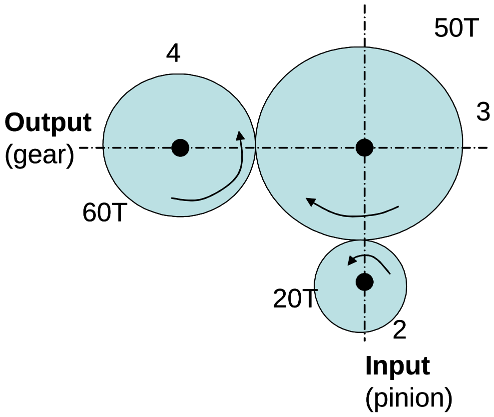
Gear Design



Problem: Pinion 2 is shown in the figure runs at 1750 rpm and transmits 2.5 kW to idler gear 3. The teeth are cut on the 20^0 full-depth system and have a module of m = 2.5 mm.

a) Draw a free-body diagram of gear 3 and show all the forces and compute the torques on the output gear 4

$$T = W_t * \frac{d_p}{2}$$

W_t: tangential force

d_p: pitch diameter

$$H = T * \omega$$

H: Power

ω: angular rotation

$$H = W_t * \frac{d_p}{2} * \left(\frac{2\pi}{60} * \omega_2 \right)$$

$$\omega \text{ (rev/min)} = \frac{\omega}{60} \text{ (rev/sec)} = 2\pi \frac{\omega}{60} \text{ (rad/sec)}$$

$$H = \frac{d_p \pi * \omega_2 * W_t}{60 * 10^3}$$

where, H : (kN)

ω_2: (rev/min)

d_p: (mm)

W_t: (kN)

$$W_t = \frac{60 * 10^3 * H}{\pi d_p * \omega_2}$$

d_p= N*m:

N: number of teeth

m: module (mm)

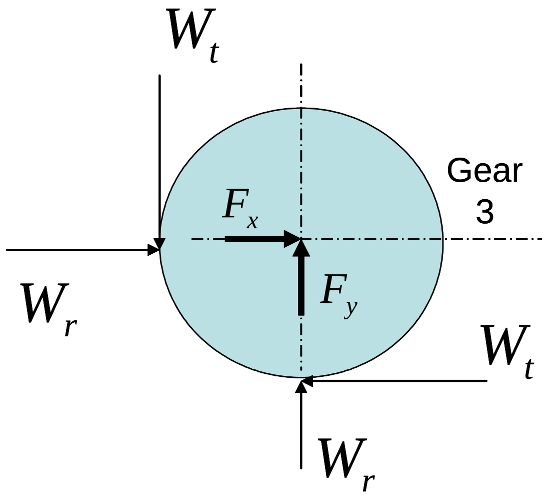
$$W_t = \frac{60 * 10^3 * H}{\pi (N * m) * \omega_2}$$

$$W_t = \frac{60 * 10^3 * (2.5)}{\pi (20 * 2.5) * 1750} = 0.546 \text{ kN}$$

Thus, the tangential force of gear 2 on gear 3 is 0.546 kN. The radial force is;

$$W_r = W_t * \tan 20^0 = 0.546 * \tan 20^0 = 0.199 \ kN$$

The tangential reaction of gear 4 on gear 3 is also equal to W_t



Forces on shaft of gear 3;

F_X = - (- 0.546 + 0.199) = 0.347 kN

F_Y = - (0.199-0.546) = 0.347 kN

The output Torque:

$$\frac{\omega_4}{\omega_2} = \frac{r_2}{r_4} = \frac{N_2}{N_4}$$

$$\frac{N_2}{N_4} = (-\frac{N_2}{N_3}) * (-\frac{N_3}{N_4}) = \frac{20}{60}$$

$$T_2 = W_t * r_2$$

$$T_4 = W_t * r_4$$

$$T_4 = \frac{r_4}{r_2} * T_2 = \frac{N_4}{N_2} * T_2$$

$$T_4 = \frac{60}{20} * T_2 = 3 * (0.546 * \frac{50}{2})$$

T_2 =13.65 Nm
T_4 =40.95 Nm

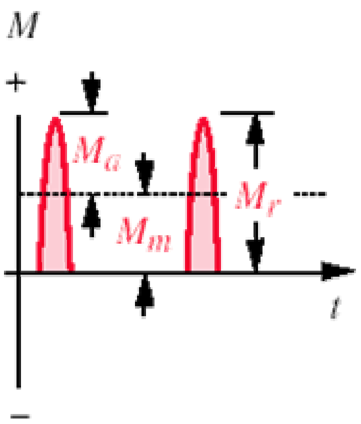
b) Compute the mean and alternating loads on 1) pinion, 2) gear and, 3) idler.

<u>The repeated loads on pinion and gear:</u>

R = 0 = $\frac{\sigma_{\min}}{\sigma_{\max}}$ (since $\sigma_{\min} = 0$)

$$W_{t_{mean}} = \frac{W_t}{2} = \frac{0.546}{2} = 0.273\, kN$$

$$W_{t_{alternating}} = \frac{W_t}{2} = \frac{0.546}{2} = 0.273\, kN$$



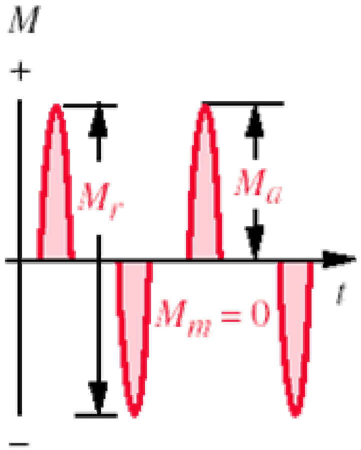
<u>The fully-reversed loads on the idler:</u>

R = -1 = $\frac{\sigma_{\min}}{\sigma_{\max}}$

$|\sigma_{\min}| = |\sigma_{\max}|$

$$W_{t_{alternating}} = W_t = 0.546\ kN$$

$$W_{t_{mean}} = 0 \text{ kN}$$



c) Determine a suitable face width and the bending stresses in the idler gear ("3")

$$\sigma_b = \frac{W_t * P_d}{F * J} * \frac{K_a * K_m}{K_v} * K_S * K_B * K_I$$

Recommended face-width range is

$$\frac{\delta}{P_d} < F < \frac{16}{P_d}$$

P_d = diametral pitch = 25.4/m

Take $F = \dfrac{12}{P_d}$

$$\frac{8m}{25.4} < F < \frac{16m}{25.4} \quad (\text{ SI units })$$

The relation between P_d (U.S. specification) and the module "m" (S.I. unit) is

$$m = \frac{25.4}{P_d} = 2.5 \text{ mm/teeth}$$

Hence, P_d = 10.16 teeth/inches.

<u>The face width</u> (F):

$$F = \frac{12}{10.16} = 1.81 \text{ inches} = 3 \text{ cm} = 30 \text{ mm}$$

<u>The load distribution factor</u> (K_m)

Using Table 11 – 16

K_m=1.6 (for F=1.81 inches = 30 mm)

The application factor:

K_a = 1 (assume uniform loading; table 11 -17)

The velocity factor:

$$K_V = \left(\frac{A}{A+\sqrt{200*V_t}}\right)^B$$

V_t : pitch-line velocity

$$V_t = \frac{d_p}{2}*\omega = \frac{d_3}{2}*\omega_3$$

Gear 3:

d_3 = m N_3 = 2.5*50 = 125 mm

$$\omega_3 = \frac{N_2}{N_3}*\omega_2$$

$$\omega_3 = \frac{20}{50}*1750$$

$$\omega_3 = 700rpm*\frac{rev}{\min}$$

$$\omega_3 = 700*\frac{2\pi}{60} = 73.266 \text{ rad/sec}$$

V_t = 4.58 m/s = 274.76 m/min

Note that $V_t(ft/\min) = V_t(m/\sec)*196.85$ since 1 ft = 12 inches = 30.48 cm

V_t = 901.5 ft/min

The quality index (Q_v)

Using Table 11-7 (page 710)

V_t	Q_v
0- 800	6-8
901.5 ft/min	**?**
800-2000	8-10

Take Q_v = 8

$$B = \frac{(12-8)^{2/3}}{4} = 0.63$$

A = 50 +56(1-B) =7 0.72

$$K_V = \left(\frac{70.72}{70.72+\sqrt{200*4.58}}\right)^{0.63} = 0.8$$

<u>The size factor</u> K_S=1 for all gears

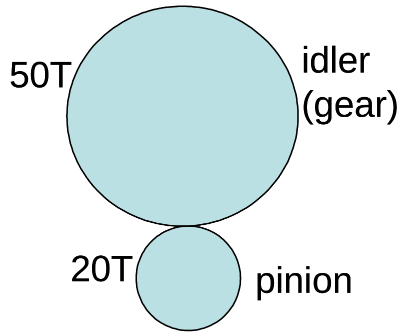
<u>The rim thickness factor</u> K_B= 1 (assume no rim)

<u>The idler factor</u> K_I = 1 for gears “2” and “4”, K_I = 1.42 for the gear “3”.

<u>The bending geometry factor</u> J for 20^0 presume angle (full depth teeth with HPSTC loading)

pinion : 20 T (20^0)
gear : 60 T (20^0)
idler : 50 T (20^0)

First, calculate the J_{pinion}, and J_{gear} based on pinion (gear “2” in the figure) and idler (gear “3” in the figure) interactions:



Using Table 11- 9 (p.716)

$J_{pinion} \cong 0.34$
$J_{gear(idler)} \cong 0.39$

$$\sigma_b = \frac{W_t * P_d}{F * J} * \frac{K_a * K_m}{K_v} * K_S * K_B * K_I$$

since F= $\frac{12}{P_d}$

$$\sigma_b = \frac{W_t * P_d^2}{12J} * \frac{K_a * K_m}{K_v} * K_S * K_B * K_I$$

also,

$$P_d = \frac{25.4}{m}$$

$$\sigma_b = \frac{W_t * \left(\frac{25.4}{m}\right)^2}{12J} * \frac{K_a * K_m}{K_v} * K_S * K_B * K_I$$

$$\sigma_b = \frac{W_t * (53.76)}{m^2 * J} * \frac{K_a * K_m}{K_v} * K_S * K_B * K_I$$

m = 2.5 mm

W_t = 0.546 kN → then σ_b is in MPa

$$\sigma_{b_{pinion}} = \frac{0.546 * (53.76)}{(2.5)^2 * (0.34)} * \frac{1 * 1.6}{0.8} * 1 * 1 * 1$$

$$\sigma_{b_{pinion}} = 27.63 \text{ MPa}$$

$$\sigma_{b_{pinion}} = \frac{0.546 * (53.76)}{(2.5)^2 * (0.39)} * \frac{1 * 1.6}{0.8} * 1 * 1 * 1.42$$

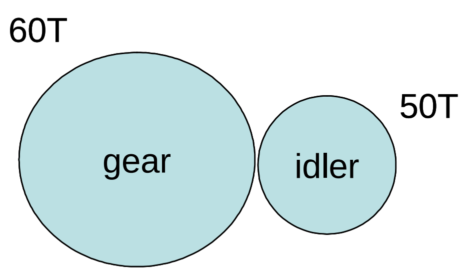
$$\sigma_{b_{gear}} = 34.2 \text{ MPa}$$

Second, calculate the J_{pinion}, and J_{gear} based on idler (“3”) and gear (“4”) interactions:

Using Table 11- 9 (p.716)

$J_{pinion(idler)} \cong 0.42$

$J_{gear} \cong 0.43$



Note: The idler has a slightly different J factor when considered to be the "gear" in mesh with the smaller pinion (0.42) than when considered to be the "pinion" in mesh with the larger gear (0.43).

$$\sigma_{b_{idler}} = \frac{0.546*(53.76)}{(2.5)^2*(0.42)}*\frac{1*1.6}{0.8}*1*1*1.42$$

= 31.75 MPa

$$\sigma_{b_{gear}} = \frac{0.546*(53.76)}{(2.5)^2*(0.43)}*\frac{1*1.6}{0.8}*1*1*1$$

= 21. 84 MPa

d) All gears are made of steel (AGMA grade 2 with Brinell Hardness number of 250 HB). The service life required is 5 years of one-shift operation. Operating temperature is 200^0 F. Calculate the safety factor for bending stress (Assume 99 % reliability).

$$S_{f_b} = \frac{K_L}{K_T * K_R} * S'_{f_b}$$

S'_{f_b} = 280 MPa (from the Figure 11-25, pp.733 for HB = 250)

ω_{piniom} =1750 rpm

ω_{idler} =700 rpm

ω_{gear} =583.3 rpm

$$N_{pinion} = 1750 rpm \frac{60 \min}{hr} * \frac{2080 hr}{shift-year} * 5 yr$$

N_{pinion} =1.092*10^9 cycles

N_{idler} = 0.4368*10^9 cycles

N_{gear} = 0.364*10^9 cycles

The life factor:
$K_{L(pinion)}$= 1.3558*$N^{-0.0178}$ (Fig. 11-24, N>10^8 commercial app)

= 0.936
$K_{L(idler)}$ = 0.951
$K_{L(gear)}$ = 0.955

The temperature factor:
K_T=1 (T<250^0)

The reliability factor:
K_R=1 (%99 reliability)

$$S_{f\,b\,pinion} = 262 \text{ MPa} \qquad N_{b\,pinion} = \frac{262}{27.63} = 9.48$$

$$S_{f\,b\,idler} = 266 \text{ MPa} \qquad N_{b\,idler} = \frac{266}{34.2} = 7.77$$

$$S_{f\,b\,gear} = 267.4 \text{ MPa} \qquad N_{b\,gear} = \frac{267.4}{21.84} = 12.24$$

Hence, idler is the most critical (it is exposed to fully reversed stresses).

e) Determine the surface stresses for

1) pinion- idler mesh

2) idler-gear mesh

and the safety factor for surface stresses.

$$\sigma_c = c_p \sqrt{\frac{\omega_t}{F*I*d} \frac{C_a*C_m*C_s*C_f}{C_v}}$$

$C_a = K_a = 1$

$C_m = K_m = 1.6$

$C_v = K_v = 0.8$

F = 30 mm

<u>Notes</u>:

- d (pitch diameter) is the smaller of mating gears
- "I" is the smaller of mating gears
- Be careful with the +/- signs in equations 11.22a and 11.22b.

    Eq. 11.22a (+ : external, -: internal)

    Eq. 11.22b (- : external, +: internal)

C_s= 1 (size factor)

C_f = 1 (surface finish factor)

<u>The elastic coefficient</u>:

$$C_p = \sqrt{\frac{1}{\pi\left[(\frac{1-v_p^2}{E_p})+(\frac{1-v_g^2}{E_g})\right]}}$$

All gears are made of steel ⟶ v =0.3 (Table 11-18 assumes that v = 0.3)

$E_p = 30*10^6$ psi = $2*10^5$ MPa

Instead of calculating the C_p using the equation; you can directly get the value from the table if the material is available.

pinion ⟶ steel				<u>UNIT</u>
	⟶		C_p=2300	$\sqrt{psi}$
gear ⟶ steel				
			C_p=191	$\sqrt{MPa}$

<u>Surface Geometry Factor (I)</u>

$$I = \frac{\cos\phi}{\left(\frac{1}{\rho_p} \pm \frac{1}{\rho_g}\right)*d_p}$$

<u>use "+" sign for external gears and "-" for internal gears</u>

$$\rho_p = \sqrt{(r_p + \frac{1+x_p}{p_d})^2 - (r_p * \cos\phi)^2} - \frac{\pi}{p_d} * \cos\phi$$

$$\rho_g = c\sin\phi \mp \rho_p$$

<u>-: external gears + : internal gears</u>

pinion-idler mesh

$$r_p = \frac{d_p}{2} = \frac{m * N_{pinion}}{2} = \frac{2.5 * 20}{2} = 25 \text{ mm}$$

$$r_{idler} = \frac{d_{idler}}{2} = \frac{2.5 * 50}{2} = \frac{125}{2} \text{ mm}$$

$$\phi = 20^0$$

$$m = \frac{25.4}{p_d} \longrightarrow P_d = \frac{25.4}{m} = 10.16$$

The pinion addendum coefficient x_p= 0
(x_p= 0 for full-depth teeth and x_p= 0.25 for 25 % long-addendum teeth)

$$\rho_p = \sqrt{(25 + \frac{1}{10.16})^2 - (25 * \cos 20^0)^2} - \frac{\pi}{10.16} * \cos 20^0$$

ρ_p = 8.54 mm

$$\rho_{gear} = c \sin\phi - \rho_1 = (25 + \frac{125}{2}) * \sin 20^0 - 8.54$$

ρ_{gear} =21.39 mm

$$I = \frac{\cos\phi}{\left(\frac{1}{\rho_p} + \frac{1}{\rho_g}\right) * d_p} = 0.115$$

Idler –gear mesh

idler

$$r_p = \frac{125}{2} m$$

$$r_p = \frac{150}{2} mm$$

$$\phi = 20^0$$

$$\rho_p = \sqrt{(62.5 + \frac{1}{10.16})^2 - (62.5 * \cos 20^0)^2} - \frac{\pi}{10.16} * \cos 20^0$$

ρ_p =21.37 mm

$$\rho_g = c \sin\phi \mp \rho_p = (62.5 + 75) * \sin 20^0 - 21.37$$

ρ_g = 25.7 mm

$$I_{idler(pinion)} = \frac{\cos\phi}{\left(\frac{1}{21.37} + \frac{1}{25.7}\right) * 62.5} = 0.175$$

The surface stress for pinion-idler mesh:

$$\sigma_{cp} = c_p \sqrt{\frac{\omega_t}{F * I_{pinion} * d_p} \frac{C_a * C_m * C_s * C_f}{C_v}}$$

$$=191 * \sqrt{\frac{0.546 * 10^3 N}{30 * 0.115 * 50} \frac{1*1.6*1*1}{0.8}}$$

σ_{cp} = 480.56 MPa

The surface stress for idler-gear mesh:

$$\sigma_{cidler} = 191 * \sqrt{\frac{0.546 * 10^3}{30 * 0.175 * 62.5} \frac{1*1.6*1*1}{0.8}}$$

σ_{cidler} = 348.4 MPa

$$S_{fc} = \frac{C_L * C_H}{C_T * C_R} * S'_{fc}$$

$S'_{fc} \cong 820 MPa$ (for HB= 250 and Grade II) Table 11-27, page 737

C_T=1 (T< 250^0 F)

C_R=1 (%99 reliability)

C_H=1 (Hardness Ratio factor)

$$C_{L\,pinion} = 1.4488N^{-0.023} = 1.4488(1.092 * 10^{9})^{-0.023}$$ (Fig. 11-26, pp.734)

$$=0.898$$

$$C_{L\,idler} = 0.917$$

$$C_{L\,gear} = 0.92$$

$$S_{f\,c_{pinion}} = 820 * 0.898 = 736 MPa$$

$$S_{f\,c_{idler}} = 752 \text{ MPa}$$

$$S_{f\,c_{gear}} = 754 \text{ MPa}$$

$$N_{c\,pinion-idler} = \left[\frac{736}{480.56}\right]^2 = 2.4$$

$$N_{c\,idler-gear} = \left[\frac{752}{348.4}\right]^2 = 4.7$$

Hence; pinion-idler surface is more critical.

Note: we take the square of the safety factor because the surface stress is related to the square root of the load.