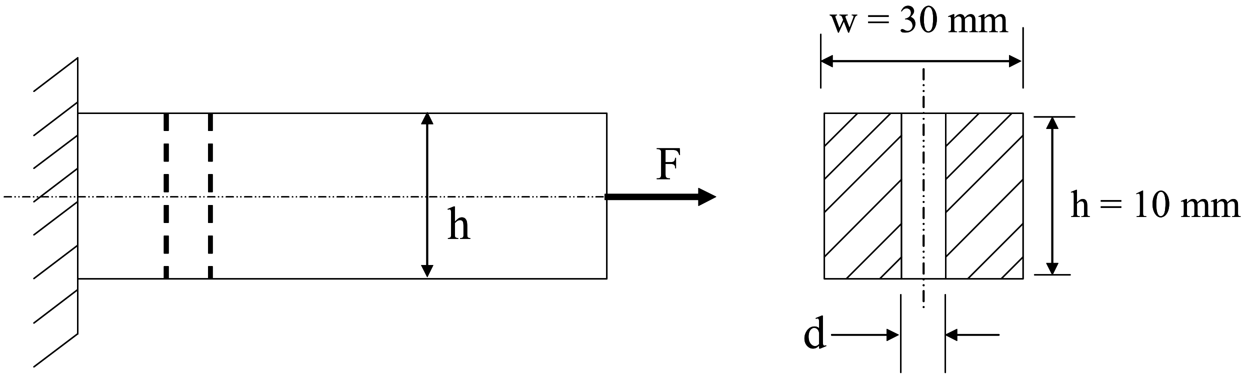
Sample Problem (Ch 6, Fatigue Analysis)

The rectangular beam shown in the figure is subjected to a fluctuating axial force. The axial force fluctuates between 5 kN and 15 kN keeping its direction. Find the max permissable diameter of the transverse hole for an infinite operation life in a working environment of 100°C. Material of the beam is AISI 1040 CD Steel ($S_{ut} = 590\ MPa, S_y = 490\ MPa$). The surface finish of the hole is ground. Take reliability as 0.99 and use a factor of safety of N = 1.2.



Critical section of the beam is at the transverse hole due to stress concentration at this point. First, we calculate the correction factors:

Type of Fatigue Loading: Fluctuating (mean stress is not zero)

Loading Factor:
Axial loading
$C_{load} = 0.7$

Size Factor:
Rectangular Cross Section
$A_{95} = 0.05wh = 0.05*30*10 = 15\ mm^2$

$d_{equivalent} = \sqrt{\frac{A_{95}}{0.076}} = 14\ mm$

$C_{size} = 1.189d^{-0.097} = 1.189(14)^{-0.097} = 0.92$

Surface Factor:

$C_{surface} = A(S_{ut})^b$

$A = 1.58\ , b = -0.085\ (ground, Table\ 6-3,\ Norton,\ pp.\ 350)$

$C_{surface} = 0.919$

Temperature Factor:
$C_{temp} = 1\ \ (T < 450°)$

Reliability Factor:
$C_{reliab} = 0.814\quad (Table\ 6-4, Norton,\ pp.\ 353)$

$S'_e = 0.5.S_{ut} = 0.5 * 590 = 295\ MPa$

$S_e = C_{load} C_{size} C_{surf} C_{temp} C_{reliab} S'_e$

$S_e = 0.7 * 0.92 * 0.919 * 1 * 0.814 * 295 = 142.12 \quad MPa$

$F_{\max} = 15000N$

$F_{\min} = 5000N$

$F_m = (F_{\max} + F_{\min})/2 = 10000N$

$F_a = (F_{\max} - F_{\min})/2 = 5000N$

Critical section is at the <u>hole</u>.

$\sigma_m = K_f (F_m / A) \quad \sigma_a = K_f (F_a / A)$

$A = h(w-d) = 300 - 10d$

$\sigma_m = K_f (10000/(300-10d))$

$\sigma_a = K_f (5000/(300-10d))$

Note that $K_f = 1 + q(K_t - 1)$ and K_t is a function of hole diameter (see Figure E-13, pp. 1000, Norton). However, we do not know the diameter of the hole. Hence, we will do iterations. Assume that K_f is 1.0 (Also assume that $K_f * |\sigma_{\max}| < S_y; \quad hence\ K_{fm} = K_f$)

Then,

$\sigma_m = (10000/(300-10d))$

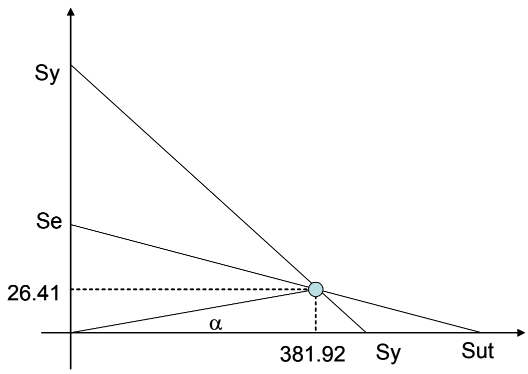
$\sigma_a = (5000/(300-10d))$

We can calculate the hole diameter using the modified Goodman Diagram.
First, find the intersection point of Yield and Goodman lines;

$$\frac{\sigma'_a}{S_y} + \frac{\sigma'_m}{S_y} = \frac{1}{n} \quad (yield)$$

$$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$\sigma'_a = 26.41 \quad MPa$
$\sigma'_m = 381.92 \quad MPa$ (intersection point)



$$\alpha = \tan^{-1}\left(\frac{\sigma'_a}{\sigma'_m}\right) = 3.95°$$

Slope of the load line:

$$\beta = \tan^{-1}(\frac{\sigma_a}{\sigma_m}) = \left(\frac{5000/(300-10d)}{10000/(300-10d)}\right) = 26.57°$$

$\alpha < \beta$ failure occurs due to fatigue, so we will use Goodman line to calculate the diameter of the hole.

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$$\frac{\frac{5000}{(300-10d)}}{142} + \frac{\frac{10000}{(300-10d)}}{590} = \frac{1}{1.2}$$

$d = 23.7\ mm$ (this will be an initial assumption for our calculations)

Now, we can calculate K_t and then K_f.

$$K_t = 3.0039 - 3.753(\frac{d}{w}) + 7.973(\frac{d}{w})^2 + ...$$

where, $\frac{d}{w} = \frac{23.74}{30} = 0.79$

$$K_t = 5.03$$

$$K_f = 1 + q(K_t - 1)$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$S_{ut} = 85.57\ ksi\,(590\ MPa)$$

$$\sqrt{a} \cong 0.075\ inch^{0.5} \quad (Table\ 6-6)$$

$$r = 0.47\ inch\ (r = 23.7/2\ mm)$$

$$q = \frac{1}{1 + \frac{0.075}{\sqrt{0.47}}} = 0.9$$

$$K_f = 1 + 0.9*(5.03-1) = 4.63$$

$$\sigma_m = 4.63*(10000/(300-10*d))$$

$$\sigma_a = 4.63*(5000/(300-10*d))$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$$\frac{\frac{4.63*5000}{(300-10d)}}{142} + \frac{\frac{4.63*10000}{(300-10d)}}{590} = \frac{1}{1.2}$$

$$d = 1.02mm$$

Iterate one more time:

$$K_t = 3.0039 - 3.753(\frac{d}{w}) + 7.973(\frac{d}{w})^2 + ...$$

where, $\frac{d}{w} = \frac{1.02}{30} = 0.034$

$$K_t = 2.88$$

$$K_f = 1 + q(K_t - 1)$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$S_{ut} = 85.57\ ksi\,(590\ MPa)$$

$$\sqrt{a} \cong 0.075\ inch^{0.5} \quad (Table\ 6-6)$$

$$r = 0.02\ inch\ (r = 0.06\ mm)$$

$$q = \frac{1}{1 + \frac{0.075}{\sqrt{0.02}}} = 0.65$$

$$K_f = 1 + 0.65(2.83 - 1) = 2.19$$

$$\sigma_m = 2.19*(10000/(300 - 10*d))$$

$$\sigma_a = 2.19*(5000/(300 - 10*d))$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$$\frac{\frac{2.19*5000}{(300-10d)}}{142} + \frac{\frac{2.19*10000}{(300-10d)}}{590} = \frac{1}{1.2}$$

$$d = 16.23\ mm$$

$$K_t = 3.27$$

$$K_f = 1 + q(K_t - 1)$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$S_{ut} = 85.57\ ksi\,(590\ MPa)$$

$$\sqrt{a} \cong 0.075\ inch^{0.5} \quad (Table\ 6-6)$$

$$r = 0.32\ inch\ (r = 16.23/2\ mm)$$

$$q = \frac{1}{1 + \frac{0.075}{\sqrt{0.32}}} = 0.66$$

$$K_f = 1 + 0.88(3.27 - 1) = 3.00$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$$\frac{\frac{3.0 * 5000}{(300 - 10d)}}{142} + \frac{\frac{3.0 * 10000}{(300 - 10d)}}{590} = \frac{1}{1.2}$$

$$d = 11.2\ mm$$

$$K_t = 2.71$$

$$K_f = 1 + q(K_t - 1)$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$S_{ut} = 85.57\ ksi\,(590\ MPa)$$

$$\sqrt{a} \cong 0.075\ inch^{0.5} \quad (Table\ 6-6)$$

$$r = 0.22\ inch\ (r = 11.2/2\ mm)$$

$$q = \frac{1}{1 + \frac{0.075}{\sqrt{0.22}}} = 0.86$$

$$K_f = 1 + 0.86(2.7 - 1) = 2.46$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$$\frac{\frac{2.46 * 5000}{(300 - 10d)}}{142} + \frac{\frac{2.46 * 10000}{(300 - 10d)}}{590} = \frac{1}{1.2}$$

$$d = 14.6\ mm$$

$$K_t = 3.07$$

$$K_f = 1 + q(K_t - 1)$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$S_{ut} = 85.57\ ksi\,(590\ MPa)$$

$$\sqrt{a} \cong 0.075\ inch^{0.5} \quad (Table\ 6-6)$$

$$r = 0.28\ inch\ (r = 14.6/2\ mm)$$

$$q = \frac{1}{1 + \frac{0.075}{\sqrt{0.28}}} = 0.88$$

$$K_f = 1 + 0.88(3.07 - 1) = 2.82$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$$\frac{\frac{2.82 * 5000}{(300 - 10d)}}{142} + \frac{\frac{2.82 * 10000}{(300 - 10d)}}{590} = \frac{1}{1.2}$$

$$d = 12.3\ mm$$

$$K_t = 2.81$$

$$K_f = 1 + q(K_t - 1)$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$S_{ut} = 85.57\ ksi\,(590\ MPa)$$

$$\sqrt{a} \cong 0.075\ inch^{0.5} \quad (Table\ 6-6)$$

$$r = 0.24\ inch\ (r = 12.3/2\ mm)$$

$$q = \frac{1}{1 + \frac{0.075}{\sqrt{0.24}}} = 0.88$$

$$K_f = 1 + 0.87(2.81 - 1) = 2.57$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$$\frac{\frac{2.57 * 5000}{(300 - 10d)}}{142} + \frac{\frac{2.57 * 10000}{(300 - 10d)}}{590} = \frac{1}{1.2}$$

$$d = 13.9\ mm$$

$$K_t = 2.98$$

$$K_f = 1 + q(K_t - 1)$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$S_{ut} = 85.57\ ksi\,(590\ MPa)$$

$$\sqrt{a} \cong 0.075\ inch^{0.5} \quad (Table\ 6-6)$$

$$r = 0.27\ inch\ (r = 13.9/2\ mm)$$

$$q = \frac{1}{1 + \frac{0.075}{\sqrt{0.27}}} = 0.87$$

$$K_f = 1 + 0.87(2.98 - 1) = 2.72$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$$\frac{\frac{2.72*5000}{(300-10d)}}{142} + \frac{\frac{2.72*10000}{(300-10d)}}{590} = \frac{1}{1.2}$$

$$d = 13.0\ mm$$

$$K_t = 2.87$$

$$K_f = 1 + q(K_t - 1)$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

$$S_{ut} = 85.57\ ksi\,(590\ MPa)$$

$$\sqrt{a} \cong 0.075\ inch^{0.5} \quad (Table\ 6-6)$$

$$r = 0.26\ inch\ (r = 13.0/2\ mm)$$

$$q = \frac{1}{1 + \frac{0.075}{\sqrt{0.26}}} = 0.87$$

$$K_f = 1 + 0.87(2.87 - 1) = 2.63$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad (Goodman)$$

$$\frac{\frac{2.63*5000}{(300-10d)}}{142} + \frac{\frac{2.63*10000}{(300-10d)}}{590} = \frac{1}{1.2}$$

$$d = 13.54\ mm$$

We can stop the iterations at this point because the difference between the diameters is getting small. Hence, we can say that the diameter of the traverse hole is 14 mm. Here, we can check if the following assumption that we made earlier is correct;

$$K_f * |\sigma_{\max}| < S_y \quad \Rightarrow K_{fm} = K_f$$

$$K_f * |F_{\max} / A| < S_y$$

$$2.63 * |15.000 / (300 - 10 * 14)| < S_y$$

$$246.5\ MPa < 490\ MPa$$