Appendices to the paper "On regular embedding of H-designs into G-designs"

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We give the constructions of some small designs necessary in the proofs of the paper "On regular embedding of H-designs into G-designs". For notations, definitions references see the mentioned paper.

1   Appendix 1

In this appendix we show the existence of the sets X, Y, Z and T verifying the statements of Lemma 2.5 for $(\alpha, \beta) \in \{(0,0),(0,1),(4,2),(4,3),(8,2),(8,4)\}$. Let $|X| = \alpha$, $|Y| = \beta$, $Z = \{0, 1, \ldots, 11\}$ and $T = \{a_0, a_1, a_2\}$.

Case $(\alpha, \beta) = (0,0)$. The set $\mathcal{E}$ is given by the following P_4s:
$[2, 1, 0, a_2]$, $[0, 2, 3, a_2]$, $[0, 3, 1, a_2]$, $[6, 5, 4, a_2]$, $[4, 6, 7, a_2]$, $[4, 7, 5, a_2]$, $[10, 9, 8, a_2]$, $[8, 10, 11, a_2]$, $[8, 11, 9, a_2]$, $[1, 4, 0, a_0]$, $[1, 5, 0, a_1]$, $[3, 6, 2, a_0]$, $[3, 7, 2, a_1]$, $[5, 2, 4, a_0]$, $[5, 3, 4, a_1]$, $[7, 0, 6, a_0]$, $[7, 1, 6, a_1]$, $[0, 8, 1, a_0]$, $[0, 9, 1, a_1]$, $[2, 10, 3, a_0]$, $[2, 11, 3, a_1]$, $[8, 2, 9, a_0]$, $[8, 3, 9, a_1]$, $[11, 0, 10, a_1]$, $[11, 1, 10, a_2]$, $[4, 8, 5, a_0]$, $[4, 9, 5, a_1]$, $[6, 10, 7, a_0]$, $[6, 11, 7, a_1]$, $[10, 4, 11, a_0]$, $[10, 5, 11, a_1]$, $[9, 6, 8, a_0]$, $[9, 7, 8, a_1]$, $[2, a_2, a_1, a_0]$, $[6, a_2, a_0, 10]$.

Case $(\alpha, \beta) = (0,1)$. Let $Y = \{\chi\}$. The set $\mathcal{E}$ is given by the following P_4s:
$[2, 1, 0, \chi]$, $[0, 2, 3, \chi]$, $[0, 3, 1, \chi]$, $[6, 5, 4, a_0]$, $[4, 6, 7, \chi]$, $[4, 7, 5, a_0]$, $[10, 9, 8, \chi]$, $[8, 10, 11, a_0]$, $[8, 11, 9, \chi]$, $[1, 4, 0, a_0]$, $[1, 5, 0, a_1]$, $[3, 6, 2, a_1]$, $[3, 7, 2, a_2]$, $[5, 2, 4, a_1]$, $[5, 3, 4, a_2]$, $[7, 0, 6, \chi]$, $[7, 1, 6, a_1]$, $[0, 8, 1, a_0]$, $[0, 9, 1, a_1]$, $[2, 10, 3, a_0]$, $[2, 11, 3, a_2]$, $[9, 2, 8, a_1]$, $[9, 3, 8, a_2]$, $[11, 0, 10, a_0]$, $[11, 1, 10, a_1]$, $[4, 8, 5, a_1]$, $[4, 9, 5, a_2]$, $[6, 10, 7, a_0]$, $[6, 11, 7, a_2]$, $[10, 4, 11, a_1]$, $[10, 5, 11, a_2]$, $[8, 6, 9, a_0]$, $[8, 7, 9, a_2]$, $[2, \chi, a_0, 6]$, $[4, \chi, a_1, 3]$, $[5, \chi, a_2, 0]$, $[2, a_0, a_1, 7]$, $[8, a_0, a_2, 1]$, $[9, a_1, a_2, 6]$, $[a_2, 10, \chi, 11]$.

Case $(\alpha, \beta) = (4,2)$. Let $X = \{\infty_0, \infty_1, \infty_2, \infty_3\}$ and $Y = \{\chi_0, \chi_1\}$. The set $\mathcal{D}$ is given by the following P_4s:
$[\infty_2, \infty_1, \infty_0, \chi_0]$, $[\infty_3, \infty_2, \infty_0, \chi_1]$, $[\infty_0, \infty_3, \infty_1, \chi_0]$, $[\chi_0, \infty_3, \chi_1, \infty_2]$, $[\infty_2, \chi_0, \chi_1, \infty_1]$.

The set $\mathcal{E} \setminus \mathcal{D}$ is given by the following P_4s:

$[0,1,2,a_1]$, $[0,2,3,a_2]$, $[1,3,0,a_2]$, $[6,5,4,a_2]$, $[4,6,7,a_2]$, $[4,7,5,a_2]$, $[8,9,10,\chi_1]$, $[8,10,11,\chi_1]$,
$[8,11,9,\chi_0]$, $[1,\infty_0,0,\chi_0]$, $[1,\infty_1,0,\chi_1]$, $[5,\infty_0,4,\chi_0]$, $[5,\infty_1,4,\chi_1]$, $[\infty_1,2,\infty_0,a_0]$, $[\infty_1,3,\infty_0,a_1]$,
$[\infty_1,6,\infty_0,a_2]$, $[\infty_0,7,\infty_1,a_0]$, $[3,\infty_2,2,\chi_0]$, $[3,\infty_3,2,\chi_1]$, $[7,\infty_2,6,\chi_0]$, $[7,\infty_3,6,a_0]$,
$[\infty_3,0,\infty_2,a_0]$, $[\infty_3,1,\infty_2,a_1]$, $[\infty_3,4,\infty_2,a_2]$, $[\infty_2,5,\infty_3,a_0]$, $[9,\infty_0,8,\chi_1]$, $[9,\infty_1,8,a_0]$,
$[\infty_0,10,\infty_1,a_1]$, $[\infty_0,11,\infty_1,a_2]$, $[11,\infty_2,10,a_0]$, $[11,\infty_3,10,a_1]$, $[\infty_2,8,\infty_3,a_1]$, $[\infty_2,9,\infty_3,a_2]$,
$[5,0,4,a_0]$, $[4,1,5,\chi_0]$, $[0,6,1,a_0]$, $[0,7,1,a_1]$, $[7,2,6,a_2]$, $[6,3,7,\chi_0]$, $[3,4,2,a_0]$, $[2,5,3,\chi_0]$,
$[9,0,8,a_2]$, $[8,1,9,a_0]$, $[0,10,1,a_2]$, $[0,11,1,\chi_1]$, $[11,2,10,\chi_0]$, $[10,3,11,a_0]$, $[2,8,3,\chi_1]$, $[2,9,3,a_1]$,
$[8,4,9,a_1]$, $[8,5,9,a_2]$, $[4,10,5,\chi_1]$, $[4,11,5,a_1]$, $[10,6,11,a_1]$, $[10,7,11,a_2]$, $[6,8,7,\chi_1]$, $[6,9,7,a_1]$,
$[0,a_0,a_1,4]$, $[3,a_0,a_2,2]$, $[0,a_1,a_2,\chi_1]$, $[5,a_0,\chi_0,1]$, $[7,a_0,\chi_1,6]$, $[6,a_1,\chi_0,8]$, $[8,a_1,\chi_1,9]$,
$[10,a_2,\chi_0,11]$.

Case $(\alpha,\beta) = (4,3)$. Let $X = \{\infty_0,\infty_1,\infty_2,\infty_3\}$ and $Y = \{\chi_0,\chi_1,\chi_2\}$. The set $\mathcal{D}$ is given by the following P_4s:

$[\infty_2,\infty_1,\infty_0,\chi_0]$, $[\infty_3,\infty_2,\infty_0,\chi_1]$, $[\infty_1,\infty_3,\infty_0,\chi_2]$, $[\chi_0,\infty_1,\chi_1,\infty_2]$, $[\infty_1,\chi_2,\chi_0,\chi_1]$,
$[\chi_0,\infty_2,\chi_2,\infty_3]$, $[\chi_0,\infty_3,\chi_1,\chi_2]$.

The set $\mathcal{E} \setminus \mathcal{D}$ is given by the following P_4s:

$[2,1,0,\chi_0]$, $[0,2,3,\chi_0]$, $[0,3,1,\chi_0]$, $[6,5,4,\chi_1]$, $[4,6,7,\chi_0]$, $[4,7,5,\chi_0]$, $[10,9,8,\chi_0]$, $[8,10,11,\chi_1]$,
$[8,11,9,\chi_1]$, $[a_0,\infty_0,0,\infty_1]$, $[a_1,\infty_0,1,\infty_1]$, $[a_0,\infty_2,2,\infty_3]$, $[a_1,\infty_2,3,\infty_3]$,$[1,\infty_2,0,a_0]$,
$[0,\infty_3,1,\chi_1]$, $[3,\infty_0,2,\chi_1]$, $[3,\infty_1,2,\chi_2]$, $[a_1,\infty_1,4,\infty_0]$, $[a_2,\infty_1,5,\infty_0]$, $[a_1,\infty_3,6,\infty_2]$,
$[a_2,\infty_3,7,\infty_2]$, $[5,\infty_2,4,\chi_2]$, $[5,\infty_3,4,a_1]$, $[7,\infty_0,6,\chi_1]$, $[7,\infty_1,6,\chi_2]$, $[\infty_1,8,\infty_0,a_2]$,
$[\infty_0,9,\infty_1,a_0]$, $[\infty_3,10,\infty_2,a_2]$, $[\infty_2,11,\infty_3,a_0]$, $[9,\infty_2,8,a_0]$, $[9,\infty_3,8,a_1]$, $[11,\infty_0,10,\chi_0]$,
$[11,\infty_1,10,a_0]$, $[0,4,1,\chi_2]$, $[0,5,1,a_0]$, $[3,6,2,a_2]$, $[2,7,3,\chi_1]$, $[4,2,5,a_0]$, $[4,3,5,a_1]$, $[7,0,6,a_2]$,
$[6,1,7,\chi_1]$, $[1,8,0,\chi_1]$, $[1,9,0,\chi_2]$, $[2,10,3,\chi_2]$, $[2,11,3,a_2]$, $[9,2,8,a_2]$, $[8,3,9,\chi_2]$, $[11,0,10,a_1]$,
$[10,1,11,a_1]$, $[5,8,4,a_2]$, $[4,9,5,a_2]$, $[6,10,7,\chi_2]$, $[6,11,7,a_2]$, $[8,6,9,a_0]$, $[8,7,9,a_1]$, $[10,4,11,\chi_2]$,
$[10,5,11,a_0]$, $[0,a_1,a_0,2]$, $[0,a_2,a_0,3]$, $[1,a_2,a_1,2]$, $[6,a_0,\chi_0,4]$, $[4,a_0,\chi_1,5]$, $[7,a_0,\chi_2,5]$,
$[1,a_1,\chi_0,9]$, $[3,a_1,\chi_1,8]$, $[7,a_1,\chi_2,8]$, $[10,a_2,\chi_0,11]$, $[9,a_2,\chi_1,10]$, $[11,a_2,\chi_2,10]$, $[2,\chi_0,6,a_1]$.

Case $(\alpha,\beta) = (8,2)$. Let $X = \{\infty_0,\ldots,\infty_7\}$ and $Y = \{\chi_0,\chi_1\}$. The set $\mathcal{D}$ is given by the following P_4s:

$[\infty_0,\infty_1,\infty_2,\chi_1]$, $[\infty_0,\infty_2,\infty_3,\chi_1]$, $[\infty_0,\infty_3,\infty_1,\chi_0]$, $[\infty_4,\infty_5,\infty_6,\chi_1]$, $[\infty_4,\infty_6,\infty_7,\chi_1]$,
$[\infty_4,\infty_7,\infty_5,\chi_1]$, $[\infty_1,\infty_4,\infty_0,\chi_0]$, $[\infty_1,\infty_5,\infty_0,\chi_1]$, $[\infty_3,\infty_6,\infty_2,\chi_0]$, $[\infty_2,\infty_7,\infty_3,\chi_0]$,
$[\infty_5,\infty_2,\infty_4,\chi_0]$, $[\infty_5,\infty_3,\infty_4,\chi_1]$, $[\infty_7,\infty_0,\infty_6,\chi_0]$, $[\infty_6,\infty_1,\infty_7,\chi_0]$, $[\infty_1,\chi_1,\chi_0,\infty_5]$.

The set $\mathcal{E} \setminus \mathcal{D}$ is given by the following P_4s:

$[2,1,0,\chi_0]$, $[0,2,3,\chi_0]$, $[0,3,1,\chi_0]$, $[6,5,4,\chi_0]$, $[4,6,7,\chi_0]$, $[4,7,5,\chi_0]$, $[10,9,8,\chi_0]$, $[8,10,11,\chi_0]$,
$[8,11,9,\chi_0]$, $[\infty_1,0,\infty_0,a_0]$, $[\infty_1,1,\infty_0,a_1]$, $[\infty_3,2,\infty_2,a_0]$, $[\infty_3,3,\infty_2,a_1]$, $[1,\infty_2,0,\chi_1]$,
$[1,\infty_3,0,a_0]$, $[3,\infty_0,2,\chi_0]$, $[3,\infty_1,2,a_0]$, $[\infty_5,0,\infty_4,a_0]$, $[\infty_5,1,\infty_4,a_1]$, $[\infty_7,2,\infty_6,a_0]$,
$[\infty_7,3,\infty_6,a_1]$, $[0,\infty_6,1,a_1]$, $[0,\infty_7,1,a_2]$, $[2,\infty_4,3,\chi_1]$, $[2,\infty_5,3,a_0]$, $[\infty_1,4,\infty_0,a_2]$,
$[\infty_0,5,\infty_1,a_0]$, $[\infty_3,6,\infty_2,a_2]$, $[\infty_2,7,\infty_3,a_0]$, $[5,\infty_2,4,\chi_1]$, $[5,\infty_3,4,a_0]$, $[7,\infty_0,6,\chi_1]$,
$[7,\infty_1,6,a_0]$, $[\infty_5,4,\infty_4,a_2]$, $[\infty_4,5,\infty_5,a_0]$, $[\infty_7,6,\infty_6,a_2]$, $[\infty_6,7,\infty_7,a_0]$, $[4,\infty_6,5,\chi_1]$,
$[4,\infty_7,5,a_0]$, $[6,\infty_4,7,\chi_1]$, $[6,\infty_5,7,a_0]$, $[\infty_0,8,\infty_1,a_1]$, $[\infty_0,9,\infty_1,a_2]$, $[\infty_2,10,\infty_3,a_1]$,
$[\infty_2,11,\infty_3,a_2]$, $[9,\infty_2,8,\chi_1]$, $[9,\infty_3,8,a_0]$, $[10,\infty_0,11,\chi_1]$, $[10,\infty_1,11,a_0]$, $[\infty_4,8,\infty_5,a_1]$,

$[\infty_4, 9, \infty_5, a_2]$, $[\infty_6, 10, \infty_7, a_1]$, $[\infty_6, 11, \infty_7, a_2]$, $[9, \infty_6, 8, a_1]$, $[9, \infty_7, 8, a_2]$, $[11, \infty_4, 10, a_1]$,
$[11, \infty_5, 10, a_2]$, $[1, 4, 0, a_1]$, $[1, 5, 0, a_2]$, $[3, 6, 2, a_1]$, $[3, 7, 2, a_2]$, $[5, 2, 4, a_1]$, $[5, 3, 4, a_2]$, $[6, 0, 7, a_1]$,
$[6, 1, 7, a_2]$, $[0, 8, 1, \chi_1]$, $[0, 9, 1, a_0]$, $[2, 10, 3, a_1]$, $[2, 11, 3, a_2]$, $[8, 2, 9, a_1]$, $[8, 3, 9, a_2]$, $[10, 0, 11, a_1]$,
$[10, 1, 11, a_2]$, $[4, 8, 5, a_1]$, $[4, 9, 5, a_2]$, $[7, 10, 6, a_1]$, $[7, 11, 6, a_2]$, $[8, 6, 9, \chi_1]$, $[8, 7, 9, a_0]$, $[11, 4, 10, \chi_0]$,
$[11, 5, 10, \chi_1]$, $[\chi_1, a_0, a_1, \chi_0]$, $[a_0, a_2, \chi_1, 2]$, $[\chi_1, a_1, a_2, \chi_0]$, $[10, a_0, \chi_0, 6]$.

Case $(\alpha, \beta) = (8, 4)$. Let $X = \{\infty_0, \dots, \infty_7\}$ and $Y = \{\chi_0, \chi_1, \chi_2, \chi_3\}$. The set $\mathcal{D}$ is given by the following P_4s:
$[\infty_2, \infty_1, \infty_0, \chi_0]$, $[\infty_0, \infty_2, \infty_3, \chi_3]$, $[\infty_0, \infty_3, \infty_1, \chi_1]$, $[\infty_4, \infty_5, \infty_6, \chi_0]$, $[\infty_4, \infty_6, \infty_7, \chi_1]$,
$[\infty_4, \infty_7, \infty_5, \chi_1]$, $[\infty_1, \infty_4, \infty_0, \chi_1]$, $[\infty_0, \infty_5, \infty_1, \chi_3]$, $[\infty_3, \infty_6, \infty_2, \chi_3]$, $[\infty_2, \infty_7, \infty_3, \chi_2]$,
$[\infty_5, \infty_2, \infty_4, \chi_3]$, $[\infty_4, \infty_3, \infty_5, \chi_2]$, $[\infty_7, \infty_0, \infty_6, \chi_1]$, $[\infty_6, \infty_1, \infty_7, \chi_2]$, $[\infty_1, \chi_0, \chi_1, \infty_2]$,
$[\infty_2, \chi_0, \chi_2, \infty_0]$, $[\infty_3, \chi_0, \chi_3, \infty_6]$, $[\infty_3, \chi_1, \chi_2, \infty_1]$, $[\infty_4, \chi_1, \chi_3, \infty_5]$, $[\infty_6, \chi_2, \chi_3, \infty_0]$,
$[\infty_2, \chi_2, \infty_4, \chi_0]$, $[\infty_5, \chi_0, \infty_7, \chi_3]$.

The set $\mathcal{E} \setminus \mathcal{D}$ is given by the following P_4s:
$[2, 1, 0, \chi_0]$, $[0, 2, 3, \chi_0]$, $[0, 3, 1, \chi_1]$, $[6, 5, 4, \chi_0]$, $[4, 6, 7, \chi_0]$, $[4, 7, 5, \chi_0]$, $[10, 9, 8, a_1]$, $[8, 10, 11, \chi_0]$,
$[8, 11, 9, \chi_0]$, $[\infty_1, 0, \infty_0, a_0]$, $[\infty_1, 1, \infty_0, a_1]$, $[\infty_3, 2, \infty_2, a_0]$, $[\infty_3, 3, \infty_2, a_1]$, $[1, \infty_2, 0, \chi_1]$,
$[1, \infty_3, 0, \chi_2]$, $[3, \infty_0, 2, a_2]$, $[3, \infty_1, 2, \chi_0]$, $[\infty_5, 0, \infty_4, a_0]$, $[\infty_5, 1, \infty_4, a_1]$, $[\infty_7, 2, \infty_6, a_0]$,
$[\infty_7, 3, \infty_6, a_1]$, $[0, \infty_6, 1, \chi_2]$, $[0, \infty_7, 1, \chi_3]$, $[3, \infty_4, 2, \chi_1]$, $[3, \infty_5, 2, \chi_2]$, $[\infty_1, 4, \infty_0, a_2]$,
$[\infty_0, 5, \infty_1, a_0]$, $[\infty_3, 6, \infty_2, a_2]$, $[\infty_2, 7, \infty_3, a_0]$, $[5, \infty_2, 4, \chi_2]$, $[5, \infty_3, 4, \chi_3]$, $[7, \infty_0, 6, a_0]$,
$[7, \infty_1, 6, a_1]$, $[\infty_5, 4, \infty_4, a_2]$, $[\infty_4, 5, \infty_5, a_0]$, $[\infty_7, 6, \infty_6, a_2]$, $[\infty_6, 7, \infty_7, a_0]$, $[5, \infty_6, 4, a_0]$,
$[4, \infty_7, 5, \chi_1]$, $[7, \infty_4, 6, \chi_1]$, $[7, \infty_5, 6, \chi_3]$, $[\infty_0, 8, \infty_1, a_1]$, $[\infty_0, 9, \infty_1, a_2]$, $[\infty_2, 10, \infty_3, a_1]$,
$[\infty_2, 11, \infty_3, a_2]$, $[9, \infty_2, 8, \chi_1]$, $[9, \infty_3, 8, \chi_2]$, $[11, \infty_0, 10, a_0]$, $[11, \infty_1, 10, a_1]$, $[\infty_4, 8, \infty_5, a_1]$,
$[\infty_4, 9, \infty_5, a_2]$, $[\infty_6, 10, \infty_7, a_1]$, $[\infty_6, 11, \infty_7, a_2]$, $[9, \infty_6, 8, a_2]$, $[9, \infty_7, 8, \chi_0]$, $[11, \infty_4, 10, \chi_1]$,
$[11, \infty_5, 10, \chi_2]$, $[1, 4, 0, \chi_3]$, $[0, 5, 1, a_0]$, $[2, 6, 3, \chi_1]$, $[2, 7, 3, \chi_2]$, $[4, 2, 5, a_2]$, $[4, 3, 5, \chi_3]$, $[7, 0, 6, a_2]$,
$[6, 1, 7, \chi_1]$, $[0, 8, 1, a_1]$, $[0, 9, 1, a_2]$, $[2, 10, 3, \chi_3]$, $[2, 11, 3, a_1]$, $[8, 2, 9, a_0]$, $[8, 3, 9, a_2]$, $[11, 0, 10, \chi_3]$,
$[10, 1, 11, a_1]$, $[4, 8, 5, a_0]$, $[4, 9, 5, a_1]$, $[6, 10, 7, \chi_2]$, $[6, 11, 7, a_1]$, $[8, 6, 9, \chi_2]$, $[8, 7, 9, \chi_3]$, $[10, 4, 11, \chi_3]$,
$[10, 5, 11, a_0]$, $[0, a_0, \chi_0, 1]$, $[3, a_0, \chi_1, 4]$, $[7, a_0, \chi_2, 5]$, $[8, a_0, \chi_3, 2]$, $[0, a_1, \chi_0, 6]$, $[2, a_1, \chi_1, 11]$,
$[4, a_1, \chi_2, 6]$, $[9, a_1, \chi_3, 7]$, $[0, a_2, \chi_0, 10]$, $[7, a_2, \chi_1, 9]$, $[10, a_2, \chi_2, 11]$, $[11, a_2, \chi_3, 8]$, $[2, a_0, a_2, 3]$,
$[4, a_2, a_1, a_0]$.

2   Appendix 2

In this appendix we show the existence of the sets X, Y, Z and T verifying the statements of Lemma 2.14 for α and β as follows:

α	0	1	4	8	8
β	0	2	2	2	4

Moreover we present a $K_{1,3}$-design of order 22 embedding a P_3-design of order 17. Let $|X| = \alpha$, $|Y| = \beta$, $Z = \mathbb{Z}_{12}$ and $T = \{a_0, a_1, a_2\}$.

Case $\alpha = \beta = 0$. The set $\mathcal{E}$ is given by the following $K_{1,3}$s (note that in this case we have a P_3-design on Z embedded into a $K_{1,3}$-design on $Z \cup T$): $[i; 2 + i, 3 + i, a_0]$, $i \in Z \setminus \{6\}$;

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$[i; 4+i, 5+i, a_1]$, $i \in Z$; $[i; 1+i, 6+i, a_2]$, $i \in \{0, 1, \ldots, 5\}$; $[7+2i; 6+2i, 8+2i, a_2]$, $i \in \{0, 1, 2\}$; $[6; 8, 9, a_2]$, $[a_0; 6, a_1, a_2]$, $[a_2; 8, 10, a_1]$.

Case $\alpha = 1$, $\beta = 2$. Let $X = \{\infty_0\}$, $Y = \{\chi_0, \chi_1\}$. The set $\mathcal{E}$ is given by the following $K_{1,3}$s: $[i; 1+i, 2+i, a_0]$, $i \in \{0, \ldots, 9\}$; $[i; 3+i, 4+i, a_1]$, $i \in \{0, \ldots, 7\}$; $[i; 5+i, 6+i, a_2]$, $i \in \{0, 1, \ldots, 5\}$; $[8+i; i, 1+i, a_2]$, $[\chi_1; 3i, 1+3i, 2+3i]$, $i \in \{0, 1, 2, 3\}$; $[\chi_0; 3i, 1+3i, 2+3i]$, $i \in \{0, 1, 2\}$; $[10; 11, \infty_0, a_0]$, $[11; 0, \infty_0, a_0]$, $[\infty_0; 0, 1, a_0]$, $[9; 0, \infty_0, a_1]$, $[\infty_0; 2, 3, a_1]$, $[6; 11, \infty_0, a_2]$, $[7; 0, \infty_0, a_2]$, $[\infty_0; 4, 5, a_2]$, $[a_1; 10, 11, a_2]$, $[10; 0, 1, \chi_0]$, $[11; 1, 2, \chi_0]$, $[\chi_1; a_0, a_1, a_2]$, $[\chi_0; 9, a_1, a_2]$, $[a_0; \chi_0, a_1, a_2]$.

The set $\mathcal{L}$ is given by the following $K_{1,3}$s:
$[i; 1+i, 2+i, a_0]$, $i \in \{0, \ldots, 9\}$; $[i; 3+i, 4+i, a_1]$, $i \in \{0, \ldots, 7\}$; $[i; 5+i, 6+i, a_2]$, $i \in \{0, 1, \ldots, 5\}$; $[8+i; i, 1+i, a_2]$, $[\chi_0; 3i, 1+3i, 2+3i]$, $[\chi_1; 3i, 1+3i, 2+3i]$, $i \in \{0, 1, 2, 3\}$; $[10; 11, \infty_0, a_0]$, $[11; 0, \infty_0, a_0]$, $[\infty_0; 0, 1, a_0]$, $[8; 11, \infty_0, a_1]$, $[9; 0, \infty_0, a_1]$, $[10; 0, 1, a_1]$, $[11; 1, 2, a_1]$, $[6; 11, \infty_0, a_2]$, $[7; 0, \infty_0, a_2]$, $[\infty_0; 4, 5, a_2]$, $[a_0; a_1, a_2, \chi_0]$, $[\chi_1; a_0, a_1, a_2]$, $[\chi_0; \infty_0, a_2, \chi_1]$, $[a_1; \infty_0, a_2, \chi_0]$, $[\infty_0; 2, 3, \chi_1]$.

Case $\alpha = 4$, $\beta = 2$. Let $X = \{\infty_0, \ldots, \infty_3\}$, $Y = \{\chi_0, \chi_1\}$. The set $\mathcal{D}$ is given by the following $K_{1,3}$s: $[\infty_1; \infty_0, \infty_2, \chi_0]$, $[\infty_2; \infty_0, \infty_3, \chi_0]$, $[\infty_3; \infty_0, \infty_1, \chi_1]$, $[\chi_0; \infty_0, \infty_3, \chi_1]$, $[\chi_1; \infty_0, \infty_1, \infty_2]$.

The set $\mathcal{E}$ is given by the following $K_{1,3}$s: $[i; 2+i, 3+i, \chi_0]$, $i \in \{0, \ldots, 8\}$; $[i; 4+i, 5+i, \chi_1]$, $i \in \{0, \ldots, 6\}$; $[i; 6+i, 7+i, a_0]$, $i \in \{0, 1, \ldots, 4\}$; $[i; 1+i, 8+i, a_1]$, $[4+i; 5+i, \infty_i, a_1]$, $[\infty_i; i, 1+i, a_2]$, $[\infty_i; 2+i, 3+i, a_0]$, $i \in \{0, 1, 2, 3\}$; $[a_2; 1+3i, 2+3i, 3+3i]$, $[6+i; \infty_i, \infty_{i+1}, a_0]$, $[8+i; \infty_i, \infty_{i+1}, \chi_1]$, $i \in \{0, 1, 2\}$; $[9; 11, \infty_0, \chi_0]$, $[10; \infty_0, \infty_1, \chi_0]$, $[11; \infty_1, \infty_2, \chi_0]$, $[\infty_2; 0, 1, a_1]$, $[\infty_3; 1, 2, a_1]$, $[11; 0, \infty_3, \chi_1]$, $[7; 11, \infty_0, \chi_1]$, $[5; 11, \infty_0, a_0]$, $[9; 0, \infty_3, a_0]$, $[10; 0, 1, a_0]$, $[11; 1, 2, a_0]$, $[9; 8, 10, a_1]$, $[11; 10, \infty_0, a_1]$, $[a_2; 10, 11, a_1]$, $[a_1; 8, 10, \infty_0]$, $[a_0; \chi_0, \chi_1, a_1]$, $[a_1; \chi_0, \chi_1, \infty_1]$, $[a_2; \chi_0, \chi_1, a_0]$, $[0; \infty_1, \infty_2, a_2]$.

Case $\alpha = 8$, $\beta = 2$. Let $X = \{\infty_0, \ldots, \infty_7\}$, $Y = \{\chi_0, \chi_1\}$. The set $\mathcal{D}$ is given by the following $K_{1,3}$s: $[\infty_i; \infty_{2+i}, \infty_{3+i}, \chi_0]$, $i \in \{0, \ldots, 7\}$; $[\infty_i; \infty_{1+i}, \infty_{4+i}, \chi_1]$, $i \in \{0, \ldots, 3\}$; $[\infty_5; \infty_4, \infty_6, \chi_1]$, $[\infty_7; \infty_0, \infty_6, \chi_1]$, $[\chi_1; \infty_4, \infty_6, \chi_0]$.

The set $\mathcal{E}$ is given by the following $K_{1,3}$s: $[i; 2+i, 3+i, \chi_0]$, $i \in \{0, \ldots, 8\}$; $[2+i; 3+i, \infty_i, \chi_1]$, $i \in \{0, \ldots, 7\}$; $[i; 4+i, 5+i, a_1]$, $i \in \{0, \ldots, 6\}$; $[\infty_i; i, 1+i, a_0]$, $[\infty_{2+i}; i, 1+i, a_2]$, $i \in \{0, \ldots, 4\}$; $[\infty_{4+i}; i, 1+i, a_1]$, $[8+i; \infty_{2+i}, \infty_{3+i}, a_2]$, $[7+i; \infty_{3+i}, \infty_{4+i}, a_0]$, $[1+i; 7+i, 8+i, a_2]$ $i \in \{0, \ldots, 3\}$; $[9; \infty_{1+i}, \infty_{2+i}, a_1]$, $[i; 8+i, 9+i, a_0]$, $i \in \{0, 1, 2\}$; $[9; 11, \infty_0, \chi_0]$, $[10; \infty_0, \infty_1, \chi_0]$, $[11; \infty_1, \infty_2, \chi_0]$, $[7; 11, \infty_6, a_1]$, $[8; \infty_1, \infty_7, a_1]$, $[0; 7, \infty_1, a_2]$, $[5; 11, \infty_5, a_2]$, $[6; \infty_6, \infty_7, a_2]$, $[7; \infty_2, \infty_7, a_2]$, $[\infty_7; 2, 5, a_2]$, $[3; 11, \infty_0, a_0]$, $[4; \infty_0, \infty_1, a_0]$, $[5; \infty_1, \infty_2, a_0]$, $[6; 0, \infty_2, a_0]$, $[11; 0, \infty_7, a_0]$, $[\infty_5; 0, 6, a_0]$, $[\infty_6; 0, 1, a_0]$, $[\infty_7; 0, 1, a_0]$, $[0; 1, 10, \chi_1]$, $[1; 2, 11, \chi_1]$, $[11; 10, \infty_0, \chi_1]$, $[a_1; \infty_1, \infty_2, a_2]$, $[\infty_0; 5, 8, a_1]$, $[\infty_0; 6, 7, a_2]$, $[\infty_1; 6, 7, a_2]$, $[\infty_3; 0, 6, a_1]$, $[a_0; a_1, a_2, \chi_1]$, $[\chi_0; a_0, a_1, a_2]$, $[\chi_1; 10, a_1, a_2]$.

Case $\alpha = 8$, $\beta = 4$. Let $X = \{\infty_0, \ldots, \infty_7\}$, $Y = \{\chi_0, \ldots, \chi_3\}$. The set $\mathcal{D}$ is given by the following $K_{1,3}$s: $[\infty_i; \infty_{2+i}, \infty_{3+i}, \chi_0]$, $i \in \{0, \ldots, 7\}$; $[\infty_i; \infty_{1+i}, \infty_{4+i}, \chi_1]$, $i \in \{0, \ldots, 3\}$;

$[\infty_5; \infty_4, \infty_6, \chi_1]$, $[\infty_7; \infty_0, \infty_6, \chi_1]$, $[\chi_1; \infty_4, \infty_6, \chi_2]$, $[\chi_2, \infty_6, \infty_7, \chi_3]$, $[\chi_3, \infty_6, \infty_7, \chi_1]$, $[\chi_2, \infty_0, \infty_1, \infty_2]$, $[\chi_2, \infty_3, \infty_4, \infty_5]$, $[\chi_3, \infty_0, \infty_1, \infty_2]$, $[\chi_3, \infty_3, \infty_4, \infty_5]$, $[\chi_0; \chi_1, \chi_2, \chi_3]$.

The set $\mathcal{E}$ is given by the following $K_{1,3}$s: $[i; 2+i, 3+i, \chi_0]$, $i \in \{0, \ldots, 8\}$; $[2+i; 3+i, \infty_i, a_0]$, $i \in \{0, \ldots, 7\}$; $[i; 4+i, 5+i, \chi_1]$, $[4+i; \infty_i, \infty_{1+i}, \chi_3]$, $i \in \{0, \ldots, 6\}$; $[\infty_i; i, 1+i, a_0]$, $[\infty_{2+i}; i, 1+i, a_2]$, $i \in \{0, \ldots, 5\}$; $[i; 6+i, 7+i, \chi_2]$, $i \in \{0, \ldots, 4\}$; $[\infty_{4+i}; i, 1+i, a_1]$, $[8+i; \infty_{2+i}, \infty_{3+i}, \chi_2]$, $[\chi_i; a_0, a_1, a_2]$, $i \in \{0, \ldots, 3\}$; $[9+i; \infty_{1+i}, \infty_{2+i}, \chi_1]$, $[i; 8+i, 9+i, \chi_3]$, $[a_1; 1+3i, 2+3i, 3+3i]$, $[a_2; 1+3i, 2+3i, 3+3i]$, $i \in \{0, 1, 2\}$; $[9; 11, \infty_0, \chi_0]$, $[10; \infty_0, \infty_1, \chi_0]$, $[11; \infty_1, \infty_2, \chi_0]$, $[7; 11, \infty_7, \chi_1]$, $[8; \infty_1, \infty_7, \chi_1]$, $[0; 1, 10, a_0]$, $[5; 11, \infty_0, \chi_2]$, $[6; \infty_0, \infty_6, \chi_2]$, $[7; \infty_2, \infty_6, \chi_2]$, $[\infty_6; 0, 1, a_0]$, $[\infty_7; 1, 2, a_0]$, $[3; 11, \infty_0, \chi_3]$, $[0; \infty_1, \infty_3, a_2]$, $[0; \infty_5, \infty_7, a_1]$, $[1; 2, 11, a_0]$, $[11; 0, \infty_7, \chi_3]$, $[\infty_0; 7, 8, a_2]$, $[11; 10, \infty_0, a_0]$, $[\infty_1; 6, 7, a_2]$, $[a_0; a_1, a_2, 10]$, $[a_1; \infty_1, \infty_2, \infty_3]$, $[a_1; 10, 11, \infty_0]$, $[a_2; 10, 11, a_1]$.

A $K_{1,3}$-design $(V \cup W, \mathcal{B})$ of order 22 embedding a P_3-design of order 17 on V. Let $V = \mathbb{Z}_{12} \cup \{\infty_0, \ldots, \infty_4\}$ and $W = \{\chi_0, \chi_1\} \cup \{a_0, a_1, a_2\}$. Place in $\mathcal{B}$:

- the blocks of a $K_{1,3}$-design on $\{\infty_0, \ldots, \infty_4\} \cup \{\chi_0, \chi_1\}$ embedding a P_3-design on $\{\infty_0, \ldots, \infty_4\}$;

- $[i; 1+i, 2+i, \chi_0]$, $i \in \{0, \ldots, 9\}$; $[i; 3+i, 4+i, a_2]$, $i \in \{0, \ldots, 7\}$; $[i; 5+i, 6+i, a_1]$, $i \in \{0, \ldots, 5\}$; $[\infty_i; i, 1+i, a_1]$, $[\infty_i; 2+i, 3+i, a_0]$, $i \in \{0, \ldots, 4\}$; $[i; 7+i, 8+i, a_0]$, $[5+i; \infty_i, \infty_{1+i}, a_0]$, $[7+i; \infty_i, \infty_{1+i}, a_1]$, $i \in \{0, \ldots, 3\}$; $[9+i; \infty_i, \infty_{1+i}, a_2]$, $[\chi_1; 2+3i, 3+3i, 4+3i]$, $i \in \{0, 1, 2\}$; $[10; 11, \infty_0, \chi_0]$, $[8; 11, \infty_0, a_2]$, $[6; 11, \infty_0, a_1]$, $[11; 0, \infty_4, a_1]$, $[11; \infty_0, \infty_1, a_0]$, $[9; 0, \infty_4, a_0]$, $[10; 0, 1, a_0]$, $[11; 1, 2, \chi_0]$, $[0; \infty_1, \infty_4, \chi_1]$, $[1; \infty_3, \infty_4, \chi_1]$, $[\infty_2; 0, 1, a_2]$, $[\infty_3; 0, 2, a_2]$, $[\infty_4; 2, 3, a_2]$, $[4; 11, \infty_0, a_0]$, $[a_2; \infty_0, \infty_1, \chi_0]$, $[\chi_1; 11, a_0, a_2]$, $[a_0; a_1, a_2, \chi_0]$, $[a_1; a_2, \chi_0, \chi_1]$, $[\chi_1; 2+3i, 3+3i, 4+3i]$, $i = 0, 1, 2$.

3 Appendix 3

In this appendix we give a P_4-design $(V \cup W, \mathcal{B})$ of order n which P_3-regularly embeds a K_2-design of order $\left\lfloor \frac{3+\sqrt{12n^2-12n+9}}{6} \right\rfloor$ on V, for each $n \in \{4, 6, 7, 9, 13, 15\}$.

Case $n = 4$. Let $V = \{v_0, v_1\}$ and $W = \{w_0, w_1\}$. The set $\mathcal{B}$ is given by the following P_4s: $[v_0, v_1, w_0, w_1]$, $[w_0, v_0, w_1, v_1]$.

Case $n = 6$. Let $V = \{v_0, v_1, v_2\}$ and $W = \{w_0, w_1, w_2\}$. The set $\mathcal{B}$ is given by the following P_4s:
$[v_0, v_1, w_1, w_0]$, $[v_1, v_2, w_2, w_1]$, $[v_2, v_0, w_0, w_2]$, $[v_0, w_1, v_2, w_0]$, $[w_0, v_1, w_2, v_0]$.

Case $n = 7$. Let $V = \{v_0, v_1, v_2, v_3\}$ and $W = \{w_0, w_1, w_2\}$. The set $\mathcal{B}$ is given by the following P_4s:
$[v_1, v_0, w_1, v_2]$, $[v_2, v_1, w_2, v_0]$, $[v_0, v_2, w_0, v_1]$, $[v_0, v_3, w_1, v_1]$, $[v_1, v_3, w_2, v_2]$, $[v_2, v_3, w_0, w_1]$, $[v_0, w_0, w_2, w_1]$.

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Case $n = 9$. Let $V = \{v_0, v_1, v_2, v_3, v_4\}$ and $W = \{w_0, w_1, w_2, w_3\}$. The set $\mathcal{B}$ is given by the following P_4s:

$[v_1, v_0, w_1, v_2]$, $[v_2, v_1, w_2, v_3]$, $[v_3, v_2, w_3, v_0]$, $[v_0, v_3, w_0, v_1]$, $[v_0, v_4, w_0, w_1]$, $[v_1, v_4, w_1, w_2]$,
$[v_2, v_4, w_2, w_3]$, $[v_3, v_4, w_3, w_0]$, $[v_2, v_0, w_0, w_2]$, $[v_3, v_1, w_1, w_3]$, $[v_0, w_2, v_2, w_0]$, $[v_1, w_3, v_3, w_1]$.

Case $n = 13$. Let $V = \{v_0, v_1, \ldots, v_6\}$ and $W = \{w_0, w_1, \ldots, w_5\}$. The set $\mathcal{B}$ is given by the following P_4s:

$[v_1, v_0, w_1, v_2]$, $[v_2, v_1, w_2, v_3]$, $[v_3, v_2, w_3, v_4]$, $[v_4, v_3, w_4, v_5]$, $[v_5, v_4, w_5, v_0]$, $[v_0, v_5, w_0, v_1]$,
$[v_2, v_0, w_2, v_4]$, $[v_3, v_1, w_3, v_5]$, $[v_4, v_2, w_4, v_0]$, $[v_5, v_3, w_5, v_1]$, $[v_0, v_4, w_0, v_2]$, $[v_1, v_5, w_1, v_3]$,
$[v_0, v_6, w_0, w_2]$, $[v_1, v_6, w_1, w_3]$, $[v_2, v_6, w_2, w_4]$, $[v_3, v_6, w_3, w_5]$, $[v_4, v_6, w_4, w_0]$, $[v_5, v_6, w_5, w_1]$,
$[v_3, v_0, w_0, w_3]$, $[v_4, v_1, w_1, w_4]$, $[v_5, v_2, w_2, w_5]$, $[v_0, w_3, v_3, w_0]$, $[v_1, w_4, v_4, w_1]$, $[v_2, w_5, v_5, w_2]$,
$[w_0, w_1, w_2, w_3]$, $[w_3, w_4, w_5, w_0]$.

Case $n = 15$. Let $V = \{v_0, v_1, \ldots, v_7\}$ and $W = \{w_0, w_1, \ldots, w_6\}$. The set $\mathcal{B}$ is given by the following P_4s:

$[v_1, v_0, w_1, v_2]$, $[v_2, v_1, w_2, v_3]$, $[v_3, v_2, w_3, v_4]$, $[v_4, v_3, w_4, v_5]$, $[v_5, v_4, w_5, v_6]$, $[v_6, v_5, w_6, v_0]$,
$[v_0, v_6, w_0, v_1]$, $[v_2, v_0, w_2, v_4]$, $[v_3, v_1, w_3, v_5]$, $[v_4, v_2, w_4, v_6]$, $[v_5, v_3, w_5, v_0]$, $[v_6, v_4, w_6, v_1]$,
$[v_0, v_5, w_0, v_2]$, $[v_1, v_6, w_1, v_3]$, $[v_3, v_0, w_3, v_6]$, $[v_4, v_1, w_4, v_0]$, $[v_5, v_2, w_5, v_1]$, $[v_6, v_3, w_6, v_2]$,
$[v_0, v_4, w_0, v_3]$, $[v_1, v_5, w_1, v_4]$, $[v_2, v_6, w_2, v_5]$, $[v_0, v_7, w_1, v_1]$, $[v_1, v_7, w_2, v_2]$, $[v_2, v_7, w_3, v_3]$,
$[v_3, v_7, w_4, v_4]$, $[v_4, v_7, w_5, v_5]$, $[v_5, v_7, w_6, v_6]$, $[v_6, v_7, w_0, v_0]$, $[w_0, w_1, w_3, w_6]$, $[w_1, w_2, w_4, w_0]$,
$[w_2, w_3, w_5, w_1]$, $[w_3, w_4, w_6, w_2]$, $[w_4, w_5, w_0, w_3]$, $[w_5, w_6, w_1, w_4]$, $[w_6, w_0, w_2, w_5]$.

4   Appendix 4

In this appendix we show the existence of the sets X, Y, Z and T verifying the statements of Lemma 4.3 for α and β as follows:

α	0	1	3	4	7	7	10	10	13	13
β	0	0	5	5	9	10	14	15	19	20

Let $|X| = \alpha$, $|Y| = \beta$, $Z = \{1, 2, \ldots, 16\}$ and $T = \{a_1, a_2, \ldots, a_{24}\}$.

Case $\alpha = \beta = 0$. The set $\mathcal{D}$ is given by the following kites:

$[a_1, 2, 1 \bowtie a_{22}]$, $[a_1, 4, 3 \bowtie a_{22}]$, $[a_1, 5, 6 \bowtie a_{23}]$, $[a_1, 8, 7 \bowtie a_{23}]$, $[a_1, 9, 10 \bowtie a_{22}]$, $[a_1, 12, 11 \bowtie a_{23}]$, $[a_1, 13, 14 \bowtie a_{23}]$, $[a_1, 16, 15 \bowtie a_{23}]$, $[a_2, 4, 1 \bowtie a_{23}]$, $[a_2, 3, 2 \bowtie a_{23}]$, $[a_2, 8, 5 \bowtie a_{23}]$, $[a_2, 6, 7 \bowtie a_{22}]$, $[a_2, 12, 9 \bowtie a_{23}]$, $[a_2, 10, 11 \bowtie a_{22}]$, $[a_2, 16, 13 \bowtie a_{23}]$, $[a_2, 14, 15 \bowtie a_{22}]$, $[a_3, 3, 1 \bowtie a_{21}]$, $[a_3, 4, 2 \bowtie a_{21}]$, $[a_3, 7, 5 \bowtie a_{22}]$, $[a_3, 8, 6 \bowtie a_{22}]$, $[a_3, 11, 9 \bowtie a_{22}]$, $[a_3, 12, 10 \bowtie a_{23}]$, $[a_3, 15, 13 \bowtie a_{22}]$, $[a_3, 16, 14 \bowtie a_{22}]$, $[a_4, 1, 8 \bowtie a_{21}]$, $[a_4, 2, 5 \bowtie a_{21}]$, $[a_4, 3, 6 \bowtie a_{21}]$, $[a_4, 4, 7 \bowtie a_{21}]$, $[a_4, 9, 16 \bowtie a_{21}]$, $[a_4, 10, 13 \bowtie a_{21}]$, $[a_4, 11, 14 \bowtie a_{21}]$, $[a_4, 12, 15 \bowtie a_{21}]$, $[a_5, 7, 1 \bowtie a_{20}]$, $[a_5, 8, 2 \bowtie a_{20}]$, $[a_5, 5, 3 \bowtie a_{20}]$, $[a_5, 6, 4 \bowtie a_{20}]$, $[a_5, 15, 9 \bowtie a_{20}]$, $[a_5, 16, 10 \bowtie a_{21}]$, $[a_5, 13, 11 \bowtie a_{21}]$, $[a_5, 14, 12 \bowtie a_{21}]$, $[a_6, 1, 6 \bowtie a_{20}]$, $[a_6, 2, 7 \bowtie a_{20}]$, $[a_6, 3, 8 \bowtie a_{19}]$, $[a_6, 4, 5 \bowtie a_{20}]$, $[a_6, 9, 14 \bowtie a_{20}]$, $[a_6, 10, 15 \bowtie a_{20}]$, $[a_6, 11, 16 \bowtie a_{20}]$, $[a_6, 12, 13 \bowtie a_{20}]$, $[a_7, 5, 1 \bowtie a_{19}]$, $[a_7, 6, 2 \bowtie a_{19}]$, $[a_7, 7, 3 \bowtie a_{19}]$, $[a_7, 8, 4 \bowtie a_{19}]$, $[a_7, 9, 13 \bowtie a_{19}]$, $[a_7, 14, 10 \bowtie a_{20}]$, $[a_7, 15, 11 \bowtie a_{19}]$, $[a_7, 16, 12 \bowtie$

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$a_{19}]$, $[a_8, 1, 16 \bowtie a_{19}]$, $[a_8, 2, 9 \bowtie a_{19}]$, $[a_8, 3, 10 \bowtie a_{19}]$, $[a_8, 4, 11 \bowtie a_{18}]$, $[a_8, 5, 12 \bowtie a_{18}]$, $[a_8, 6, 13 \bowtie a_{18}]$, $[a_8, 7, 14 \bowtie a_{19}]$, $[a_8, 8, 15 \bowtie a_{19}]$, $[a_9, 15, 1 \bowtie a_{18}]$, $[a_9, 16, 2 \bowtie a_{18}]$, $[a_9, 9, 3 \bowtie a_{18}]$, $[a_9, 10, 4 \bowtie a_{18}]$, $[a_9, 11, 5 \bowtie a_{18}]$, $[a_9, 12, 6 \bowtie a_{18}]$, $[a_9, 13, 7 \bowtie a_{18}]$, $[a_9, 14, 8 \bowtie a_{18}]$, $[a_{10}, 1, 14 \bowtie a_{18}]$, $[a_{10}, 2, 15 \bowtie a_{18}]$, $[a_{10}, 3, 16 \bowtie a_{18}]$, $[a_{10}, 4, 9 \bowtie a_{24}]$, $[a_{10}, 5, 10 \bowtie a_{24}]$, $[a_{10}, 6, 11 \bowtie a_{24}]$, $[a_{10}, 7, 12 \bowtie a_{24}]$, $[a_{10}, 8, 13 \bowtie a_{24}]$, $[a_{11}, 13, 1 \bowtie a_{24}]$, $[a_{11}, 14, 2 \bowtie a_{24}]$, $[a_{11}, 15, 3 \bowtie a_{24}]$, $[a_{11}, 16, 4 \bowtie a_{24}]$, $[a_{11}, 9, 5 \bowtie a_{24}]$, $[a_{11}, 10, 6 \bowtie a_{24}]$, $[a_{11}, 11, 7 \bowtie a_{24}]$, $[a_{11}, 12, 8 \bowtie a_{24}]$, $[a_{12}, 1, 12 \bowtie a_{17}]$, $[a_{12}, 2, 13 \bowtie a_{17}]$, $[a_{12}, 3, 14 \bowtie a_{17}]$, $[a_{12}, 4, 15 \bowtie a_{17}]$, $[a_{12}, 5, 16 \bowtie a_{17}]$, $[a_{12}, 6, 9 \bowtie a_{17}]$, $[a_{12}, 7, 10 \bowtie a_{17}]$, $[a_{12}, 8, 11 \bowtie a_{17}]$, $[a_{13}, 11, 1 \bowtie a_{17}]$, $[a_{13}, 12, 2 \bowtie a_{17}]$, $[a_{13}, 13, 3 \bowtie a_{17}]$, $[a_{13}, 14, 4 \bowtie a_{17}]$, $[a_{13}, 15, 5 \bowtie a_{17}]$, $[a_{13}, 16, 6 \bowtie a_{17}]$, $[a_{13}, 9, 7 \bowtie a_{17}]$, $[a_{13}, 10, 8 \bowtie a_{17}]$, $[a_{14}, 1, 10 \bowtie a_{16}]$, $[a_{14}, 2, 11 \bowtie a_{16}]$, $[a_{14}, 3, 12 \bowtie a_{16}]$, $[a_{14}, 4, 13 \bowtie a_{16}]$, $[a_{14}, 5, 14 \bowtie a_{16}]$, $[a_{14}, 6, 15 \bowtie a_{16}]$, $[a_{14}, 7, 16 \bowtie a_{16}]$, $[a_{14}, 8, 9 \bowtie a_{16}]$, $[a_{15}, 9, 1 \bowtie a_{16}]$, $[a_{15}, 10, 2 \bowtie a_{16}]$, $[a_{15}, 11, 3 \bowtie a_{16}]$, $[a_{15}, 12, 4 \bowtie a_{16}]$, $[a_{15}, 13, 5 \bowtie a_{16}]$, $[a_{15}, 14, 6 \bowtie a_{16}]$, $[a_{15}, 15, 7 \bowtie a_{16}]$, $[a_{15}, 16, 8 \bowtie a_{16}]$, $[a_4, a_9, a_{24} \bowtie 14]$, $[a_{14}, a_{10}, a_{19} \bowtie 5]$, $[a_{15}, a_{11}, a_{20} \bowtie 8]$, $[a_{16}, a_{12}, a_{21} \bowtie 3]$, $[a_2, a_7, a_{22} \bowtie 2]$, $[a_3, a_8, a_{23} \bowtie 3]$, $[a_{11}, a_{10}, a_{22} \bowtie 4]$, $[a_{12}, a_{11}, a_{23} \bowtie 4]$, $[a_{13}, a_{12}, a_{24} \bowtie 15]$, $[a_8, a_2, a_{24} \bowtie 16]$, $[a_{10}, a_{12}, a_{18} \bowtie 9]$, $[a_{11}, a_{13}, a_{19} \bowtie 6]$, $[a_{12}, a_{14}, a_{20} \bowtie 11]$, $[a_{13}, a_{15}, a_{21} \bowtie 4]$, $[a_{14}, a_{16}, a_{22} \bowtie 8]$, $[a_{15}, a_{17}, a_{23} \bowtie 8]$, $[a_{16}, a_{24}, a_{18} \bowtie 10]$, $[a_{17}, a_1, a_{19} \bowtie 7]$, $[a_{18}, a_2, a_{20} \bowtie 12]$, $[a_{19}, a_3, a_{21} \bowtie 9]$, $[a_{20}, a_4, a_{22} \bowtie 12]$, $[a_{21}, a_5, a_{23} \bowtie 12]$, $[a_6, a_{24}, a_{22} \bowtie 16]$, $[a_7, a_1, a_{23} \bowtie 16]$, $[a_3, a_{10}, a_{24} \bowtie a_1]$, $[a_4, a_{11}, a_1 \bowtie a_{14}]$, $[a_5, a_{12}, a_2 \bowtie a_{15}]$, $[a_6, a_{13}, a_3 \bowtie a_{16}]$, $[a_7, a_{14}, a_4 \bowtie a_{17}]$, $[a_8, a_{15}, a_5 \bowtie a_{18}]$, $[a_9, a_{16}, a_6 \bowtie a_{19}]$, $[a_{10}, a_{17}, a_7 \bowtie a_{20}]$, $[a_{11}, a_{18}, a_8 \bowtie a_{21}]$, $[a_{12}, a_{19}, a_9 \bowtie a_{22}]$, $[a_{13}, a_{20}, a_{10} \bowtie a_{23}]$, $[a_{14}, a_{21}, a_{11} \bowtie a_{24}]$, $[a_{15}, a_{22}, a_{12} \bowtie a_1]$, $[a_{16}, a_{23}, a_{13} \bowtie a_{14}]$, $[a_{17}, a_{24}, a_{14} \bowtie a_{15}]$, $[a_{18}, a_1, a_{15} \bowtie a_{16}]$, $[a_{19}, a_2, a_{16} \bowtie a_{17}]$, $[a_{20}, a_3, a_{17} \bowtie a_{18}]$, $[a_{21}, a_4, a_{18} \bowtie a_{19}]$, $[a_{22}, a_5, a_{19} \bowtie a_{20}]$, $[a_{23}, a_6, a_{20} \bowtie a_{21}]$, $[a_{24}, a_7, a_{21} \bowtie a_{22}]$, $[a_1, a_8, a_{22} \bowtie a_{23}]$, $[a_2, a_9, a_{23} \bowtie a_{24}]$, $[a_5, a_{10}, a_1 \bowtie a_{13}]$, $[a_6, a_{11}, a_2 \bowtie a_{14}]$, $[a_7, a_{12}, a_3 \bowtie a_{15}]$, $[a_8, a_{13}, a_4 \bowtie a_{16}]$, $[a_9, a_{14}, a_5 \bowtie a_{17}]$, $[a_{10}, a_{15}, a_6 \bowtie a_{18}]$, $[a_{11}, a_{16}, a_7 \bowtie a_{19}]$, $[a_{12}, a_{17}, a_8 \bowtie a_{20}]$, $[a_{13}, a_{18}, a_9 \bowtie a_{21}]$, $[a_{17}, a_{22}, a_{13} \bowtie a_2]$, $[a_{18}, a_{23}, a_{14} \bowtie a_3]$, $[a_{19}, a_{24}, a_{15} \bowtie a_4]$, $[a_{20}, a_1, a_{16} \bowtie a_5]$, $[a_{21}, a_2, a_{17} \bowtie a_6]$, $[a_{22}, a_3, a_{18} \bowtie a_7]$, $[a_{23}, a_4, a_{19} \bowtie a_8]$, $[a_{24}, a_5, a_{20} \bowtie a_9]$, $[a_1, a_6, a_{21} \bowtie a_{10}]$, $[a_9, a_3, a_1 \bowtie a_2]$, $[a_{10}, a_4, a_2 \bowtie a_3]$, $[a_{11}, a_5, a_3 \bowtie a_4]$, $[a_{12}, a_6, a_4 \bowtie a_5]$, $[a_{13}, a_7, a_5 \bowtie a_6]$, $[a_{14}, a_8, a_6 \bowtie a_7]$, $[a_{15}, a_9, a_7 \bowtie a_8]$, $[a_{16}, a_{10}, a_8 \bowtie a_9]$, $[a_{17}, a_{11}, a_9 \bowtie a_{10}]$.

Case $\alpha = 1$ and $\beta = 0$. Let $X = \{\infty\}$. The set $\mathcal{D}$ is given by the blocks of a kite system on T and the following kites:

$[a_1, \infty, 2 \bowtie a_{18}]$, $[a_1, 16, 3 \bowtie a_{18}]$, $[a_1, 15, 4 \bowtie a_{18}]$, $[a_1, 14, 5 \bowtie a_{18}]$, $[a_1, 13, 6 \bowtie a_{18}]$, $[a_1, 12, 7 \bowtie a_{18}]$, $[a_1, 11, 8 \bowtie a_{18}]$, $[a_1, 10, 9 \bowtie a_{18}]$, $[a_2, 3, 1 \bowtie a_{18}]$, $[a_2, 4, \infty \bowtie a_{18}]$, $[a_2, 5, 16 \bowtie a_{18}]$, $[a_2, 6, 15 \bowtie a_{18}]$, $[a_2, 7, 14 \bowtie a_{18}]$, $[a_2, 8, 13 \bowtie a_{18}]$, $[a_2, 9, 12 \bowtie a_{18}]$, $[a_2, 11, 10 \bowtie a_{18}]$, $[a_3, 2, 4 \bowtie a_{19}]$, $[a_3, 1, 5 \bowtie a_{19}]$, $[a_3, \infty, 6 \bowtie a_{19}]$, $[a_3, 16, 7 \bowtie a_{19}]$, $[a_3, 8, 15 \bowtie a_{15}]$, $[a_3, 14, 9 \bowtie a_{19}]$, $[a_3, 13, 10 \bowtie a_{19}]$, $[a_3, 12, 11 \bowtie a_{19}]$, $[a_4, 5, 3 \bowtie a_{19}]$, $[a_4, 6, 2 \bowtie a_{19}]$, $[a_4, 7, 1 \bowtie a_{19}]$, $[a_4, 8, \infty \bowtie a_{19}]$, $[a_4, 9, 16 \bowtie a_{19}]$, $[a_4, 10, 15 \bowtie a_{19}]$, $[a_4, 11, 14 \bowtie a_{19}]$, $[a_4, 12, 13 \bowtie a_{19}]$, $[a_5, 4, 6 \bowtie a_{20}]$, $[a_5, 3, 7 \bowtie a_{20}]$, $[a_5, 2, 8 \bowtie a_{20}]$, $[a_5, 1, 9 \bowtie a_{20}]$, $[a_5, \infty, 10 \bowtie a_{20}]$, $[a_5, 11, 16 \bowtie a_{16}]$, $[a_5, 15, 12 \bowtie a_{20}]$, $[a_5, 14, 13 \bowtie a_{20}]$, $[a_6, 7, 5 \bowtie a_{20}]$, $[a_6, 8, 4 \bowtie a_{20}]$, $[a_6, 9, 3 \bowtie a_{20}]$, $[a_6, 10, 2 \bowtie a_{20}]$, $[a_6, 11, 1 \bowtie a_{20}]$, $[a_6, 12, \infty \bowtie a_{20}]$, $[a_6, 13, 16 \bowtie a_{20}]$, $[a_6, 14, 15 \bowtie a_{20}]$, $[a_7, 6, 8 \bowtie a_{21}]$, $[a_7, 5, 9 \bowtie a_{21}]$, $[a_7, 4, 10 \bowtie a_{21}]$, $[a_7, 3, 11 \bowtie a_{21}]$, $[a_7, 2, 12 \bowtie a_{21}]$, $[a_7, 1, 13 \bowtie a_{21}]$, $[a_7, \infty, 14 \bowtie a_{21}]$, $[a_7, 16, 15 \bowtie a_{21}]$, $[a_8, 9, 7 \bowtie a_{21}]$, $[a_8, 10, 6 \bowtie a_{21}]$, $[a_8, 11, 5 \bowtie a_{21}]$, $[a_8, 12, 4 \bowtie a_{21}]$, $[a_8, 13, 3 \bowtie a_{21}]$, $[a_8, 14, 2 \bowtie a_{21}]$, $[a_8, 15, 1 \bowtie a_{21}]$, $[a_8, 16, \infty \bowtie a_{21}]$, $[a_9, 8, 10 \bowtie a_{22}]$, $[a_9, 7, 11 \bowtie a_{22}]$,

$[a_9, 6, 12 \bowtie a_{22}]$, $[a_9, 5, 13 \bowtie a_{22}]$, $[a_9, 4, 14 \bowtie a_{22}]$, $[a_9, 3, 15 \bowtie a_{22}]$, $[a_9, 2, 16 \bowtie a_{22}]$, $[a_9, 1, \infty \bowtie a_{22}]$, $[a_{10}, 11, 9 \bowtie a_{22}]$, $[a_{10}, 12, 8 \bowtie a_{22}]$, $[a_{10}, 13, 7 \bowtie a_{22}]$, $[a_{10}, 14, 6 \bowtie a_{22}]$, $[a_{10}, 15, 5 \bowtie a_{22}]$, $[a_{10}, 16, 4 \bowtie a_{22}]$, $[a_{10}, \infty, 3 \bowtie a_{22}]$, $[a_{10}, 1, 2 \bowtie a_{22}]$, $[a_{11}, 10, 12 \bowtie a_{23}]$, $[a_{11}, 9, 13 \bowtie a_{23}]$, $[a_{11}, 8, 14 \bowtie a_{23}]$, $[a_{11}, 7, 15 \bowtie a_{23}]$, $[a_{11}, 6, 16 \bowtie a_{23}]$, $[a_{11}, 5, \infty \bowtie a_{23}]$, $[a_{11}, 4, 1 \bowtie a_{23}]$, $[a_{11}, 2, 3 \bowtie a_{23}]$, $[a_{12}, 13, 11 \bowtie a_{23}]$, $[a_{12}, 14, 10 \bowtie a_{23}]$, $[a_{12}, 15, 9 \bowtie a_{23}]$, $[a_{12}, 16, 8 \bowtie a_{23}]$, $[a_{12}, \infty, 7 \bowtie a_{23}]$, $[a_{12}, 1, 6 \bowtie a_{23}]$, $[a_{12}, 2, 5 \bowtie a_{23}]$, $[a_{12}, 3, 4 \bowtie a_{23}]$, $[a_{13}, 12, 14 \bowtie a_{24}]$, $[a_{13}, 11, 15 \bowtie a_{24}]$, $[a_{13}, 10, 16 \bowtie a_{24}]$, $[a_{13}, 9, \infty \bowtie a_{24}]$, $[a_{13}, 8, 1 \bowtie a_{24}]$, $[a_{13}, 7, 2 \bowtie a_{24}]$, $[a_{13}, 6, 3 \bowtie a_{24}]$, $[a_{13}, 4, 5 \bowtie a_{24}]$, $[a_{14}, 15, 13 \bowtie a_{24}]$, $[a_{14}, 16, 12 \bowtie a_{24}]$, $[a_{14}, \infty, 11 \bowtie a_{24}]$, $[a_{14}, 1, 10 \bowtie a_{24}]$, $[a_{14}, 2, 9 \bowtie a_{24}]$, $[a_{14}, 3, 8 \bowtie a_{24}]$, $[a_{14}, 4, 7 \bowtie a_{24}]$, $[a_{14}, 5, 6 \bowtie a_{24}]$, $[a_{15}, 14, 16 \bowtie a_{21}]$, $[a_{15}, \infty, 13 \bowtie a_{13}]$, $[a_{15}, 12, 1 \bowtie a_{22}]$, $[a_{15}, 2, 11 \bowtie a_{11}]$, $[a_{15}, 10, 3 \bowtie a_3]$, $[a_{15}, 9, 4 \bowtie a_{24}]$, $[a_{15}, 8, 5 \bowtie a_5]$, $[a_{15}, 7, 6 \bowtie a_6]$, $[a_{16}, 15, \infty \bowtie a_{17}]$, $[a_{16}, 1, 14 \bowtie a_{20}]$, $[a_{16}, 13, 2 \bowtie a_{23}]$, $[a_{16}, 3, 12 \bowtie a_{19}]$, $[a_{16}, 4, 11 \bowtie a_{20}]$, $[a_{16}, 5, 10 \bowtie a_{10}]$, $[a_{16}, 6, 9 \bowtie a_9]$, $[a_{16}, 7, 8 \bowtie a_{19}]$, $[a_{17}, 16, 1 \bowtie a_1]$, $[a_{17}, 15, 2 \bowtie a_2]$, $[a_{17}, 3, 14 \bowtie a_{14}]$, $[a_{17}, 13, 4 \bowtie a_4]$, $[a_{17}, 5, 12 \bowtie a_{12}]$, $[a_{17}, 6, 11 \bowtie a_{18}]$, $[a_{17}, 10, 7 \bowtie a_7]$, $[a_{17}, 9, 8 \bowtie a_8]$.

Case $\alpha = 3$ and $\beta = 5$. Let $X = \{\infty_1, \infty_2, \infty_3\}$, $Y = \{\chi_1, \chi_2, \ldots, \chi_5\}$. The set $\mathcal{A}$ is given by the following kites:

$[\chi_3, \infty_2, \infty_1 \bowtie \chi_5]$, $[\chi_2, \infty_1, \infty_3 \bowtie \chi_5]$, $[\chi_1, \infty_3, \infty_2 \bowtie \chi_4]$, $[\chi_1, \chi_2, \chi_3 \bowtie \infty_3]$, $[\chi_3, \chi_5, \chi_4 \bowtie \infty_3]$, $[\infty_1, \chi_1, \chi_4 \bowtie \chi_2]$, $[\infty_2, \chi_2, \chi_5 \bowtie \chi_1]$.

The set $\mathcal{D} \setminus \mathcal{A}$ is given by the following kites:

$[a_1, 14, \infty_1 \bowtie a_5]$, $[a_1, 2, 1 \bowtie \chi_1]$, $[a_1, 5, 6 \bowtie \chi_1]$, $[a_1, 16, 10 \bowtie a_{10}]$, $[a_1, 11, 13 \bowtie \chi_1]$, $[a_1, 9, 4 \bowtie \chi_1]$, $[a_1, 12, 15 \bowtie \chi_1]$, $[a_2, 2, \infty_1 \bowtie a_{10}]$, $[a_2, 14, 13 \bowtie a_3]$, $[a_2, 7, 5 \bowtie \chi_1]$, $[a_2, 6, 8 \bowtie a_4]$, $[a_2, 9, 11 \bowtie \chi_1]$, $[a_2, 10, 15 \bowtie \chi_2]$, $[a_2, 12, 16 \bowtie a_4]$, $[a_3, 3, \infty_1 \bowtie a_{23}]$, $[a_3, 2, \infty_2 \bowtie a_{23}]$, $[a_3, 14, 15 \bowtie a_{13}]$, $[a_3, 8, 5 \bowtie \chi_2]$, $[a_3, 6, 9 \bowtie \chi_5]$, $[a_3, 7, 16 \bowtie a_{13}]$, $[a_3, 10, 11 \bowtie \chi_2]$, $[a_4, 4, \infty_1 \bowtie a_{22}]$, $[a_4, 2, \infty_3 \bowtie a_{22}]$, $[a_4, 13, 12 \bowtie a_9]$, $[a_4, 9, 5 \bowtie \chi_3]$, $[a_4, 6, 10 \bowtie \chi_1]$, $[a_4, 7, 14 \bowtie a_9]$, $[a_4, 11, 15 \bowtie \chi_3]$, $[a_5, 4, \infty_2 \bowtie a_{18}]$, $[a_5, 3, \infty_3 \bowtie a_{24}]$, $[a_5, 16, 14 \bowtie a_{13}]$, $[a_5, 15, 1 \bowtie \chi_2]$, $[a_5, 7, 11 \bowtie \chi_3]$, $[a_5, 8, 10 \bowtie \chi_2]$, $[a_5, 9, 12 \bowtie \chi_1]$, $[a_6, 6, \infty_1 \bowtie a_{19}]$, $[a_6, 5, \infty_2 \bowtie a_{24}]$, $[a_6, 4, \infty_3 \bowtie a_{19}]$, $[a_6, 3, 1 \bowtie \chi_3]$, $[a_6, 7, 15 \bowtie \chi_4]$, $[a_6, 8, 16 \bowtie a_{11}]$, $[a_6, 9, 14 \bowtie a_{11}]$, $[a_7, 5, \infty_3 \bowtie a_{20}]$, $[a_7, 14, 12 \bowtie a_8]$, $[a_7, 4, 1 \bowtie a_2]$, $[a_7, 2, 3 \bowtie a_1]$, $[a_7, 11, 8 \bowtie \chi_1]$, $[a_7, 9, 15 \bowtie \chi_5]$, $[a_7, 10, \infty_1 \bowtie a_{20}]$, $[a_8, 8, \infty_1 \bowtie a_{21}]$, $[a_8, 7, \infty_2 \bowtie a_1]$, $[a_8, 6, \infty_3 \bowtie a_1]$, $[a_8, 1, 5 \bowtie \chi_4]$, $[a_8, 4, 2 \bowtie a_5]$, $[a_8, 9, 16 \bowtie \chi_1]$, $[a_8, 14, 10 \bowtie a_6]$, $[a_9, 9, \infty_1 \bowtie a_{14}]$, $[a_9, 8, \infty_2 \bowtie a_2]$, $[a_9, 7, \infty_3 \bowtie a_2]$, $[a_9, 1, 6 \bowtie \chi_2]$, $[a_9, 2, 5 \bowtie \chi_5]$, $[a_9, 4, 3 \bowtie \chi_1]$, $[a_9, 10, 13 \bowtie \chi_2]$, $[a_{10}, \infty_2, 9 \bowtie a_{20}]$, $[a_{10}, 8, \infty_3 \bowtie a_3]$, $[a_{10}, 1, 7 \bowtie \chi_1]$, $[a_{10}, 6, 2 \bowtie \chi_1]$, $[a_{10}, 3, 5 \bowtie a_{12}]$, $[a_{10}, 11, 14 \bowtie \chi_1]$, $[a_{10}, 13, 15 \bowtie a_{12}]$, $[a_{11}, \infty_1, 11 \bowtie a_8]$, $[a_{11}, 10, \infty_2 \bowtie a_{12}]$, $[a_{11}, 9, \infty_3 \bowtie a_{21}]$, $[a_{11}, 1, 8 \bowtie \chi_2]$, $[a_{11}, 2, 7 \bowtie \chi_2]$, $[a_{11}, 3, 6 \bowtie \chi_3]$, $[a_{11}, 5, 4 \bowtie \chi_2]$, $[a_{12}, \infty_1, 12 \bowtie a_6]$, $[a_{12}, \infty_3, 10 \bowtie a_{17}]$, $[a_{12}, 13, 16 \bowtie \chi_2]$, $[a_{12}, 1, 9 \bowtie \chi_1]$, $[a_{12}, 8, 2 \bowtie \chi_2]$, $[a_{12}, 3, 7 \bowtie \chi_3]$, $[a_{12}, 4, 6 \bowtie \chi_4]$, $[a_{13}, \infty_1, 13 \bowtie a_7]$, $[a_{13}, 12, \infty_2 \bowtie a_7]$, $[a_{13}, \infty_3, 11 \bowtie a_{18}]$, $[a_{13}, 1, 10 \bowtie \chi_3]$, $[a_{13}, 2, 9 \bowtie \chi_2]$, $[a_{13}, 3, 8 \bowtie \chi_3]$, $[a_{13}, 7, 4 \bowtie \chi_3]$, $[a_{14}, \infty_2, 13 \bowtie a_{11}]$, $[a_{14}, \infty_3, 12 \bowtie a_3]$, $[a_{14}, 16, 5 \bowtie a_{15}]$, $[a_{14}, 1, 11 \bowtie \chi_4]$, $[a_{14}, 10, 2 \bowtie \chi_3]$, $[a_{14}, 3, 9 \bowtie \chi_3]$, $[a_{14}, 4, 8 \bowtie \chi_4]$, $[a_{15}, \infty_1, 15 \bowtie a_{14}]$, $[a_{15}, 14, \infty_2 \bowtie a_4]$, $[a_{15}, \infty_3, 13 \bowtie a_5]$, $[a_{15}, 1, 12 \bowtie \chi_2]$, $[a_{15}, 2, 11 \bowtie \chi_5]$, $[a_{15}, 3, 10 \bowtie \chi_4]$, $[a_{15}, 6, 16 \bowtie \chi_3]$, $[a_{16}, \infty_1, 16 \bowtie a_{10}]$, $[a_{16}, \infty_2, 15 \bowtie a_9]$, $[a_{16}, \infty_3, 14 \bowtie a_{12}]$, $[a_{16}, 13, 1 \bowtie \chi_4]$, $[a_{16}, 12, 2 \bowtie \chi_4]$, $[a_{16}, 11, 3 \bowtie a_{22}]$, $[a_{16}, 10, 4 \bowtie \chi_5]$, $[a_{17}, \infty_1, 7 \bowtie a_{16}]$, $[a_{17}, \infty_2, 16 \bowtie a_9]$, $[a_{17}, \infty_3, 15 \bowtie a_{11}]$, $[a_{17}, 14, 1 \bowtie a_{21}]$, $[a_{17}, 13, 2 \bowtie \chi_5]$, $[a_{17}, 12, 3 \bowtie a_8]$, $[a_{17}, 4, 11 \bowtie$

$a_{19}]$, $[a_{18}, \infty_3, 16 \bowtie a_{23}]$, $[a_{18}, 8, 7 \bowtie \chi_4]$, $[a_{18}, 15, 6 \bowtie a_{13}]$, $[a_{18}, 2, 14 \bowtie a_{14}]$, $[a_{18}, 13, 3 \bowtie \chi_2]$, $[a_{18}, 12, 4 \bowtie a_{24}]$, $[a_{18}, \infty_1, 5 \bowtie a_5]$, $[a_{19}, \infty_2, 6 \bowtie a_{14}]$, $[a_{19}, 1, 16 \bowtie \chi_4]$, $[a_{19}, 15, 2 \bowtie a_{22}]$, $[a_{19}, 14, 3 \bowtie \chi_3]$, $[a_{19}, 13, 4 \bowtie a_2]$, $[a_{19}, 5, 12 \bowtie \chi_3]$, $[a_{19}, 7, 9 \bowtie \chi_4]$, $[a_{20}, \infty_2, 11 \bowtie a_6]$, $[a_{20}, 2, 16 \bowtie \chi_5]$, $[a_{20}, 15, 3 \bowtie \chi_4]$, $[a_{20}, 4, 14 \bowtie \chi_2]$, $[a_{20}, 5, 13 \bowtie a_6]$, $[a_{20}, 7, 6 \bowtie a_{16}]$, $[a_{20}, 8, 12 \bowtie \chi_4]$, $[a_{21}, \infty_2, 3 \bowtie a_{24}]$, $[a_{21}, 11, 16 \bowtie a_7]$, $[a_{21}, 15, 4 \bowtie a_{15}]$, $[a_{21}, 5, 14 \bowtie \chi_3]$, $[a_{21}, 12, 6 \bowtie \chi_5]$, $[a_{21}, 13, 8 \bowtie a_{15}]$, $[a_{21}, 9, 10 \bowtie \chi_5]$, $[a_{22}, \infty_2, 1 \bowtie a_{18}]$, $[a_{22}, 16, 4 \bowtie \chi_4]$, $[a_{22}, 15, 5 \bowtie a_{17}]$, $[a_{22}, 6, 13 \bowtie \chi_3]$, $[a_{22}, 10, 7 \bowtie a_7]$, $[a_{22}, 8, 14 \bowtie \chi_4]$, $[a_{22}, 11, 12 \bowtie \chi_5]$, $[a_{23}, \infty_3, 1 \bowtie a_{20}]$, $[a_{23}, 10, 5 \bowtie a_{16}]$, $[a_{23}, 11, 6 \bowtie a_{17}]$, $[a_{23}, 12, 7 \bowtie \chi_5]$, $[a_{23}, 15, 8 \bowtie a_{16}]$, $[a_{23}, 9, 13 \bowtie \chi_4]$, $[a_{23}, 16, 3 \bowtie \chi_5]$, $[a_{24}, \infty_1, 1 \bowtie \chi_5]$, $[a_{24}, 11, 5 \bowtie a_{13}]$, $[a_{24}, 6, 14 \bowtie \chi_5]$, $[a_{24}, 7, 13 \bowtie \chi_5]$, $[a_{24}, 9, 8 \bowtie \chi_5]$, $[a_{24}, 12, 10 \bowtie a_{18}]$, $[a_{24}, 16, 15 \bowtie a_8]$, $[a_7, a_{14}, a_1 \bowtie a_{22}]$, $[a_8, a_{15}, a_2 \bowtie a_{17}]$, $[a_9, a_{16}, a_3 \bowtie a_4]$, $[a_4, a_{17}, a_{10} \bowtie a_{18}]$, $[a_{11}, a_{18}, a_5 \bowtie a_{16}]$, $[a_6, a_{19}, a_{12} \bowtie a_{14}]$, $[a_{13}, a_{20}, a_7 \bowtie a_{19}]$, $[a_{14}, a_{21}, a_8 \bowtie a_{10}]$, $[a_{15}, a_{22}, a_9 \bowtie a_{14}]$, $[a_{16}, a_{23}, a_{10} \bowtie a_{20}]$, $[a_{17}, a_{24}, a_{11} \bowtie a_{20}]$, $[a_{13}, a_1, a_{19} \bowtie a_{21}]$, $[a_{14}, a_2, a_{20} \bowtie a_{18}]$, $[a_{21}, a_3, a_{15} \bowtie a_{23}]$, $[a_{22}, a_4, a_{16} \bowtie a_8]$, $[a_{23}, a_5, a_{17} \bowtie a_9]$, $[a_{24}, a_6, a_{18} \bowtie a_{12}]$, $[a_1, a_8, a_{20} \bowtie a_{12}]$, $[a_2, a_9, a_{21} \bowtie a_{24}]$, $[a_3, a_{10}, a_{22} \bowtie a_{20}]$, $[a_{23}, a_4, a_{11} \bowtie a_{19}]$, $[a_{24}, a_5, a_{12} \bowtie a_{22}]$, $[a_1, a_{10}, a_{15} \bowtie a_7]$, $[a_2, a_{11}, a_{16} \bowtie a_{14}]$, $[a_3, a_{12}, a_{17} \bowtie a_{15}]$, $[a_4, a_{18}, a_{13} \bowtie a_5]$, $[a_{14}, a_{19}, a_5 \bowtie a_{15}]$, $[a_{15}, a_{20}, a_6 \bowtie a_{16}]$, $[a_7, a_{21}, a_{16} \bowtie a_{18}]$, $[a_8, a_{22}, a_{17} \bowtie a_{19}]$, $[a_{18}, a_{23}, a_9 \bowtie a_{19}]$, $[a_{24}, a_2, a_3 \bowtie a_{11}]$, $[a_{22}, a_2, a_{13} \bowtie a_{15}]$, $[a_{23}, a_3, a_{14} \bowtie a_6]$, $[a_{15}, a_{24}, a_4 \bowtie a_{14}]$, $[a_1, a_{17}, a_6 \bowtie a_{13}]$, $[a_2, a_{18}, a_7 \bowtie a_{17}]$, $[a_3, a_{19}, a_8 \bowtie a_{18}]$, $[a_{20}, a_9, a_4 \bowtie a_{12}]$, $[a_5, a_{10}, a_{21} \bowtie a_{11}]$, $[a_6, a_{22}, a_{11} \bowtie a_{13}]$, $[a_7, a_{23}, a_{12} \bowtie a_{21}]$, $[a_8, a_{24}, a_{13} \bowtie a_3]$, $[a_{11}, a_9, a_1 \bowtie a_{21}]$, $[a_2, a_{10}, a_{12} \bowtie a_1]$, $[a_{13}, a_{21}, a_{23} \bowtie 4]$, $[a_{14}, a_{22}, a_{24} \bowtie 2]$, $[a_{16}, a_{24}, a_1 \bowtie 8]$, $[a_3, a_1, a_{18} \bowtie 9]$, $[a_{19}, a_4, a_2 \bowtie 3]$, $[a_{20}, a_5, a_3 \bowtie 1]$, $[a_6, a_4, a_{21} \bowtie 7]$, $[a_{22}, a_5, a_7 \bowtie 6]$, $[a_{23}, a_6, a_8 \bowtie 13]$, $[a_{24}, a_7, a_9 \bowtie 11]$, $[a_1, a_5, a_4 \bowtie 1]$, $[a_2, a_5, a_6 \bowtie 2]$, $[a_7, a_6, a_3 \bowtie 4]$, $[a_8, a_7, a_4 \bowtie 3]$, $[a_9, a_8, a_5 \bowtie 6]$, $[a_6, a_9, a_{10} \bowtie 12]$, $[a_7, a_{11}, a_{10} \bowtie 4]$, $[a_8, a_{12}, a_{11} \bowtie 12]$, $[a_9, a_{13}, a_{12} \bowtie 11]$, $[a_{10}, a_{13}, a_{14} \bowtie 7]$, $[a_{11}, a_{14}, a_{15} \bowtie 7]$, $[a_{12}, a_{15}, a_{16} \bowtie 9]$, $[a_{13}, a_{16}, a_{17} \bowtie 8]$, $[a_{14}, a_{18}, a_{17} \bowtie 9]$, $[a_{19}, a_{18}, a_{15} \bowtie 9]$, $[a_{16}, a_{20}, a_{19} \bowtie 10]$, $[a_{17}, a_{21}, a_{20} \bowtie 10]$, $[a_{18}, a_{22}, a_{21} \bowtie 2]$, $[a_{19}, a_{23}, a_{22} \bowtie 9]$, $[a_{20}, a_{24}, a_{23} \bowtie 2]$, $[a_{23}, a_2, a_1 \bowtie 7]$, $[a_{10}, a_{24}, a_{19} \bowtie 8]$.

Case $\alpha = 4$ and $\beta = 5$. Let $X = \{\infty_1, \infty_2, \infty_3, \infty_4\}$, $Y = \{\chi_1, \chi_2, \ldots, \chi_5\}$. The set $\mathcal{A}$ is given by the following kites:
$[\chi_1, \infty_2, \infty_1 \bowtie \chi_4]$, $[\chi_1, \infty_4, \infty_3 \bowtie \chi_5]$, $[\chi_2, \infty_3, \infty_1 \bowtie \chi_5]$, $[\chi_2, \infty_2, \infty_4 \bowtie \chi_5]$, $[\chi_3, \infty_1, \infty_4 \bowtie \chi_4]$, $[\chi_3, \infty_2, \infty_3 \bowtie \chi_4]$, $[\chi_1, \chi_4, \chi_2 \bowtie \chi_3]$, $[\chi_1, \chi_5, \chi_3 \bowtie \chi_4]$, $[\infty_2, \chi_4, \chi_5 \bowtie \chi_2]$.

The set $\mathcal{D} \setminus \mathcal{A}$ is given by the following kites:
$[a_1, \infty_1, 1 \bowtie \chi_1]$, $[a_1, 3, 2 \bowtie \chi_1]$, $[a_1, 5, 4 \bowtie a_{12}]$, $[a_1, 16, 6 \bowtie \chi_1]$, $[a_1, 9, 15 \bowtie a_7]$, $[a_1, 14, 7 \bowtie a_6]$, $[a_1, 13, 10 \bowtie \chi_1]$, $[a_1, 8, 12 \bowtie \chi_1]$, $[a_2, 2, \infty_1 \bowtie a_{24}]$, $[a_2, \infty_2, 1 \bowtie \chi_2]$, $[a_2, 4, 3 \bowtie a_{14}]$, $[a_2, 5, 15 \bowtie a_{12}]$, $[a_2, 16, 7 \bowtie a_{12}]$, $[a_2, 14, 11 \bowtie \chi_1]$, $[a_2, 9, 13 \bowtie a_{24}]$, $[a_2, 6, 8 \bowtie \chi_1]$, $[a_3, 3, \infty_1 \bowtie a_{19}]$, $[a_3, 2, \infty_2 \bowtie a_{24}]$, $[a_3, \infty_3, 1 \bowtie \chi_3]$, $[a_3, 6, 4 \bowtie a_{15}]$, $[a_3, 14, 5 \bowtie a_9]$, $[a_3, 16, 8 \bowtie \chi_2]$, $[a_3, 15, 10 \bowtie a_{24}]$, $[a_3, 13, 11 \bowtie \chi_2]$, $[a_4, 4, \infty_1 \bowtie a_{20}]$, $[a_4, 3, \infty_2 \bowtie a_1]$, $[a_4, 2, \infty_3 \bowtie a_{24}]$, $[a_4, \infty_4, 1 \bowtie \chi_4]$, $[a_4, 5, 13 \bowtie \chi_1]$, $[a_4, 9, 16 \bowtie \chi_1]$, $[a_4, 15, 14 \bowtie \chi_1]$, $[a_4, 7, 6 \bowtie a_{16}]$, $[a_5, 5, \infty_1 \bowtie a_{21}]$, $[a_5, 4, \infty_2 \bowtie a_{13}]$, $[a_5, 3, \infty_3 \bowtie a_1]$, $[a_5, 2, \infty_4 \bowtie a_{24}]$, $[a_5, 9, 1 \bowtie \chi_5]$, $[a_5, 7, 8 \bowtie \chi_3]$, $[a_5, 10, 12 \bowtie a_{24}]$, $[a_5, 6, 11 \bowtie \chi_3]$, $[a_6, 6, \infty_1 \bowtie a_{23}]$, $[a_6, 5, \infty_2 \bowtie a_{18}]$, $[a_6, 4, \infty_3 \bowtie a_2]$, $[a_6, 3, \infty_4 \bowtie a_1]$, $[a_6, 1, 10 \bowtie \chi_2]$, $[a_6, 16, 2 \bowtie \chi_2]$, $[a_6, 13, 15 \bowtie a_{24}]$, $[a_6, 12, 11 \bowtie \chi_4]$, $[a_7, \infty_1, 7 \bowtie a_{24}]$, $[a_7, 6, \infty_2 \bowtie a_{19}]$, $[a_7, \infty_3, 5 \bowtie \chi_1]$, $[a_7, 4, \infty_4 \bowtie a_2]$, $[a_7, 1, 12 \bowtie \chi_2]$, $[a_7, 3, 9 \bowtie \chi_1]$, $[a_7, 16, 14 \bowtie a_5]$, $[a_7, 10, 11 \bowtie \chi_5]$, $[a_8, 8, \infty_1 \bowtie a_{22}]$, $[a_8, 7, \infty_2 \bowtie a_{21}]$, $[a_8, 6, \infty_3 \bowtie a_{19}]$, $[a_8, 5, \infty_4 \bowtie a_3]$, $[a_8, 10, 2 \bowtie \chi_3]$,

$[a_8, 3, 13 \bowtie a_5]$, $[a_8, 11, 4 \bowtie a_{17}]$, $[a_8, 12, 9 \bowtie \chi_2]$, $[a_9, \infty_1, 9 \bowtie \chi_3]$, $[a_9, 8, \infty_2 \bowtie a_{22}]$, $[a_9, 7, \infty_3 \bowtie$
$a_{22}]$, $[a_9, 6, \infty_4 \bowtie a_{13}]$, $[a_9, 16, 1 \bowtie a_{24}]$, $[a_9, 12, 2 \bowtie \chi_4]$, $[a_9, 13, 4 \bowtie a_{24}]$, $[a_9, 11, 15 \bowtie a_5]$,
$[a_{10}, \infty_1, 10 \bowtie \chi_3]$, $[a_{10}, \infty_2, 9 \bowtie a_{24}]$, $[a_{10}, 8, \infty_3 \bowtie a_{23}]$, $[a_{10}, 7, \infty_4 \bowtie a_{21}]$, $[a_{10}, 1, 15 \bowtie \chi_1]$,
$[a_{10}, 11, 2 \bowtie \chi_5]$, $[a_{10}, 14, 4 \bowtie a_{16}]$, $[a_{10}, 16, 12 \bowtie a_4]$, $[a_{11}, \infty_1, 11 \bowtie a_{24}]$, $[a_{11}, \infty_2, 10 \bowtie \chi_4]$,
$[a_{11}, 6, 9 \bowtie \chi_4]$, $[a_{11}, 8, \infty_4 \bowtie a_{22}]$, $[a_{11}, 1, 14 \bowtie \chi_2]$, $[a_{11}, 15, 2 \bowtie a_{24}]$, $[a_{11}, 4, 12 \bowtie \chi_3]$, $[a_{11}, 7, 3 \bowtie$
$\chi_1]$, $[a_{12}, 12, \infty_1 \bowtie a_{17}]$, $[a_{12}, \infty_2, 11 \bowtie a_{20}]$, $[a_{12}, \infty_3, 10 \bowtie \chi_5]$, $[a_{12}, \infty_4, 9 \bowtie \chi_5]$, $[a_{12}, 13, 1 \bowtie$
$a_{13}]$, $[a_{12}, 16, 3 \bowtie a_{24}]$, $[a_{12}, 8, 5 \bowtie \chi_2]$, $[a_{12}, 14, 6 \bowtie a_{15}]$, $[a_{13}, 13, \infty_1 \bowtie a_{18}]$, $[a_{13}, 12, 6 \bowtie \chi_2]$,
$[a_{13}, 11, \infty_3 \bowtie a_{20}]$, $[a_{13}, 10, 8 \bowtie a_{24}]$, $[a_{13}, 14, 2 \bowtie a_7]$, $[a_{13}, 15, 3 \bowtie \chi_2]$, $[a_{13}, 4, 9 \bowtie a_3]$,
$[a_{13}, 5, 16 \bowtie a_8]$, $[a_{14}, \infty_1, 14 \bowtie a_{24}]$, $[a_{14}, \infty_2, 13 \bowtie a_7]$, $[a_{14}, \infty_3, 12 \bowtie \chi_4]$, $[a_{14}, \infty_4, 11 \bowtie$
$a_4]$, $[a_{14}, 2, 9 \bowtie a_{17}]$, $[a_{14}, 6, 5 \bowtie \chi_3]$, $[a_{14}, 4, 8 \bowtie \chi_4]$, $[a_{14}, 7, 15 \bowtie a_8]$, $[a_{15}, \infty_1, 15 \bowtie \chi_2]$,
$[a_{15}, \infty_2, 14 \bowtie \chi_3]$, $[a_{15}, \infty_3, 13 \bowtie a_{11}]$, $[a_{15}, \infty_4, 12 \bowtie a_{18}]$, $[a_{15}, 2, 8 \bowtie \chi_5]$, $[a_{15}, 10, 3 \bowtie \chi_3]$,
$[a_{15}, 7, 5 \bowtie a_{24}]$, $[a_{15}, 11, 16 \bowtie a_5]$, $[a_{16}, \infty_1, 16 \bowtie a_{24}]$, $[a_{16}, \infty_2, 15 \bowtie \chi_3]$, $[a_{16}, \infty_3, 14 \bowtie \chi_4]$,
$[a_{16}, \infty_4, 13 \bowtie a_{10}]$, $[a_{16}, 11, 1 \bowtie a_{14}]$, $[a_{16}, 12, 3 \bowtie \chi_4]$, $[a_{16}, 9, 5 \bowtie a_{21}]$, $[a_{16}, 10, 7 \bowtie a_3]$,
$[a_{17}, \infty_2, 16 \bowtie a_{11}]$, $[a_{17}, 15, \infty_3 \bowtie a_{11}]$, $[a_{17}, \infty_4, 14 \bowtie a_9]$, $[a_{17}, 8, 1 \bowtie a_{15}]$, $[a_{17}, 7, 2 \bowtie a_{12}]$,
$[a_{17}, 11, 3 \bowtie \chi_5]$, $[a_{17}, 10, 5 \bowtie \chi_4]$, $[a_{17}, 12, 13 \bowtie \chi_2]$, $[a_{18}, \infty_3, 16 \bowtie \chi_2]$, $[a_{18}, \infty_4, 15 \bowtie \chi_5]$,
$[a_{18}, 1, 7 \bowtie \chi_1]$, $[a_{18}, 2, 6 \bowtie a_{24}]$, $[a_{18}, 9, 8 \bowtie a_4]$, $[a_{18}, 10, 4 \bowtie \chi_1]$, $[a_{18}, 5, 11 \bowtie a_1]$, $[a_{18}, 14, 13 \bowtie$
$\chi_3]$, $[a_{19}, \infty_4, 16 \bowtie \chi_3]$, $[a_{19}, 1, 6 \bowtie a_{17}]$, $[a_{19}, 13, 2 \bowtie a_{23}]$, $[a_{19}, 9, 7 \bowtie \chi_2]$, $[a_{19}, 15, 4 \bowtie \chi_2]$,
$[a_{19}, 5, 12 \bowtie \chi_5]$, $[a_{19}, 14, 10 \bowtie a_2]$, $[a_{19}, 3, 8 \bowtie a_6]$, $[a_{20}, 2, 1 \bowtie a_8]$, $[a_{20}, 3, 5 \bowtie \chi_5]$, $[a_{20}, 4, 7 \bowtie$
$\chi_3]$, $[a_{20}, 10, 16 \bowtie \chi_4]$, $[a_{20}, 6, 15 \bowtie \chi_4]$, $[a_{20}, 14, 9 \bowtie a_6]$, $[a_{20}, 8, 13 \bowtie \chi_4]$, $[a_{20}, 12, \infty_2 \bowtie$
$a_{23}]$, $[a_{21}, 1, 3 \bowtie a_{10}]$, $[a_{21}, 2, 4 \bowtie \chi_3]$, $[a_{21}, 15, 16 \bowtie \chi_5]$, $[a_{21}, 14, 12 \bowtie a_3]$, $[a_{21}, 13, 7 \bowtie \chi_4]$,
$[a_{21}, 11, 8 \bowtie a_{16}]$, $[a_{21}, 10, 6 \bowtie \chi_3]$, $[a_{21}, \infty_3, 9 \bowtie a_{15}]$, $[a_{22}, 1, 4 \bowtie \chi_4]$, $[a_{22}, 2, 5 \bowtie a_{10}]$, $[a_{22}, 3, 6 \bowtie$
$\chi_4]$, $[a_{22}, 11, 7 \bowtie a_{13}]$, $[a_{22}, 15, 12 \bowtie a_2]$, $[a_{22}, 8, 14 \bowtie \chi_5]$, $[a_{22}, 9, 10 \bowtie a_4]$, $[a_{22}, 16, 13 \bowtie \chi_5]$,
$[a_{23}, 10, \infty_4 \bowtie a_{20}]$, $[a_{23}, 1, 5 \bowtie a_{11}]$, $[a_{23}, 16, 4 \bowtie \chi_5]$, $[a_{23}, 14, 3 \bowtie a_{18}]$, $[a_{23}, 15, 8 \bowtie a_7]$,
$[a_{23}, 13, 6 \bowtie \chi_5]$, $[a_{23}, 12, 7 \bowtie \chi_5]$, $[a_{23}, 9, 11 \bowtie a_{19}]$, $[a_7, a_{14}, a_1 \bowtie \chi_2]$, $[a_8, a_{15}, a_2 \bowtie \chi_3]$,
$[a_3, a_9, a_{16} \bowtie \chi_4]$, $[a_{10}, a_{17}, a_4 \bowtie \chi_4]$, $[a_{11}, a_{18}, a_5 \bowtie \chi_2]$, $[a_{12}, a_{19}, a_6 \bowtie \chi_3]$, $[a_{13}, a_{20}, a_7 \bowtie \chi_3]$,
$[a_8, a_{21}, a_{14} \bowtie 16]$, $[a_9, a_{22}, a_{15} \bowtie \chi_3]$, $[a_{16}, a_{23}, a_{10} \bowtie \chi_3]$, $[a_{17}, a_{24}, a_{11} \bowtie \chi_2]$, $[a_{13}, a_1, a_{19} \bowtie \chi_2]$,
$[a_{14}, a_2, a_{20} \bowtie \chi_4]$, $[a_{21}, a_3, a_{15} \bowtie \chi_2]$, $[a_{22}, a_4, a_{16} \bowtie \chi_2]$, $[a_{23}, a_5, a_{17} \bowtie \chi_3]$, $[a_{24}, a_6, a_{18} \bowtie \chi_4]$,
$[a_1, a_8, a_{20} \bowtie \chi_3]$, $[a_2, a_9, a_{21} \bowtie \chi_5]$, $[a_3, a_{10}, a_{22} \bowtie \chi_3]$, $[a_4, a_{23}, a_{11} \bowtie \chi_5]$, $[a_5, a_{24}, a_{12} \bowtie \chi_3]$,
$[a_{10}, a_{15}, a_1 \bowtie \chi_4]$, $[a_{11}, a_{16}, a_2 \bowtie \chi_4]$, $[a_{12}, a_{17}, a_3 \bowtie \chi_5]$, $[a_4, a_{18}, a_{13} \bowtie \chi_4]$, $[a_5, a_{14}, a_{19} \bowtie \chi_3]$,
$[a_{15}, a_{20}, a_6 \bowtie \chi_4]$, $[a_{16}, a_{21}, a_7 \bowtie \chi_5]$, $[a_8, a_{22}, a_{17} \bowtie \chi_4]$, $[a_{18}, a_{23}, a_9 \bowtie \chi_5]$, $[a_{19}, a_{24}, a_{10} \bowtie \chi_5]$,
$[a_{21}, a_1, a_{12} \bowtie \chi_4]$, $[a_{22}, a_2, a_{13} \bowtie \chi_3]$, $[a_{23}, a_3, a_{14} \bowtie \chi_3]$, $[a_{24}, a_4, a_{15} \bowtie \chi_4]$, $[a_1, a_6, a_{17} \bowtie \chi_5]$,
$[a_2, a_7, a_{18} \bowtie \chi_5]$, $[a_3, a_{19}, a_8 \bowtie 14]$, $[a_4, a_{20}, a_9 \bowtie 3]$, $[a_5, a_{21}, a_{10} \bowtie 6]$, $[a_{22}, a_{11}, a_6 \bowtie \chi_5]$,
$[a_7, a_{12}, a_{23} \bowtie \chi_4]$, $[a_{24}, a_{13}, a_8 \bowtie \chi_3]$, $[a_{11}, a_9, a_1 \bowtie \chi_5]$, $[a_2, a_{10}, a_{12} \bowtie \chi_5]$, $[a_3, a_{13}, a_{11} \bowtie \chi_4]$,
$[a_5, a_{15}, a_{13} \bowtie \chi_5]$, $[a_6, a_{16}, a_{14} \bowtie \chi_4]$, $[a_7, a_{17}, a_{15} \bowtie \chi_5]$, $[a_{18}, a_{16}, a_8 \bowtie \chi_5]$, $[a_{19}, a_{17}, a_9 \bowtie \chi_4]$,
$[a_{11}, a_{21}, a_{19} \bowtie \chi_5]$, $[a_{12}, a_{22}, a_{20} \bowtie \chi_5]$, $[a_{21}, a_4, a_6 \bowtie 14]$, $[a_1, a_5, a_4 \bowtie \chi_3]$, $[a_6, a_5, a_2 \bowtie \chi_5]$,
$[a_7, a_6, a_3 \bowtie \chi_4]$, $[a_8, a_7, a_4 \bowtie \chi_5]$, $[a_9, a_8, a_5 \bowtie \chi_5]$, $[a_6, a_{10}, a_9 \bowtie 10]$, $[a_7, a_{10}, a_{11} \bowtie \chi_3]$,
$[a_{12}, a_{11}, a_8 \bowtie \chi_4]$, $[a_{13}, a_{12}, a_9 \bowtie \chi_3]$, $[a_{14}, a_{13}, a_{10} \bowtie \chi_4]$, $[a_{11}, a_{15}, a_{14} \bowtie \chi_5]$, $[a_{12}, a_{15}, a_{16} \bowtie \chi_3]$,
$[a_{13}, a_{17}, a_{16} \bowtie 2]$, $[a_{18}, a_{17}, a_{14} \bowtie 10]$, $[a_{15}, a_{18}, a_{19} \bowtie \chi_4]$, $[a_{20}, a_{19}, a_{16} \bowtie \chi_5]$, $[a_{20}, a_{23}, a_{24} \bowtie \chi_4]$,
$[a_1, a_2, a_{23} \bowtie \chi_3]$, $[a_3, a_2, a_{24} \bowtie \chi_5]$, $[\chi_1, a_6, a_{13} \bowtie a_{21}]$, $[\chi_1, a_{12}, a_{18} \bowtie a_1]$, $[\chi_1, a_7, a_{19} \bowtie a_2]$,
$[\chi_1, a_9, a_{14} \bowtie a_{22}]$, $[\chi_1, a_{11}, a_{20} \bowtie a_3]$, $[\chi_1, a_5, a_{16} \bowtie a_{24}]$, $[\chi_1, a_8, a_{10} \bowtie a_{18}]$, $[\chi_1, a_2, a_{17} \bowtie a_{20}]$,
$[\chi_1, a_{15}, a_{23} \bowtie a_6]$, $[\chi_1, a_3, a_4 \bowtie a_{12}]$, $[\chi_1, a_{21}, a_{24} \bowtie a_7]$, $[\chi_1, a_1, a_{22} \bowtie a_5]$, $[\chi_3, a_{21}, a_{18} \bowtie a_{22}]$,
$[\chi_4, a_{21}, a_{22} \bowtie a_{19}]$, $[\chi_5, a_{22}, a_{23} \bowtie a_{19}]$, $[\chi_2, a_{13}, a_{23} \bowtie a_{21}]$, $[\chi_2, a_{18}, a_3 \bowtie a_1]$, $[\chi_2, a_2, a_4 \bowtie a_{19}]$,
$[\chi_2, a_{22}, a_{24} \bowtie a_{14}]$, $[\chi_3, a_3, a_5 \bowtie a_{20}]$, $[\chi_3, a_{24}, a_1 \bowtie a_{16}]$, $[\chi_2, a_{10}, a_{20} \bowtie a_{18}]$, $[\chi_2, a_{17}, a_{21} \bowtie a_{20}]$,

$[\chi_2, a_6, a_8 \bowtie a_{23}]$, $[\chi_2, a_{12}, a_{14} \bowtie a_4]$, $[\chi_2, a_7, a_9 \bowtie a_{24}]$, $[\chi_4, a_5, a_7 \bowtie a_{22}]$.

Case $\alpha = 7$ and $\beta = 9$. Let $X = \{\infty_1, \infty_2, \ldots, \infty_7\}$, $Y = \{\chi_1, \chi_2, \ldots, \chi_9\}$. The set $\mathcal{A}$ is given by the blocks of a kite system on Y and the following kites:

$[\chi_1, \infty_2, \infty_7 \bowtie \chi_7]$, $[\chi_1, \infty_3, \infty_6 \bowtie \chi_8]$, $[\chi_1, \infty_4, \infty_5 \bowtie \chi_9]$, $[\chi_2, \infty_3, \infty_1 \bowtie \chi_1]$, $[\chi_2, \infty_4, \infty_7 \bowtie \chi_8]$, $[\chi_2, \infty_5, \infty_6 \bowtie \chi_9]$, $[\chi_3, \infty_4, \infty_2 \bowtie \chi_2]$, $[\chi_3, \infty_5, \infty_1 \bowtie \chi_8]$, $[\chi_3, \infty_6, \infty_7 \bowtie \chi_9]$, $[\chi_4, \infty_5, \infty_3 \bowtie \chi_3]$, $[\chi_4, \infty_6, \infty_2 \bowtie \chi_8]$, $[\chi_4, \infty_7, \infty_1 \bowtie \chi_9]$, $[\chi_5, \infty_6, \infty_4 \bowtie \chi_4]$, $[\chi_5, \infty_7, \infty_3 \bowtie \chi_8]$, $[\chi_5, \infty_1, \infty_2 \bowtie \chi_9]$, $[\chi_6, \infty_7, \infty_5 \bowtie \chi_5]$, $[\chi_6, \infty_1, \infty_4 \bowtie \chi_8]$, $[\chi_6, \infty_2, \infty_3 \bowtie \chi_9]$, $[\chi_7, \infty_1, \infty_6 \bowtie \chi_6]$, $[\chi_7, \infty_2, \infty_5 \bowtie \chi_8]$, $[\chi_7, \infty_3, \infty_4 \bowtie \chi_9]$.

The set $\mathcal{D} \setminus \mathcal{A}$ is given by the following kites:

$[a_1, \infty_2, 16 \bowtie \chi_1]$, $[a_1, 15, \infty_3 \bowtie a_2]$, $[a_1, \infty_4, 14 \bowtie \chi_1]$, $[a_1, \infty_5, 13 \bowtie \chi_1]$, $[a_1, 12, \infty_6 \bowtie a_4]$, $[a_1, 11, \infty_7 \bowtie a_4]$, $[a_1, 10, 1 \bowtie \chi_1]$, $[a_1, 2, 9 \bowtie a_{16}]$, $[a_1, 8, 3 \bowtie a_{10}]$, $[a_1, 7, 4 \bowtie \chi_1]$, $[a_1, 5, 6 \bowtie \chi_1]$, $[a_2, 16, \infty_4 \bowtie a_4]$, $[a_2, 15, \infty_5 \bowtie a_3]$, $[a_2, \infty_6, 14 \bowtie \chi_2]$, $[a_2, \infty_7, 13 \bowtie \chi_2]$, $[a_2, 12, 1 \bowtie \chi_2]$, $[a_2, 11, 2 \bowtie \chi_1]$, $[a_2, 3, 10 \bowtie \chi_1]$, $[a_2, 9, 4 \bowtie \chi_2]$, $[a_2, 8, 5 \bowtie a_{12}]$, $[a_2, 7, 6 \bowtie \chi_2]$, $[a_3, 16, \infty_6 \bowtie a_5]$, $[a_3, \infty_7, 15 \bowtie \chi_1]$, $[a_3, 14, 1 \bowtie \chi_3]$, $[a_3, 13, 2 \bowtie \chi_2]$, $[a_3, 3, 12 \bowtie a_{19}]$, $[a_3, 11, 4 \bowtie \chi_3]$, $[a_3, 5, 10 \bowtie \chi_2]$, $[a_3, 9, 6 \bowtie \chi_3]$, $[a_3, 8, 7 \bowtie a_{14}]$, $[a_4, 16, 1 \bowtie \chi_4]$, $[a_4, 15, 2 \bowtie \chi_3]$, $[a_4, 3, 14 \bowtie a_{21}]$, $[a_4, 4, 13 \bowtie a_{24}]$, $[a_4, 12, 5 \bowtie \chi_1]$, $[a_4, 11, 6 \bowtie \chi_4]$, $[a_4, 7, 10 \bowtie \chi_3]$, $[a_4, 9, 8 \bowtie \chi_1]$, $[a_5, \infty_2, 1 \bowtie \chi_5]$, $[a_5, 2, \infty_1 \bowtie a_{24}]$, $[a_5, 16, 3 \bowtie \chi_1]$, $[a_5, 15, 4 \bowtie \chi_4]$, $[a_5, 14, 5 \bowtie \chi_2]$, $[a_5, 6, 13 \bowtie \chi_3]$, $[a_5, 12, 7 \bowtie \chi_1]$, $[a_5, 8, 11 \bowtie \chi_1]$, $[a_5, 9, 10 \bowtie \chi_4]$, $[a_6, \infty_4, 1 \bowtie \chi_6]$, $[a_6, \infty_3, 2 \bowtie \chi_4]$, $[a_6, 3, \infty_2 \bowtie a_{15}]$, $[a_6, \infty_1, 4 \bowtie \chi_5]$, $[a_6, 5, 16 \bowtie \chi_2]$, $[a_6, 6, 15 \bowtie \chi_2]$, $[a_6, 14, 7 \bowtie \chi_2]$, $[a_6, 13, 8 \bowtie \chi_2]$, $[a_6, 9, 12 \bowtie \chi_1]$, $[a_6, 11, 10 \bowtie \chi_5]$, $[a_7, 1, \infty_6 \bowtie a_{16}]$, $[a_7, \infty_5, 2 \bowtie \chi_5]$, $[a_7, \infty_4, 3 \bowtie \chi_2]$, $[a_7, \infty_3, 4 \bowtie \chi_6]$, $[a_7, 5, \infty_2 \bowtie a_2]$, $[a_7, 6, \infty_1 \bowtie a_2]$, $[a_7, 16, 7 \bowtie \chi_3]$, $[a_7, 8, 15 \bowtie \chi_3]$, $[a_7, 14, 9 \bowtie a_{24}]$, $[a_7, 13, 10 \bowtie \chi_6]$, $[a_7, 11, 12 \bowtie \chi_2]$, $[a_8, 2, \infty_7 \bowtie a_{18}]$, $[a_8, \infty_6, 3 \bowtie \chi_3]$, $[a_8, \infty_5, 4 \bowtie \chi_7]$, $[a_8, 5, \infty_4 \bowtie a_{16}]$, $[a_8, 6, \infty_3 \bowtie a_{14}]$, $[a_8, \infty_2, 7 \bowtie \chi_4]$, $[a_8, 8, \infty_1 \bowtie a_{15}]$, $[a_8, 9, 16 \bowtie a_{23}]$, $[a_8, 15, 10 \bowtie \chi_7]$, $[a_8, 14, 11 \bowtie a_{18}]$, $[a_8, 13, 12 \bowtie \chi_3]$, $[a_9, 3, 1 \bowtie \chi_7]$, $[a_9, \infty_7, 4 \bowtie \chi_8]$, $[a_9, \infty_6, 5 \bowtie \chi_3]$, $[a_9, 6, \infty_5 \bowtie a_{16}]$, $[a_9, \infty_4, 7 \bowtie \chi_5]$, $[a_9, 8, \infty_3 \bowtie a_{16}]$, $[a_9, 9, \infty_2 \bowtie a_{16}]$, $[a_9, \infty_1, 10 \bowtie \chi_8]$, $[a_9, 11, 16 \bowtie a_{24}]$, $[a_9, 15, 12 \bowtie \chi_4]$, $[a_9, 14, 13 \bowtie \chi_4]$, $[a_{10}, 4, 2 \bowtie \chi_6]$, $[a_{10}, 5, 1 \bowtie \chi_8]$, $[a_{10}, 6, \infty_7 \bowtie a_5]$, $[a_{10}, \infty_6, 7 \bowtie \chi_6]$, $[a_{10}, 8, \infty_5 \bowtie a_{15}]$, $[a_{10}, \infty_4, 9 \bowtie \chi_1]$, $[a_{10}, \infty_3, 10 \bowtie \chi_9]$, $[a_{10}, 11, \infty_2 \bowtie a_3]$, $[a_{10}, \infty_1, 12 \bowtie \chi_5]$, $[a_{10}, 16, 13 \bowtie \chi_5]$, $[a_{10}, 14, 15 \bowtie \chi_4]$, $[a_{11}, 5, 3 \bowtie \chi_4]$, $[a_{11}, 6, 2 \bowtie \chi_7]$, $[a_{11}, 1, 7 \bowtie \chi_7]$, $[a_{11}, 8, \infty_7 \bowtie a_6]$, $[a_{11}, \infty_6, 9 \bowtie \chi_2]$, $[a_{11}, 10, \infty_5 \bowtie a_4]$, $[a_{11}, \infty_4, 11 \bowtie \chi_2]$, $[a_{11}, \infty_3, 12 \bowtie \chi_6]$, $[a_{11}, \infty_2, 13 \bowtie \chi_6]$, $[a_{11}, 14, \infty_1 \bowtie a_1]$, $[a_{11}, 16, 15 \bowtie \chi_5]$, $[a_{12}, 4, 6 \bowtie \chi_5]$, $[a_{12}, 7, 3 \bowtie \chi_5]$, $[a_{12}, 8, 2 \bowtie \chi_8]$, $[a_{12}, 1, 9 \bowtie \chi_3]$, $[a_{12}, \infty_7, 10 \bowtie a_{17}]$, $[a_{12}, \infty_6, 11 \bowtie \chi_3]$, $[a_{12}, \infty_5, 12 \bowtie \chi_7]$, $[a_{12}, 13, \infty_4 \bowtie a_5]$, $[a_{12}, \infty_3, 14 \bowtie \chi_3]$, $[a_{12}, \infty_2, 15 \bowtie \chi_6]$, $[a_{12}, 16, \infty_1 \bowtie a_3]$, $[a_{13}, 7, 5 \bowtie a_{24}]$, $[a_{13}, 4, 8 \bowtie \chi_3]$, $[a_{13}, 9, 3 \bowtie \chi_6]$, $[a_{13}, 10, 2 \bowtie \chi_9]$, $[a_{13}, 1, 11 \bowtie \chi_4]$, $[a_{13}, \infty_7, 12 \bowtie \chi_8]$, $[a_{13}, 13, \infty_6 \bowtie a_6]$, $[a_{13}, \infty_5, 14 \bowtie \chi_4]$, $[a_{13}, \infty_4, 15 \bowtie \chi_7]$, $[a_{13}, 16, \infty_3 \bowtie a_4]$, $[a_{14}, 6, 8 \bowtie a_{24}]$, $[a_{14}, 9, 5 \bowtie \chi_4]$, $[a_{14}, 10, 4 \bowtie \chi_9]$, $[a_{14}, 11, 3 \bowtie \chi_7]$, $[a_{14}, 2, 12 \bowtie \chi_9]$, $[a_{14}, 13, 1 \bowtie a_8]$, $[a_{14}, \infty_7, 14 \bowtie \chi_5]$, $[a_{14}, \infty_6, 15 \bowtie \chi_8]$, $[a_{14}, 16, \infty_5 \bowtie a_5]$, $[a_{15}, 9, 7 \bowtie \chi_8]$, $[a_{15}, 10, 6 \bowtie a_{24}]$, $[a_{15}, 5, 11 \bowtie a_{24}]$, $[a_{15}, 12, 4 \bowtie a_{24}]$, $[a_{15}, 3, 13 \bowtie a_{20}]$, $[a_{15}, 2, 14 \bowtie \chi_6]$, $[a_{15}, 1, 15 \bowtie \chi_9]$, $[a_{15}, \infty_7, 16 \bowtie \chi_3]$, $[a_{16}, 10, 8 \bowtie a_{15}]$, $[a_{16}, 11, 7 \bowtie \chi_9]$, $[a_{16}, 12, 6 \bowtie a_{13}]$, $[a_{16}, 13, 5 \bowtie \chi_5]$, $[a_{16}, 4, 14 \bowtie \chi_7]$, $[a_{16}, 3, 15 \bowtie a_{22}]$, $[a_{16}, 2, 16 \bowtie \chi_4]$, $[a_{16}, 1, \infty_1 \bowtie a_{13}]$, $[a_{17}, 11, 9 \bowtie \chi_4]$, $[a_{17}, 12, 8 \bowtie \chi_4]$, $[a_{17}, 7, 13 \bowtie \chi_7]$, $[a_{17}, 6, 14 \bowtie \chi_8]$, $[a_{17}, 15, 5 \bowtie \chi_6]$, $[a_{17}, 4, 16 \bowtie \chi_5]$, $[a_{17}, 3, \infty_1 \bowtie a_4]$, $[a_{17}, 2, \infty_2 \bowtie a_4]$, $[a_{17}, 1, \infty_3 \bowtie a_3]$, $[a_{18}, 10, 12 \bowtie a_{24}]$, $[a_{18}, 13, 9 \bowtie \chi_5]$,

$[a_{18}, 14, 8 \bowtie \chi_5]$, $[a_{18}, 7, 15 \bowtie a_{24}]$, $[a_{18}, 6, 16 \bowtie \chi_6]$, $[a_{18}, \infty_1, 5 \bowtie \chi_7]$, $[a_{18}, 4, \infty_2 \bowtie a_{13}]$, $[a_{18}, 3, \infty_3 \bowtie a_5]$, $[a_{18}, 2, \infty_4 \bowtie a_3]$, $[a_{18}, 1, \infty_5 \bowtie a_6]$, $[a_{19}, 13, 11 \bowtie \chi_5]$, $[a_{19}, 14, 10 \bowtie a_{24}]$, $[a_{19}, 15, 9 \bowtie \chi_6]$, $[a_{19}, 16, 8 \bowtie \chi_6]$, $[a_{19}, \infty_1, 7 \bowtie a_{24}]$, $[a_{19}, \infty_2, 6 \bowtie \chi_6]$, $[a_{19}, \infty_3, 5 \bowtie \chi_8]$, $[a_{19}, 4, \infty_4 \bowtie a_{17}]$, $[a_{19}, 3, \infty_5 \bowtie a_{17}]$, $[a_{19}, 2, \infty_6 \bowtie a_{17}]$, $[a_{19}, 1, \infty_7 \bowtie a_7]$, $[a_{20}, 12, 14 \bowtie a_{24}]$, $[a_{20}, 15, 11 \bowtie \chi_6]$, $[a_{20}, 10, 16 \bowtie \chi_7]$, $[a_{20}, \infty_1, 9 \bowtie \chi_7]$, $[a_{20}, \infty_2, 8 \bowtie \chi_7]$, $[a_{20}, 7, \infty_3 \bowtie a_{24}]$, $[a_{20}, 6, \infty_4 \bowtie a_{24}]$, $[a_{20}, \infty_5, 5 \bowtie \chi_9]$, $[a_{20}, 4, \infty_6 \bowtie a_{18}]$, $[a_{20}, \infty_7, 3 \bowtie \chi_8]$, $[a_{20}, 2, 1 \bowtie a_{24}]$, $[a_{21}, 15, 13 \bowtie \chi_8]$, $[a_{21}, 12, 16 \bowtie \chi_8]$, $[a_{21}, \infty_1, 11 \bowtie \chi_7]$, $[a_{21}, 10, \infty_2 \bowtie a_{24}]$, $[a_{21}, \infty_3, 9 \bowtie \chi_8]$, $[a_{21}, \infty_4, 8 \bowtie \chi_8]$, $[a_{21}, 7, \infty_5 \bowtie a_{24}]$, $[a_{21}, \infty_6, 6 \bowtie \chi_7]$, $[a_{21}, 5, \infty_7 \bowtie a_{24}]$, $[a_{21}, 4, 1 \bowtie \chi_9]$, $[a_{21}, 2, 3 \bowtie a_{24}]$, $[a_{22}, 14, 16 \bowtie \chi_9]$, $[a_{22}, \infty_1, 13 \bowtie \chi_9]$, $[a_{22}, 12, \infty_2 \bowtie a_{14}]$, $[a_{22}, \infty_3, 11 \bowtie \chi_8]$, $[a_{22}, 10, \infty_4 \bowtie a_{14}]$, $[a_{22}, \infty_5, 9 \bowtie \chi_9]$, $[a_{22}, 8, \infty_6 \bowtie a_{24}]$, $[a_{22}, 7, \infty_7 \bowtie a_{17}]$, $[a_{22}, 1, 6 \bowtie \chi_8]$, $[a_{22}, 5, 2 \bowtie a_{24}]$, $[a_{22}, 4, 3 \bowtie \chi_9]$, $[a_{23}, 15, \infty_1 \bowtie a_{14}]$, $[a_{23}, \infty_2, 14 \bowtie \chi_9]$, $[a_{23}, 13, \infty_3 \bowtie a_{15}]$, $[a_{23}, 12, \infty_4 \bowtie a_{15}]$, $[a_{23}, \infty_5, 11 \bowtie \chi_9]$, $[a_{23}, 10, \infty_6 \bowtie a_{15}]$, $[a_{23}, 9, \infty_7 \bowtie a_{16}]$, $[a_{23}, 1, 8 \bowtie \chi_9]$, $[a_{23}, 7, 2 \bowtie a_9]$, $[a_{23}, 3, 6 \bowtie \chi_9]$, $[a_{23}, 5, 4 \bowtie a_{11}]$, $[a_7, a_{14}, a_1 \bowtie \chi_5]$, $[a_8, a_{15}, a_2 \bowtie \chi_5]$, $[a_3, a_{16}, a_9 \bowtie \chi_8]$, $[a_{10}, a_{17}, a_4 \bowtie \chi_6]$, $[a_{11}, a_{18}, a_5 \bowtie \chi_7]$, $[a_6, a_{12}, a_{19} \bowtie \chi_7]$, $[a_7, a_{13}, a_{20} \bowtie \chi_7]$, $[a_8, a_{14}, a_{21} \bowtie \chi_7]$, $[a_9, a_{22}, a_{15} \bowtie \chi_7]$, $[a_{10}, a_{16}, a_{23} \bowtie \chi_4]$, $[a_{11}, a_{24}, a_{17} \bowtie \chi_2]$, $[a_{13}, a_1, a_{19} \bowtie \chi_5]$, $[a_{14}, a_2, a_{20} \bowtie \chi_8]$, $[a_{15}, a_3, a_{21} \bowtie \chi_3]$, $[a_{16}, a_4, a_{22} \bowtie \chi_2]$, $[a_{17}, a_5, a_{23} \bowtie \chi_7]$, $[a_{18}, a_6, a_{24} \bowtie \chi_5]$, $[a_1, a_{20}, a_8 \bowtie \chi_5]$, $[a_2, a_9, a_{21} \bowtie \chi_1]$, $[a_{22}, a_{10}, a_3 \bowtie \chi_7]$, $[a_{23}, a_{11}, a_4 \bowtie \chi_7]$, $[a_{24}, a_{12}, a_5 \bowtie \chi_8]$, $[a_1, a_{15}, a_{10} \bowtie \chi_5]$, $[a_2, a_{16}, a_{11} \bowtie \chi_5]$, $[a_{12}, a_{17}, a_3 \bowtie \chi_8]$, $[a_4, a_{13}, a_{18} \bowtie \chi_5]$, $[a_5, a_{19}, a_{14} \bowtie \chi_6]$, $[a_6, a_{20}, a_{15} \bowtie \chi_4]$, $[a_7, a_{21}, a_{16} \bowtie \chi_1]$, $[a_8, a_{22}, a_{17} \bowtie \chi_3]$, $[\chi_9, a_{13}, a_6 \bowtie a_{22}]$, $[\chi_9, a_{18}, a_{12} \bowtie a_{21}]$, $[\chi_9, a_{19}, a_7 \bowtie a_{23}]$, $[\chi_9, a_{14}, a_9 \bowtie a_{18}]$, $[\chi_9, a_{20}, a_{11} \bowtie a_1]$, $[\chi_9, a_{16}, a_5 \bowtie a_{21}]$, $[\chi_9, a_8, a_{10} \bowtie a_{19}]$, $[\chi_9, a_2, a_{17} \bowtie a_1]$, $[\chi_9, a_{15}, a_{23} \bowtie a_{14}]$, $[\chi_9, a_3, a_4 \bowtie a_{20}]$, $[\chi_9, a_{21}, a_{24} \bowtie a_{15}]$, $[\chi_9, a_1, a_{22} \bowtie a_{13}]$, $[\chi_6, a_{12}, a_2 \bowtie a_{18}]$, $[\chi_6, a_{13}, a_3 \bowtie a_{19}]$, $[\chi_7, a_{18}, a_8 \bowtie a_{24}]$, $[\chi_2, a_4, a_{14} \bowtie a_{24}]$, $[\chi_1, a_{20}, a_5 \bowtie a_{15}]$, $[\chi_8, a_6, a_{10} \bowtie a_{20}]$, $[\chi_8, a_{11}, a_{21} \bowtie a_6]$, $[\chi_8, a_8, a_{12} \bowtie a_{22}]$, $[\chi_3, a_{18}, a_{23} \bowtie a_{13}]$, $[\chi_2, a_7, a_3 \bowtie a_{18}]$, $[\chi_1, a_9, a_{23} \bowtie a_8]$, $[\chi_1, a_{10}, a_{24} \bowtie a_{19}]$, $[\chi_1, a_{12}, a_1 \bowtie a_{21}]$, $[\chi_1, a_{13}, a_2 \bowtie a_{22}]$, $[\chi_1, a_{14}, a_3 \bowtie a_{23}]$, $[\chi_1, a_{15}, a_4 \bowtie a_{24}]$, $[\chi_1, a_{17}, a_6 \bowtie a_1]$, $[\chi_1, a_{18}, a_7 \bowtie a_2]$, $[\chi_1, a_{19}, a_8 \bowtie a_3]$, $[\chi_2, a_{20}, a_9 \bowtie a_4]$, $[\chi_2, a_{21}, a_{10} \bowtie a_5]$, $[\chi_1, a_{22}, a_{11} \bowtie a_6]$, $[\chi_2, a_{12}, a_{23} \bowtie a_{19}]$, $[\chi_2, a_{24}, a_{13} \bowtie a_8]$, $[\chi_3, a_1, a_9 \bowtie a_{11}]$, $[\chi_3, a_2, a_{10} \bowtie a_{12}]$, $[\chi_3, a_3, a_{11} \bowtie a_{13}]$, $[\chi_3, a_{12}, a_4 \bowtie a_7]$, $[\chi_3, a_5, a_{13} \bowtie a_{15}]$, $[\chi_3, a_6, a_{14} \bowtie a_{16}]$, $[\chi_3, a_7, a_{15} \bowtie a_{17}]$, $[\chi_3, a_8, a_{16} \bowtie a_{18}]$, $[\chi_4, a_9, a_{17} \bowtie a_{19}]$, $[\chi_4, a_{10}, a_{18} \bowtie a_{20}]$, $[\chi_4, a_{11}, a_{19} \bowtie a_{21}]$, $[\chi_4, a_{12}, a_{20} \bowtie a_{22}]$, $[\chi_4, a_{13}, a_{21} \bowtie a_{23}]$, $[\chi_4, a_{22}, a_{14} \bowtie a_{12}]$, $[\chi_4, a_{16}, a_{24} \bowtie a_1]$, $[\chi_2, a_{18}, a_1 \bowtie a_3]$, $[\chi_2, a_{19}, a_2 \bowtie a_4]$, $[\chi_5, a_{20}, a_3 \bowtie a_5]$, $[\chi_5, a_{21}, a_4 \bowtie a_6]$, $[\chi_5, a_{22}, a_5 \bowtie a_7]$, $[\chi_5, a_{23}, a_6 \bowtie a_8]$, $[\chi_5, a_7, a_9 \bowtie a_{24}]$, $[\chi_4, a_1, a_4 \bowtie a_5]$, $[\chi_4, a_2, a_5 \bowtie a_6]$, $[\chi_4, a_3, a_6 \bowtie a_7]$, $[\chi_4, a_8, a_7 \bowtie a_{24}]$, $[\chi_2, a_5, a_8 \bowtie a_9]$, $[\chi_6, a_6, a_9 \bowtie a_{10}]$, $[\chi_6, a_7, a_{10} \bowtie a_{11}]$, $[\chi_6, a_8, a_{11} \bowtie a_{12}]$, $[\chi_7, a_9, a_{12} \bowtie a_{13}]$, $[\chi_7, a_{10}, a_{13} \bowtie a_{14}]$, $[\chi_7, a_{11}, a_{14} \bowtie a_{15}]$, $[\chi_5, a_{15}, a_{12} \bowtie a_7]$, $[\chi_5, a_{13}, a_{16} \bowtie a_{17}]$, $[\chi_5, a_{14}, a_{17} \bowtie a_{18}]$, $[\chi_6, a_{15}, a_{18} \bowtie a_{19}]$, $[\chi_6, a_{16}, a_{19} \bowtie a_{20}]$, $[\chi_6, a_{17}, a_{20} \bowtie a_{21}]$, $[\chi_6, a_{22}, a_{21} \bowtie a_{18}]$, $[\chi_7, a_{24}, a_{22} \bowtie a_{18}]$, $[\chi_6, a_{24}, a_{23} \bowtie a_{20}]$, $[\chi_8, a_{23}, a_1 \bowtie a_2]$, $[\chi_8, a_{24}, a_2 \bowtie a_3]$, $[\chi_3, a_{22}, a_{19} \bowtie a_9]$, $[\chi_6, a_1, a_5 \bowtie a_9]$, $[\chi_7, a_6, a_2 \bowtie a_{23}]$, $[\chi_8, a_{19}, a_4 \bowtie a_8]$, $[\chi_8, a_{17}, a_{13} \bowtie a_9]$, $[\chi_2, a_{15}, a_{11} \bowtie a_7]$, $[\chi_8, a_{18}, a_{14} \bowtie a_{10}]$, $[\chi_7, a_1, a_{16} \bowtie a_{12}]$, $[\chi_8, a_{16}, a_{15} \bowtie a_{19}]$, $[\chi_2, a_6, a_{16} \bowtie a_{20}]$, $[\chi_7, a_7, a_{17} \bowtie a_{21}]$, $[\chi_8, a_7, a_{22} \bowtie a_{23}]$, $[\chi_3, a_{20}, a_{24} \bowtie a_3]$.

Case $\alpha = 7$ and $\beta = 10$. Let $X = \{\infty_1, \infty_2, \ldots, \infty_7\}$, $Y = \{\chi_1, \chi_2, \ldots, \chi_{10}\}$. The set $\mathcal{A}$ is given by the following kites:

$[\chi_1, \infty_2, \infty_7 \bowtie \chi_7]$, $[\chi_1, \infty_3, \infty_6 \bowtie \chi_{10}]$, $[\chi_1, \infty_5, \infty_4 \bowtie \chi_8]$, $[\chi_2, \infty_3, \infty_1 \bowtie \chi_1]$, $[\chi_2, \infty_4, \infty_7 \bowtie \chi_{10}]$, $[\chi_2, \infty_6, \infty_5 \bowtie \chi_8]$, $[\chi_3, \infty_4, \infty_2 \bowtie \chi_2]$, $[\chi_3, \infty_5, \infty_1 \bowtie \chi_9]$, $[\chi_3, \infty_7, \infty_6 \bowtie \chi_8]$, $[\chi_4, \infty_5, \infty_3 \bowtie \chi_3]$, $[\chi_4, \infty_6, \infty_2 \bowtie \chi_9]$, $[\chi_4, \infty_1, \infty_7 \bowtie \chi_8]$, $[\chi_5, \infty_6, \infty_4 \bowtie \chi_4]$, $[\chi_5, \infty_7, \infty_3 \bowtie \chi_9]$, $[\chi_5, \infty_1, \infty_2 \bowtie$

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$\chi_{10}]$, $[\chi_6, \infty_7, \infty_5 \bowtie \chi_5]$, $[\chi_6, \infty_4, \infty_1 \bowtie \chi_8]$, $[\chi_6, \infty_2, \infty_3 \bowtie \chi_8]$, $[\chi_7, \infty_1, \infty_6 \bowtie \chi_6]$, $[\chi_7, \infty_5, \infty_2 \bowtie \chi_8]$, $[\chi_7, \infty_3, \infty_4 \bowtie \chi_9]$, $[\chi_2, \chi_4, \chi_3 \bowtie \chi_1]$, $[\chi_7, \chi_6, \chi_1 \bowtie \chi_2]$, $[\chi_1, \chi_8, \chi_9 \bowtie \infty_5]$, $[\chi_1, \chi_5, \chi_{10} \bowtie \infty_3]$, $[\chi_2, \chi_6, \chi_9 \bowtie \infty_6]$, $[\chi_2, \chi_5, \chi_7 \bowtie \chi_{10}]$, $[\chi_2, \chi_8, \chi_{10} \bowtie \infty_1]$, $[\chi_3, \chi_6, \chi_5 \bowtie \chi_9]$, $[\chi_3, \chi_7, \chi_8 \bowtie \chi_6]$, $[\chi_3, \chi_9, \chi_{10} \bowtie \infty_5]$, $[\chi_4, \chi_6, \chi_{10} \bowtie \infty_4]$, $[\chi_4, \chi_7, \chi_9 \bowtie \infty_7]$, $[\chi_8, \chi_5, \chi_4 \bowtie \chi_1]$.

The set $\mathcal{D} \setminus \mathcal{A}$ is given by the following kites:

$[a_1, \infty_2, 16 \bowtie \chi_1]$, $[a_1, \infty_3, 15 \bowtie \chi_1]$, $[a_1, 14, \infty_4 \bowtie a_{15}]$, $[a_1, 13, \infty_5 \bowtie a_4]$, $[a_1, \infty_6, 12 \bowtie \chi_1]$, $[a_1, \infty_7, 11 \bowtie \chi_1]$, $[a_1, 10, 1 \bowtie \chi_1]$, $[a_1, 2, 9 \bowtie \chi_1]$, $[a_1, 8, 3 \bowtie \chi_1]$, $[a_1, 7, 4 \bowtie \chi_1]$, $[a_1, 6, 5 \bowtie a_{12}]$, $[a_2, \infty_4, 16 \bowtie \chi_2]$, $[a_2, \infty_5, 15 \bowtie \chi_2]$, $[a_2, 14, \infty_6 \bowtie a_5]$, $[a_2, \infty_7, 13 \bowtie \chi_1]$, $[a_2, 12, 1 \bowtie \chi_2]$, $[a_2, 11, 2 \bowtie a_9]$, $[a_2, 10, 3 \bowtie \chi_{10}]$, $[a_2, 4, 9 \bowtie \chi_2]$, $[a_2, 8, 5 \bowtie \chi_1]$, $[a_2, 6, 7 \bowtie a_{24}]$, $[a_3, \infty_6, 16 \bowtie \chi_3]$, $[a_3, \infty_7, 15 \bowtie \chi_3]$, $[a_3, 14, 1 \bowtie \chi_3]$, $[a_3, 2, 13 \bowtie a_{20}]$, $[a_3, 12, 3 \bowtie \chi_9]$, $[a_3, 11, 4 \bowtie a_{11}]$, $[a_3, 10, 5 \bowtie \chi_2]$, $[a_3, 6, 9 \bowtie \chi_3]$, $[a_3, 8, 7 \bowtie \chi_1]$, $[a_4, 16, 1 \bowtie \chi_4]$, $[a_4, 2, 15 \bowtie \chi_4]$, $[a_4, 14, 3 \bowtie \chi_2]$, $[a_4, 4, 13 \bowtie \chi_2]$, $[a_4, 12, 5 \bowtie \chi_3]$, $[a_4, 6, 11 \bowtie \chi_2]$, $[a_4, 10, 7 \bowtie a_{14}]$, $[a_4, 9, 8 \bowtie \chi_1]$, $[a_5, 1, \infty_2 \bowtie a_{14}]$, $[a_5, 2, \infty_1 \bowtie a_{24}]$, $[a_5, 16, 3 \bowtie \chi_3]$, $[a_5, 4, 15 \bowtie a_{22}]$, $[a_5, 5, 14 \bowtie \chi_1]$, $[a_5, 13, 6 \bowtie a_{13}]$, $[a_5, 7, 12 \bowtie a_{19}]$, $[a_5, 11, 8 \bowtie \chi_2]$, $[a_5, 9, 10 \bowtie \chi_1]$, $[a_6, \infty_4, 1 \bowtie \chi_5]$, $[a_6, 2, \infty_3 \bowtie a_2]$, $[a_6, 3, \infty_2 \bowtie a_{24}]$, $[a_6, \infty_1, 4 \bowtie \chi_2]$, $[a_6, 16, 5 \bowtie \chi_4]$, $[a_6, 15, 6 \bowtie \chi_1]$, $[a_6, 7, 14 \bowtie a_{21}]$, $[a_6, 8, 13 \bowtie a_{24}]$, $[a_6, 9, 12 \bowtie \chi_2]$, $[a_6, 11, 10 \bowtie \chi_2]$, $[a_7, \infty_6, 1 \bowtie \chi_6]$, $[a_7, 2, \infty_5 \bowtie a_{24}]$, $[a_7, 3, \infty_4 \bowtie a_3]$, $[a_7, \infty_3, 4 \bowtie \chi_3]$, $[a_7, 5, \infty_2 \bowtie a_2]$, $[a_7, \infty_1, 6 \bowtie \chi_2]$, $[a_7, 7, 16 \bowtie \chi_4]$, $[a_7, 15, 8 \bowtie a_{15}]$, $[a_7, 9, 14 \bowtie \chi_2]$, $[a_7, 10, 13 \bowtie \chi_3]$, $[a_7, 11, 12 \bowtie \chi_3]$, $[a_8, 2, \infty_7 \bowtie a_4]$, $[a_8, \infty_6, 3 \bowtie \chi_4]$, $[a_8, \infty_5, 4 \bowtie \chi_4]$, $[a_8, \infty_4, 5 \bowtie \chi_5]$, $[a_8, 6, \infty_3 \bowtie a_{15}]$, $[a_8, 7, \infty_2 \bowtie a_4]$, $[a_8, 8, \infty_1 \bowtie a_2]$, $[a_8, 9, 16 \bowtie \chi_5]$, $[a_8, 15, 10 \bowtie a_{17}]$, $[a_8, 11, 14 \bowtie a_{24}]$, $[a_8, 13, 12 \bowtie \chi_4]$, $[a_9, 1, 3 \bowtie \chi_5]$, $[a_9, \infty_7, 4 \bowtie \chi_5]$, $[a_9, \infty_6, 5 \bowtie \chi_6]$, $[a_9, 6, \infty_5 \bowtie a_3]$, $[a_9, \infty_4, 7 \bowtie \chi_2]$, $[a_9, \infty_3, 8 \bowtie \chi_3]$, $[a_9, \infty_2, 9 \bowtie \chi_4]$, $[a_9, \infty_1, 10 \bowtie \chi_3]$, $[a_9, 16, 11 \bowtie a_{18}]$, $[a_9, 12, 15 \bowtie a_{24}]$, $[a_9, 14, 13 \bowtie \chi_{10}]$, $[a_{10}, 2, 4 \bowtie \chi_6]$, $[a_{10}, 5, 1 \bowtie a_8]$, $[a_{10}, \infty_7, 6 \bowtie \chi_3]$, $[a_{10}, \infty_6, 7 \bowtie \chi_3]$, $[a_{10}, \infty_5, 8 \bowtie \chi_4]$, $[a_{10}, \infty_4, 9 \bowtie \chi_5]$, $[a_{10}, 10, \infty_3 \bowtie a_3]$, $[a_{10}, 11, \infty_2 \bowtie a_3]$, $[a_{10}, \infty_1, 12 \bowtie \chi_5]$, $[a_{10}, 13, 16 \bowtie a_{24}]$, $[a_{10}, 15, 14 \bowtie \chi_3]$, $[a_{11}, 5, 3 \bowtie \chi_6]$, $[a_{11}, 2, 6 \bowtie \chi_4]$, $[a_{11}, 7, 1 \bowtie \chi_7]$, $[a_{11}, \infty_7, 8 \bowtie \chi_5]$, $[a_{11}, \infty_6, 9 \bowtie \chi_6]$, $[a_{11}, \infty_5, 10 \bowtie \chi_4]$, $[a_{11}, \infty_4, 11 \bowtie \chi_3]$, $[a_{11}, \infty_3, 12 \bowtie \chi_6]$, $[a_{11}, \infty_2, 13 \bowtie \chi_4]$, $[a_{11}, 14, \infty_1 \bowtie a_1]$, $[a_{11}, 15, 16 \bowtie \chi_6]$, $[a_{12}, 4, 6 \bowtie \chi_5]$, $[a_{12}, 7, 3 \bowtie \chi_7]$, $[a_{12}, 8, 2 \bowtie \chi_{10}]$, $[a_{12}, 9, 1 \bowtie \chi_8]$, $[a_{12}, \infty_7, 10 \bowtie \chi_5]$, $[a_{12}, \infty_6, 11 \bowtie \chi_4]$, $[a_{12}, \infty_5, 12 \bowtie \chi_7]$, $[a_{12}, \infty_4, 13 \bowtie \chi_5]$, $[a_{12}, \infty_3, 14 \bowtie \chi_4]$, $[a_{12}, \infty_2, 15 \bowtie \chi_5]$, $[a_{12}, 16, \infty_1 \bowtie a_{14}]$, $[a_{13}, 5, 7 \bowtie \chi_4]$, $[a_{13}, 4, 8 \bowtie \chi_6]$, $[a_{13}, 9, 3 \bowtie \chi_8]$, $[a_{13}, 10, 2 \bowtie a_{24}]$, $[a_{13}, 11, 1 \bowtie \chi_9]$, $[a_{13}, \infty_7, 12 \bowtie \chi_8]$, $[a_{13}, \infty_6, 13 \bowtie \chi_6]$, $[a_{13}, \infty_5, 14 \bowtie \chi_5]$, $[a_{13}, 15, \infty_4 \bowtie a_4]$, $[a_{13}, \infty_3, 16 \bowtie \chi_7]$, $[a_{14}, 6, 8 \bowtie \chi_7]$, $[a_{14}, 9, 5 \bowtie a_{24}]$, $[a_{14}, 10, 4 \bowtie \chi_7]$, $[a_{14}, 11, 3 \bowtie a_{10}]$, $[a_{14}, 12, 2 \bowtie \chi_1]$, $[a_{14}, 1, 13 \bowtie \chi_7]$, $[a_{14}, \infty_7, 14 \bowtie \chi_6]$, $[a_{14}, \infty_6, 15 \bowtie \chi_6]$, $[a_{14}, \infty_5, 16 \bowtie \chi_8]$, $[a_{15}, 9, 7 \bowtie \chi_5]$, $[a_{15}, 10, 6 \bowtie \chi_6]$, $[a_{15}, 5, 11 \bowtie \chi_5]$, $[a_{15}, 4, 12 \bowtie \chi_9]$, $[a_{15}, 3, 13 \bowtie \chi_8]$, $[a_{15}, 14, 2 \bowtie \chi_2]$, $[a_{15}, 15, 1 \bowtie \chi_{10}]$, $[a_{15}, \infty_7, 16 \bowtie \chi_9]$, $[a_{16}, 8, 10 \bowtie \chi_6]$, $[a_{16}, 11, 7 \bowtie \chi_6]$, $[a_{16}, 12, 6 \bowtie a_{24}]$, $[a_{16}, 5, 13 \bowtie \chi_9]$, $[a_{16}, 14, 4 \bowtie \chi_8]$, $[a_{16}, 3, 15 \bowtie \chi_7]$, $[a_{16}, 16, 2 \bowtie \chi_3]$, $[a_{16}, 1, \infty_1 \bowtie a_4]$, $[a_{17}, 9, 11 \bowtie \chi_6]$, $[a_{17}, 12, 8 \bowtie a_{24}]$, $[a_{17}, 13, 7 \bowtie \chi_7]$, $[a_{17}, 14, 6 \bowtie \chi_7]$, $[a_{17}, 5, 15 \bowtie \chi_8]$, $[a_{17}, 16, 4 \bowtie \chi_9]$, $[a_{17}, 3, \infty_1 \bowtie a_{15}]$, $[a_{17}, 2, \infty_2 \bowtie a_{15}]$, $[a_{17}, 1, \infty_3 \bowtie a_{24}]$, $[a_{18}, 12, 10 \bowtie a_{24}]$, $[a_{18}, 13, 9 \bowtie a_{24}]$, $[a_{18}, 8, 14 \bowtie \chi_7]$, $[a_{18}, 15, 7 \bowtie \chi_8]$, $[a_{18}, 16, 6 \bowtie \chi_8]$, $[a_{18}, \infty_1, 5 \bowtie \chi_7]$, $[a_{18}, 4, \infty_2 \bowtie a_{16}]$, $[a_{18}, 3, \infty_3 \bowtie a_4]$, $[a_{18}, \infty_4, 2 \bowtie \chi_4]$, $[a_{18}, 1, \infty_5 \bowtie a_5]$, $[a_{19}, 13, 11 \bowtie a_{24}]$, $[a_{19}, 10, 14 \bowtie \chi_8]$, $[a_{19}, 15, 9 \bowtie a_{16}]$, $[a_{19}, 8, 16 \bowtie a_{23}]$, $[a_{19}, \infty_1, 7 \bowtie \chi_9]$, $[a_{19}, 6, \infty_2 \bowtie a_{13}]$, $[a_{19}, \infty_3, 5 \bowtie \chi_8]$, $[a_{19}, 4, \infty_4 \bowtie a_{24}]$, $[a_{19}, 3, \infty_5 \bowtie a_6]$, $[a_{19}, \infty_6, 2 \bowtie \chi_5]$, $[a_{19}, 1, \infty_7 \bowtie a_{24}]$, $[a_{20}, 14, 12 \bowtie a_{24}]$, $[a_{20}, 15, 11 \bowtie \chi_7]$, $[a_{20}, 16, 10 \bowtie \chi_7]$, $[a_{20}, \infty_1, 9 \bowtie \chi_7]$, $[a_{20}, \infty_2, 8 \bowtie \chi_8]$, $[a_{20}, 7, \infty_3 \bowtie a_5]$, $[a_{20}, 6, \infty_4 \bowtie a_5]$,

$[a_{20}, \infty_5, 5 \bowtie \chi_9]$, $[a_{20}, 4, \infty_6 \bowtie a_{24}]$, $[a_{20}, 3, \infty_7 \bowtie a_6]$, $[a_{20}, 1, 2 \bowtie \chi_6]$, $[a_{21}, 13, 15 \bowtie \chi_9]$, $[a_{21}, 12, 16 \bowtie \chi_{10}]$, $[a_{21}, \infty_1, 11 \bowtie \chi_8]$, $[a_{21}, \infty_2, 10 \bowtie \chi_8]$, $[a_{21}, \infty_3, 9 \bowtie \chi_8]$, $[a_{21}, \infty_4, 8 \bowtie \chi_9]$, $[a_{21}, \infty_5, 7 \bowtie \chi_{10}]$, $[a_{21}, 6, \infty_6 \bowtie a_4]$, $[a_{21}, \infty_7, 5 \bowtie \chi_{10}]$, $[a_{21}, 1, 4 \bowtie \chi_{10}]$, $[a_{21}, 3, 2 \bowtie \chi_7]$, $[a_{22}, 16, 14 \bowtie \chi_9]$, $[a_{22}, 13, \infty_1 \bowtie a_3]$, $[a_{22}, \infty_2, 12 \bowtie \chi_{10}]$, $[a_{22}, \infty_3, 11 \bowtie \chi_9]$, $[a_{22}, \infty_4, 10 \bowtie \chi_9]$, $[a_{22}, \infty_5, 9 \bowtie \chi_9]$, $[a_{22}, \infty_6, 8 \bowtie \chi_{10}]$, $[a_{22}, 7, \infty_7 \bowtie a_5]$, $[a_{22}, 1, 6 \bowtie \chi_9]$, $[a_{22}, 5, 2 \bowtie \chi_8]$, $[a_{22}, 4, 3 \bowtie a_{24}]$, $[a_{23}, \infty_1, 15 \bowtie \chi_{10}]$, $[a_{23}, \infty_2, 14 \bowtie \chi_{10}]$, $[a_{23}, 13, \infty_3 \bowtie a_{14}]$, $[a_{23}, 12, \infty_4 \bowtie a_{14}]$, $[a_{23}, \infty_5, 11 \bowtie \chi_{10}]$, $[a_{23}, \infty_6, 10 \bowtie \chi_{10}]$, $[a_{23}, \infty_7, 9 \bowtie \chi_{10}]$, $[a_{23}, 8, 1 \bowtie a_{24}]$, $[a_{23}, 7, 2 \bowtie \chi_9]$, $[a_{23}, 3, 6 \bowtie \chi_{10}]$, $[a_{23}, 5, 4 \bowtie a_{24}]$, $[\chi_1, a_1, a_{14} \bowtie a_7]$, $[a_2, a_8, a_{15} \bowtie \infty_6]$, $[a_3, a_9, a_{16} \bowtie \infty_3]$, $[a_4, a_{17}, a_{10} \bowtie \chi_9]$, $[a_5, a_{18}, a_{11} \bowtie \chi_9]$, $[a_{12}, a_{19}, a_6 \bowtie \infty_6]$, $[a_{13}, a_{20}, a_7 \bowtie \infty_7]$, $[\chi_1, a_8, a_{21} \bowtie a_{14}]$, $[\chi_1, a_9, a_{22} \bowtie a_{15}]$, $[a_{10}, a_{23}, a_{16} \bowtie \infty_5]$, $[a_{11}, a_{24}, a_{17} \bowtie \infty_6]$, $[a_{19}, a_1, a_{13} \bowtie \infty_1]$, $[\chi_2, a_{14}, a_2 \bowtie a_{20}]$, $[a_{21}, a_3, a_{15} \bowtie \infty_5]$, $[a_{22}, a_4, a_{16} \bowtie \infty_4]$, $[a_{23}, a_5, a_{17} \bowtie \infty_5]$, $[a_{24}, a_6, a_{18} \bowtie \infty_6]$, $[\chi_2, a_{20}, a_8 \bowtie a_1]$, $[\chi_2, a_{21}, a_9 \bowtie a_2]$, $[\chi_2, a_{22}, a_{10} \bowtie a_3]$, $[\chi_1, a_{23}, a_{11} \bowtie a_4]$, $[\chi_1, a_{24}, a_{12} \bowtie a_5]$, $[\chi_2, a_1, a_{15} \bowtie a_{10}]$, $[a_2, a_{11}, a_{16} \bowtie \infty_6]$, $[a_3, a_{12}, a_{17} \bowtie \infty_7]$, $[a_4, a_{13}, a_{18} \bowtie \infty_7]$, $[\chi_1, a_5, a_{19} \bowtie a_{14}]$, $[\chi_1, a_6, a_{20} \bowtie a_{15}]$, $[a_7, a_{21}, a_{16} \bowtie \infty_7]$, $[\chi_3, a_8, a_{22} \bowtie a_{17}]$, $[\chi_9, a_{18}, a_{23} \bowtie \chi_8]$, $[\chi_5, a_{12}, a_1 \bowtie \chi_9]$, $[a_{13}, a_{22}, a_2 \bowtie \chi_8]$, $[\chi_6, a_{15}, a_4 \bowtie \chi_7]$, $[\chi_5, a_{17}, a_6 \bowtie a_1]$, $[a_2, a_7, a_{18} \bowtie a_9]$, $[a_{19}, a_3, a_8 \bowtie a_{14}]$, $[a_{20}, a_9, a_4 \bowtie a_8]$, $[a_{21}, a_{10}, a_5 \bowtie a_{22}]$, $[a_6, a_{11}, a_{22} \bowtie a_7]$, $[a_{23}, a_{12}, a_7 \bowtie a_5]$, $[a_8, a_{13}, a_{24} \bowtie a_{10}]$, $[a_1, a_9, a_{11} \bowtie \chi_7]$, $[\chi_1, a_2, a_{10} \bowtie a_{12}]$, $[\chi_2, a_{11}, a_3 \bowtie \chi_6]$, $[a_5, a_{13}, a_{15} \bowtie \chi_9]$, $[\chi_5, a_{16}, a_{14} \bowtie a_6]$, $[a_7, a_{15}, a_{17} \bowtie \chi_9]$, $[\chi_1, a_{18}, a_{16} \bowtie a_8]$, $[\chi_2, a_{19}, a_{17} \bowtie a_9]$, $[\chi_6, a_{20}, a_{18} \bowtie a_{10}]$, $[a_{11}, a_{21}, a_{19} \bowtie \chi_9]$, $[a_{12}, a_{22}, a_{20} \bowtie \chi_9]$, $[a_{13}, a_{21}, a_{23} \bowtie a_{14}]$, $[a_{24}, a_{22}, a_{14} \bowtie a_3]$, $[a_{16}, a_1, a_{24} \bowtie \chi_4]$, $[a_3, a_1, a_{18} \bowtie \chi_4]$, $[\chi_5, a_4, a_2 \bowtie a_{19}]$, $[a_{20}, a_3, a_5 \bowtie \chi_2]$, $[\chi_4, a_6, a_4 \bowtie a_{21}]$, $[a_8, a_6, a_{23} \bowtie a_3]$, $[a_9, a_7, a_{24} \bowtie a_{19}]$, $[a_1, a_5, a_4 \bowtie a_{14}]$, $[a_6, a_5, a_2 \bowtie \chi_9]$, $[\chi_7, a_5, a_8 \bowtie a_9]$, $[a_{10}, a_9, a_6 \bowtie \chi_9]$, $[\chi_4, a_{11}, a_{10} \bowtie a_7]$, $[\chi_8, a_{12}, a_{11} \bowtie a_8]$, $[a_9, a_{13}, a_{12} \bowtie a_4]$, $[\chi_5, a_{10}, a_{13} \bowtie a_{14}]$, $[a_{11}, a_{15}, a_{14} \bowtie a_{12}]$, $[a_{12}, a_{15}, a_{16} \bowtie a_{17}]$, $[\chi_7, a_{18}, a_{17} \bowtie a_{14}]$, $[a_{15}, a_{18}, a_{19} \bowtie a_{10}]$, $[\chi_4, a_{16}, a_{19} \bowtie a_{20}]$, $[a_{21}, a_{20}, a_{17} \bowtie \infty_4]$, $[a_{18}, a_{21}, a_{22} \bowtie \chi_9]$, $[a_{20}, a_{23}, a_{24} \bowtie \chi_5]$, $[a_{23}, a_2, a_1 \bowtie a_{10}]$, $[\chi_7, a_3, a_2 \bowtie a_{24}]$, $[\chi_{10}, a_{13}, a_6 \bowtie \chi_2]$, $[\chi_{10}, a_{18}, a_{12} \bowtie \chi_2]$, $[\chi_{10}, a_{19}, a_7 \bowtie \chi_1]$, $[\chi_{10}, a_{14}, a_9 \bowtie \chi_3]$, $[\chi_{10}, a_{20}, a_{11} \bowtie \chi_3]$, $[\chi_{10}, a_5, a_{16} \bowtie \chi_6]$, $[\chi_{10}, a_8, a_{10} \bowtie \chi_3]$, $[\chi_{10}, a_{17}, a_2 \bowtie \chi_6]$, $[\chi_{10}, a_{23}, a_{15} \bowtie \chi_1]$, $[\chi_{10}, a_4, a_3 \bowtie \chi_1]$, $[\chi_{10}, a_{21}, a_{24} \bowtie \chi_2]$, $[\chi_{10}, a_1, a_{22} \bowtie \chi_9]$, $[\chi_3, a_1, a_7 \bowtie \chi_2]$, $[\chi_3, a_6, a_{15} \bowtie \chi_4]$, $[\chi_3, a_{20}, a_{14} \bowtie \chi_4]$, $[\chi_3, a_2, a_{21} \bowtie \chi_4]$, $[\chi_2, a_{23}, a_4 \bowtie \chi_1]$, $[\chi_3, a_3, a_{13} \bowtie \chi_1]$, $[\chi_9, a_9, a_5 \bowtie \chi_8]$, $[\chi_8, a_{24}, a_3 \bowtie \chi_4]$, $[\chi_9, a_{12}, a_8 \bowtie \chi_6]$, $[\chi_3, a_{24}, a_5 \bowtie \chi_5]$, $[\chi_4, a_8, a_{17} \bowtie \chi_1]$, $[\chi_4, a_2, a_{12} \bowtie \chi_3]$, $[\chi_4, a_1, a_{20} \bowtie \chi_5]$, $[\chi_5, a_3, a_{22} \bowtie \chi_4]$, $[\chi_5, a_{15}, a_9 \bowtie \chi_4]$, $[\chi_6, a_1, a_{17} \bowtie \chi_3]$, $[\chi_5, a_8, a_{18} \bowtie \chi_2]$, $[\chi_6, a_9, a_{19} \bowtie \chi_3]$, $[\chi_7, a_{20}, a_{16} \bowtie \chi_3]$, $[\chi_7, a_{24}, a_{15} \bowtie \chi_8]$, $[\chi_5, a_{19}, a_{23} \bowtie \chi_4]$, $[\chi_7, a_{19}, a_{22} \bowtie \chi_8]$, $[\chi_6, a_{22}, a_{23} \bowtie \chi_3]$, $[\chi_8, a_{14}, a_{18} \bowtie \chi_3]$, $[\chi_5, a_{11}, a_7 \bowtie \chi_4]$, $[\chi_6, a_6, a_{21} \bowtie \chi_5]$, $[\chi_8, a_{20}, a_{10} \bowtie \chi_6]$, $[\chi_8, a_{19}, a_4 \bowtie \chi_3]$, $[\chi_7, a_{21}, a_{12} \bowtie \chi_6]$, $[\chi_7, a_{10}, a_{14} \bowtie \chi_9]$, $[\chi_6, a_{11}, a_{13} \bowtie \chi_7]$, $[\chi_9, a_4, a_{24} \bowtie \chi_6]$, $[\chi_8, a_{21}, a_1 \bowtie \chi_7]$, $[\chi_7, a_{23}, a_9 \bowtie \chi_8]$, $[\chi_9, a_{16}, a_{13} \bowtie \chi_2]$, $[\chi_8, a_{17}, a_{13} \bowtie \chi_4]$, $[\chi_6, a_{14}, a_5 \bowtie \chi_4]$, $[\chi_8, a_8, a_7 \bowtie \chi_6]$, $[\chi_8, a_6, a_{16} \bowtie \chi_2]$, $[\chi_7, a_6, a_7 \bowtie a_4]$, $[\chi_9, a_7, a_3 \bowtie a_6]$.

Case $\alpha = 10$ and $\beta = 14$. Let $X = \{\infty_1, \infty_2, \ldots, \infty_{10}\}$, $Y = \{\chi_1, \chi_2, \ldots, \chi_{14}\}$. The set $\mathcal{A}$ is given by the following kites:

$[\chi_1, \infty_2, \infty_1 \bowtie \chi_{10}]$, $[\chi_1, \infty_{10}, \infty_3 \bowtie \chi_3]$, $[\chi_1, \infty_9, \infty_4 \bowtie \chi_{12}]$, $[\chi_1, \infty_8, \infty_5 \bowtie \chi_{13}]$, $[\chi_1, \infty_7, \infty_6 \bowtie \chi_{12}]$, $[\chi_2, \infty_3, \infty_1 \bowtie \chi_{11}]$, $[\chi_2, \infty_4, \infty_2 \bowtie \chi_{10}]$, $[\chi_2, \infty_{10}, \infty_5 \bowtie \chi_{14}]$, $[\chi_2, \infty_9, \infty_6 \bowtie \chi_{13}]$, $[\chi_2, \infty_7, \infty_8 \bowtie \chi_{10}]$, $[\chi_3, \infty_4, \infty_1 \bowtie \chi_{12}]$, $[\chi_3, \infty_5, \infty_3 \bowtie \chi_{14}]$, $[\chi_3, \infty_6, \infty_2 \bowtie \chi_{11}]$, $[\chi_3, \infty_7, \infty_{10} \bowtie \chi_{10}]$, $[\chi_3, \infty_9, \infty_8 \bowtie \chi_{11}]$, $[\chi_4, \infty_5, \infty_1 \bowtie \chi_{13}]$, $[\chi_4, \infty_6, \infty_4 \bowtie \chi_{13}]$, $[\chi_4, \infty_3, \infty_7 \bowtie \chi_{14}]$, $[\chi_4, \infty_8, \infty_2 \bowtie \chi_{12}]$, $[\chi_4, \infty_{10}, \infty_9 \bowtie \chi_{14}]$, $[\chi_5, \infty_6, \infty_1 \bowtie \chi_{14}]$, $[\chi_5, \infty_5, \infty_7 \bowtie \chi_{10}]$, $[\chi_5, \infty_8, \infty_4 \bowtie \chi_{14}]$, $[\chi_5, \infty_3, \infty_9 \bowtie \chi_{13}]$, $[\chi_5, \infty_{10}, \infty_2 \bowtie \chi_{13}]$, $[\chi_6, \infty_1, \infty_7 \bowtie \chi_{11}]$, $[\chi_6, \infty_8, \infty_6 \bowtie \chi_{10}]$, $[\chi_6, \infty_9, \infty_5 \bowtie \chi_{10}]$, $[\chi_6, \infty_4, \infty_{10} \bowtie \chi_{11}]$, $[\chi_6, \infty_3, \infty_2 \bowtie$

$\chi_{14}], [\chi_7, \infty_1, \infty_8 \bowtie \chi_{12}], [\chi_7, \infty_9, \infty_7 \bowtie \chi_{12}], [\chi_7, \infty_{10}, \infty_6 \bowtie \chi_{11}], [\chi_7, \infty_2, \infty_5 \bowtie \chi_{11}], [\chi_7, \infty_4, \infty_3 \bowtie$
$\chi_{10}], [\chi_8, \infty_1, \infty_9 \bowtie \chi_{12}], [\chi_8, \infty_{10}, \infty_8 \bowtie \chi_{13}], [\chi_8, \infty_2, \infty_7 \bowtie \chi_{13}], [\chi_8, \infty_6, \infty_3 \bowtie \chi_{11}], [\chi_8, \infty_5, \infty_4 \bowtie$
$\chi_{10}], [\chi_9, \infty_1, \infty_{10} \bowtie \chi_{13}], [\chi_9, \infty_2, \infty_9 \bowtie \chi_{11}], [\chi_9, \infty_8, \infty_3 \bowtie \chi_{12}], [\chi_9, \infty_7, \infty_4 \bowtie \chi_{11}], [\chi_9, \infty_6, \infty_5 \bowtie$
$\chi_{12}], [\chi_1, \chi_3, \chi_9 \bowtie \chi_7], [\chi_4, \chi_{10}, \chi_2 \bowtie \chi_8], [\chi_3, \chi_5, \chi_{11} \bowtie \chi_1], [\chi_4, \chi_{12}, \chi_6 \bowtie \chi_{13}], [\chi_5, \chi_7, \chi_{13} \bowtie$
$\infty_{10}], [\chi_6, \chi_8, \chi_{14} \bowtie \infty_{10}], [\chi_1, \chi_8, \chi_{10} \bowtie \infty_9], [\chi_9, \chi_{11}, \chi_2 \bowtie \chi_7], [\chi_{10}, \chi_{12}, \chi_3 \bowtie \chi_8], [\chi_{11}, \chi_{13}, \chi_4 \bowtie$
$\chi_9], [\chi_5, \chi_{12}, \chi_{14} \bowtie \infty_8], [\chi_1, \chi_7, \chi_{14} \bowtie \chi_4], [\chi_2, \chi_5, \chi_1 \bowtie \chi_4], [\chi_2, \chi_3, \chi_6 \bowtie \chi_1], [\chi_4, \chi_7, \chi_3 \bowtie \chi_{13}],$
$[\chi_4, \chi_5, \chi_8 \bowtie \chi_{13}], [\chi_6, \chi_9, \chi_5 \bowtie \chi_{10}], [\chi_8, \chi_7, \chi_{11} \bowtie \chi_6], [\chi_6, \chi_{10}, \chi_7 \bowtie \chi_{12}], [\chi_8, \chi_9, \chi_{12} \bowtie \chi_{11}],$
$[\chi_{10}, \chi_{13}, \chi_9 \bowtie \chi_{14}], [\chi_{10}, \chi_{11}, \chi_{14} \bowtie \chi_3], [\chi_1, \chi_{13}, \chi_{12} \bowtie \chi_2], [\chi_2, \chi_{13}, \chi_{14} \bowtie \infty_6].$

Let $\mathcal{D}_1$ be the set of following kites:

$[a_1, 1, \infty_1 \bowtie a_{17}], [a_1, 9, \infty_2 \bowtie a_9], [a_1, 8, \infty_5 \bowtie a_{10}], [a_1, 16, \infty_6 \bowtie a_2], [a_1, 7, \infty_8 \bowtie a_{11}],$
$[a_1, 15, \infty_9 \bowtie a_3], [a_2, 2, \infty_1 \bowtie a_{18}], [a_2, 10, \infty_2 \bowtie a_{10}], [a_2, 1, \infty_4 \bowtie a_1], [a_2, 9, \infty_5 \bowtie a_{11}],$
$[a_2, \infty_8, 8 \bowtie a_4], [a_2, 16, \infty_9 \bowtie a_5], [a_3, 3, \infty_1 \bowtie a_{19}], [a_3, 11, \infty_2 \bowtie a_{11}], [a_3, 2, \infty_4 \bowtie a_{18}],$
$[a_3, 10, \infty_5 \bowtie a_{12}], [a_3, 1, \infty_7 \bowtie a_1], [a_3, 9, \infty_8 \bowtie a_{12}], [a_4, 4, \infty_1 \bowtie a_{20}], [a_4, 12, \infty_2 \bowtie a_{12}],$
$[a_4, 3, \infty_4 \bowtie a_{19}], [a_4, 11, \infty_5 \bowtie a_{13}], [a_4, 2, \infty_7 \bowtie a_2], [a_4, 10, \infty_8 \bowtie a_{13}], [a_4, 1, \infty_{10} \bowtie a_1],$
$[a_5, 5, \infty_1 \bowtie a_{21}], [a_5, 13, \infty_2 \bowtie a_{13}], [a_5, 4, \infty_4 \bowtie a_{20}], [a_5, 12, \infty_5 \bowtie a_{14}], [a_5, 3, \infty_7 \bowtie a_{19}],$
$[a_5, 11, \infty_8 \bowtie a_{14}], [a_5, 2, \infty_{10} \bowtie a_2], [a_5, 16, 1 \bowtie a_7], [a_6, 6, \infty_1 \bowtie a_{22}], [a_6, 14, \infty_2 \bowtie a_{14}],$
$[a_6, \infty_4, 5 \bowtie a_1], [a_6, 13, \infty_5 \bowtie a_{15}], [a_6, 4, \infty_7 \bowtie a_{20}], [a_6, 12, \infty_8 \bowtie a_{15}], [a_6, 3, \infty_{10} \bowtie a_3],$
$[a_6, 15, 2 \bowtie a_8], [a_7, \infty_1, 7 \bowtie a_2], [a_7, 15, \infty_2 \bowtie a_{15}], [a_7, 6, \infty_4 \bowtie a_{21}], [a_7, 14, \infty_5 \bowtie a_{16}],$
$[a_7, 5, \infty_7 \bowtie a_{21}], [a_7, 13, \infty_8 \bowtie a_{16}], [a_7, 4, \infty_{10} \bowtie a_{20}], [a_8, 8, \infty_1 \bowtie a_{23}], [a_8, 16, \infty_2 \bowtie a_{16}],$
$[a_8, \infty_4, 7 \bowtie a_3], [a_8, 15, \infty_5 \bowtie a_{17}], [a_8, \infty_7, 6 \bowtie a_1], [a_8, 14, \infty_8 \bowtie a_{17}], [a_8, 5, \infty_{10} \bowtie a_{21}],$
$[a_8, 4, 13 \bowtie a_1], [a_9, 9, \infty_1 \bowtie a_{24}], [a_9, 1, \infty_3 \bowtie a_1], [a_9, \infty_4, 8 \bowtie a_3], [a_9, \infty_5, 16 \bowtie a_3], [a_9, \infty_7, 7 \bowtie$
$a_4], [a_9, 15, \infty_8 \bowtie a_{18}], [a_9, \infty_{10}, 6 \bowtie a_2], [a_9, 5, 12 \bowtie a_1], [a_{10}, \infty_1, 10 \bowtie a_1], [a_{10}, 2, \infty_3 \bowtie a_3],$
$[a_{10}, \infty_4, 9 \bowtie a_4], [a_{10}, 1, \infty_6 \bowtie a_3], [a_{10}, 8, \infty_7 \bowtie a_{22}], [a_{10}, \infty_8, 16 \bowtie a_4], [a_{10}, \infty_{10}, 7 \bowtie a_5],$
$[a_{10}, 6, 11 \bowtie a_1], [a_{11}, \infty_1, 11 \bowtie a_2], [a_{11}, 3, \infty_3 \bowtie a_4], [a_{11}, \infty_4, 10 \bowtie a_5], [a_{11}, 2, \infty_6 \bowtie a_4],$
$[a_{11}, \infty_7, 9 \bowtie a_5], [a_{11}, 1, \infty_9 \bowtie a_4], [a_{11}, \infty_{10}, 8 \bowtie a_5], [a_{12}, \infty_1, 12 \bowtie a_2], [a_{12}, 4, \infty_3 \bowtie a_2],$
$[a_{12}, 11, \infty_4 \bowtie a_{22}], [a_{12}, 3, \infty_6 \bowtie a_5], [a_{12}, \infty_7, 10 \bowtie a_6], [a_{12}, 2, \infty_9 \bowtie a_6], [a_{12}, \infty_{10}, 9 \bowtie a_6],$
$[a_{13}, \infty_1, 13 \bowtie a_2], [a_{13}, 5, \infty_3 \bowtie a_5], [a_{13}, 12, \infty_4 \bowtie a_{23}], [a_{13}, 4, \infty_6 \bowtie a_6], [a_{13}, \infty_7, 11 \bowtie a_6],$
$[a_{13}, 3, \infty_9 \bowtie a_7], [a_{13}, \infty_{10}, 10 \bowtie a_7], [a_{14}, \infty_1, 14 \bowtie a_1], [a_{14}, 6, \infty_3 \bowtie a_6], [a_{14}, 13, \infty_4 \bowtie a_{24}],$
$[a_{14}, 5, \infty_6 \bowtie a_7], [a_{14}, \infty_7, 12 \bowtie a_3], [a_{14}, 4, \infty_9 \bowtie a_8], [a_{14}, \infty_{10}, 11 \bowtie a_7], [a_{14}, 7, 10 \bowtie a_{24}],$
$[a_{15}, \infty_1, 15 \bowtie a_2], [a_{15}, 7, \infty_3 \bowtie a_7], [a_{15}, \infty_4, 14 \bowtie a_3], [a_{15}, 6, \infty_6 \bowtie a_8], [a_{15}, \infty_7, 13 \bowtie a_3],$
$[a_{15}, 5, \infty_9 \bowtie a_9], [a_{15}, \infty_{10}, 12 \bowtie a_8], [a_{15}, 9, 8 \bowtie a_{14}], [a_{16}, \infty_1, 16 \bowtie a_6], [a_{16}, 8, \infty_3 \bowtie a_8],$
$[a_{16}, \infty_4, 15 \bowtie a_3], [a_{16}, 7, \infty_6 \bowtie a_9], [a_{16}, \infty_7, 14 \bowtie a_2], [a_{16}, 6, \infty_9 \bowtie a_{10}], [a_{16}, 13, \infty_{10} \bowtie a_{22}],$
$[a_{17}, \infty_2, 1 \bowtie a_6], [a_{17}, \infty_3, 9 \bowtie a_7], [a_{17}, \infty_4, 16 \bowtie a_7], [a_{17}, \infty_6, 8 \bowtie a_6], [a_{17}, 15, \infty_7 \bowtie a_{24}],$
$[a_{17}, \infty_9, 7 \bowtie a_6], [a_{17}, 14, \infty_{10} \bowtie a_{23}], [a_{18}, \infty_2, 2 \bowtie a_7], [a_{18}, \infty_3, 10 \bowtie a_8], [a_{18}, \infty_5, 1 \bowtie a_8],$
$[a_{18}, \infty_6, 9 \bowtie a_8], [a_{18}, 16, \infty_7 \bowtie a_{23}], [a_{18}, \infty_9, 8 \bowtie a_7], [a_{18}, \infty_{10}, 15 \bowtie a_4], [a_{18}, 14, 3 \bowtie a_9],$
$[a_{19}, \infty_2, 3 \bowtie a_2], [a_{19}, \infty_3, 11 \bowtie a_8], [a_{19}, \infty_5, 2 \bowtie a_9], [a_{19}, \infty_6, 10 \bowtie a_9], [a_{19}, \infty_8, 1 \bowtie a_{12}],$
$[a_{19}, \infty_9, 9 \bowtie a_{13}], [a_{19}, 16, \infty_{10} \bowtie a_{24}], [a_{20}, \infty_2, 4 \bowtie a_2], [a_{20}, \infty_3, 12 \bowtie a_{10}], [a_{20}, \infty_5, 3 \bowtie a_8],$
$[a_{20}, \infty_6, 11 \bowtie a_{15}], [a_{20}, \infty_8, 2 \bowtie a_{13}], [a_{20}, \infty_9, 10 \bowtie a_{15}], [a_{21}, \infty_2, 5 \bowtie a_2], [a_{21}, \infty_3, 13 \bowtie a_4],$
$[a_{21}, \infty_5, 4 \bowtie a_3], [a_{21}, \infty_6, 12 \bowtie a_7], [a_{21}, \infty_8, 3 \bowtie a_7], [a_{21}, \infty_9, 11 \bowtie a_9], [a_{22}, \infty_2, 6 \bowtie a_4],$
$[a_{22}, \infty_3, 14 \bowtie a_4], [a_{22}, \infty_5, 5 \bowtie a_3], [a_{22}, \infty_6, 13 \bowtie a_9], [a_{22}, \infty_8, 4 \bowtie a_9], [a_{22}, \infty_9, 12 \bowtie a_{11}],$
$[a_{23}, \infty_2, 7 \bowtie a_{11}], [a_{23}, \infty_3, 15 \bowtie a_{10}], [a_{23}, \infty_5, 6 \bowtie a_5], [a_{23}, \infty_6, 14 \bowtie a_5], [a_{23}, \infty_8, 5 \bowtie a_4],$
$[a_{23}, \infty_9, 13 \bowtie a_{10}], [a_{24}, \infty_2, 8 \bowtie a_{12}], [a_{24}, \infty_3, 16 \bowtie a_{11}], [a_{24}, \infty_5, 7 \bowtie a_{12}], [a_{24}, \infty_6, 15 \bowtie a_5],$

$[a_{24}, \infty_8, 6 \bowtie a_3]$, $[a_{24}, \infty_9, 14 \bowtie a_9]$, $[\chi_1, 16, 2 \bowtie a_1]$, $[\chi_1, 1, 3 \bowtie a_1]$, $[\chi_1, 15, 4 \bowtie a_1]$, $[\chi_1, 14, 5 \bowtie a_{10}]$, $[\chi_1, 13, 6 \bowtie a_{11}]$, $[\chi_1, 12, 7 \bowtie a_{13}]$, $[\chi_1, 11, 8 \bowtie a_{13}]$, $[\chi_1, 10, 9 \bowtie a_{14}]$, $[\chi_2, 16, 3 \bowtie a_{10}]$, $[\chi_2, 2, 4 \bowtie a_{10}]$, $[\chi_2, 1, 5 \bowtie a_{12}]$, $[\chi_2, 6, 15 \bowtie a_{11}]$, $[\chi_2, 7, 14 \bowtie a_{10}]$, $[\chi_2, 8, 13 \bowtie a_{11}]$, $[\chi_2, 12, 9 \bowtie a_{16}]$, $[\chi_2, 11, 10 \bowtie a_{16}]$, $[\chi_3, 16, 4 \bowtie a_{11}]$, $[\chi_3, 3, 5 \bowtie a_{11}]$, $[\chi_3, 2, 6 \bowtie a_{12}]$, $[\chi_3, 7, 1 \bowtie a_{14}]$, $[\chi_3, 8, 15 \bowtie a_{12}]$, $[\chi_3, 9, 14 \bowtie a_{11}]$, $[\chi_3, 10, 13 \bowtie a_{12}]$, $[\chi_3, 12, 11 \bowtie a_{16}]$, $[\chi_4, 5, 16 \bowtie a_{12}]$, $[\chi_4, 4, 6 \bowtie a_{13}]$, $[\chi_4, 7, 3 \bowtie a_{14}]$, $[\chi_4, 8, 2 \bowtie a_{14}]$, $[\chi_4, 9, 1 \bowtie a_{13}]$, $[\chi_4, 10, 15 \bowtie a_{14}]$, $[\chi_4, 11, 14 \bowtie a_{12}]$, $[\chi_4, 12, 13 \bowtie a_{17}]$, $[\chi_5, 6, 16 \bowtie a_{13}]$, $[\chi_5, 7, 5 \bowtie a_{17}]$, $[\chi_5, 8, 4 \bowtie a_{15}]$, $[\chi_5, 9, 3 \bowtie a_{15}]$, $[\chi_5, 10, 2 \bowtie a_{15}]$, $[\chi_5, 11, 1 \bowtie a_{15}]$, $[\chi_5, 12, 15 \bowtie a_{13}]$, $[\chi_5, 13, 14 \bowtie a_{13}]$, $[\chi_6, 7, 16 \bowtie a_{14}]$, $[\chi_6, 8, 6 \bowtie a_{17}]$, $[\chi_6, 9, 5 \bowtie a_{16}]$, $[\chi_6, 10, 4 \bowtie a_{16}]$, $[\chi_6, 11, 3 \bowtie a_{16}]$, $[\chi_6, 12, 2 \bowtie a_{16}]$, $[\chi_6, 13, 1 \bowtie a_{16}]$, $[\chi_6, 15, 14 \bowtie a_{19}]$, $[\chi_7, 16, 8 \bowtie a_{19}]$, $[\chi_7, 9, 7 \bowtie a_{18}]$, $[\chi_7, 6, 10 \bowtie a_{17}]$, $[\chi_7, 5, 11 \bowtie a_{17}]$, $[\chi_7, 4, 12 \bowtie a_{16}]$, $[\chi_7, 13, 3 \bowtie a_{17}]$, $[\chi_7, 14, 2 \bowtie a_{17}]$, $[\chi_7, 1, 15 \bowtie a_{19}]$, $[\chi_8, 16, 9 \bowtie a_{21}]$, $[\chi_8, 8, 10 \bowtie a_{21}]$, $[\chi_8, 7, 11 \bowtie a_{18}]$, $[\chi_8, 6, 12 \bowtie a_{17}]$, $[\chi_8, 13, 5 \bowtie a_{18}]$, $[\chi_8, 14, 4 \bowtie a_{17}]$, $[\chi_8, 3, 15 \bowtie a_{20}]$, $[\chi_8, 2, 1 \bowtie a_{21}]$, $[\chi_9, 10, 16 \bowtie a_{20}]$, $[\chi_9, 11, 9 \bowtie a_{20}]$, $[\chi_9, 8, 12 \bowtie a_{18}]$, $[\chi_9, 7, 13 \bowtie a_{18}]$, $[\chi_9, 14, 6 \bowtie a_{18}]$, $[\chi_9, 15, 5 \bowtie a_{19}]$, $[\chi_9, 1, 4 \bowtie a_{18}]$, $[\chi_9, 3, 2 \bowtie a_{21}]$, $[\chi_{10}, 11, 16 \bowtie a_{15}]$, $[\chi_{10}, 10, 12 \bowtie a_{19}]$, $[\chi_{10}, 9, 13 \bowtie a_{19}]$, $[\chi_{10}, 14, 8 \bowtie a_{20}]$, $[\chi_{10}, 15, 7 \bowtie a_{19}]$, $[\chi_{10}, 1, 6 \bowtie a_{19}]$, $[\chi_{10}, 2, 5 \bowtie a_{20}]$, $[\chi_{10}, 4, 3 \bowtie a_{22}]$, $[\chi_{11}, 16, 12 \bowtie a_{23}]$, $[\chi_{11}, 11, 13 \bowtie a_{20}]$, $[\chi_{11}, 10, 14 \bowtie a_{20}]$, $[\chi_{11}, 9, 15 \bowtie a_{21}]$, $[\chi_{11}, 8, 1 \bowtie a_{20}]$, $[\chi_{11}, 2, 7 \bowtie a_{20}]$, $[\chi_{11}, 3, 6 \bowtie a_{20}]$, $[\chi_{11}, 5, 4 \bowtie a_{19}]$, $[\chi_{12}, 13, 16 \bowtie a_{22}]$, $[\chi_{12}, 12, 14 \bowtie a_{21}]$, $[\chi_{12}, 11, 15 \bowtie a_{22}]$, $[\chi_{12}, 10, 1 \bowtie a_{21}]$, $[\chi_{12}, 9, 2 \bowtie a_{22}]$, $[\chi_{12}, 3, 8 \bowtie a_{21}]$, $[\chi_{12}, 4, 7 \bowtie a_{21}]$, $[\chi_{12}, 5, 6 \bowtie a_{21}]$, $[\chi_{13}, 14, 16 \bowtie a_{23}]$, $[\chi_{13}, 15, 13 \bowtie a_{24}]$, $[\chi_{13}, 12, 1 \bowtie a_{23}]$, $[\chi_{13}, 2, 11 \bowtie a_{22}]$, $[\chi_{13}, 3, 10 \bowtie a_{22}]$, $[\chi_{13}, 4, 9 \bowtie a_{22}]$, $[\chi_{13}, 5, 8 \bowtie a_{22}]$, $[\chi_{13}, 6, 7 \bowtie a_{22}]$, $[\chi_{14}, 15, 16 \bowtie a_{21}]$, $[\chi_{14}, 14, 1 \bowtie a_{24}]$, $[\chi_{14}, 13, 2 \bowtie a_{23}]$, $[\chi_{14}, 12, 3 \bowtie a_{23}]$, $[\chi_{14}, 11, 4 \bowtie a_{23}]$, $[\chi_{14}, 5, 10 \bowtie a_{23}]$, $[\chi_{14}, 6, 9 \bowtie a_{23}]$, $[\chi_{14}, 7, 8 \bowtie a_{23}]$.

Note that the blocks in $\mathcal{D}_1$ cover all edges of K_{64} on $X \cup Y \cup Z \cup T$ except all edges of K_{24} on T, $K_{14,24}$ on $Y \cup T$, and the following ones: $\{a_{24}, 2\}$, $\{a_{24}, 3\}$, $\{a_{24}, 4\}$, $\{a_{24}, 5\}$, $\{a_{24}, 9\}$, $\{a_{24}, 11\}$, $\{a_{24}, 12\}$, $\{a_{23}, 11\}$. Using these missing edges we construct a set $\mathcal{D}_2$ of kites as follows. At first form the following set $\mathcal{D}_3$ of blocks:

- For $j = 1, \ldots, 6$ and $i = 1, \ldots, 12$ $[\chi_j, a_{2j+i-2}, a_{i+12} \bowtie a_{2j+i-1}]$, where the indices of a_{2j+i-2} and a_{2j+i-1} are reduced (mod 12) to the range $[1, 12]$.

- For $j = 1, 2, 3$ and $i = 1, \ldots, 6$ $[\chi_{j+6}, a_{2j+i+4}, a_i \bowtie a_{2j+i+5}]$, where the indices of a_{2j+i+4} and a_{2j+i+5} are reduced (mod 6) to the range $[7, 12]$.

- For $j = 1, 2, 3$ and $i = 1, \ldots, 6$ $[\chi_{j+6}, a_{2j+i+16}, a_{i+12} \bowtie a_{2j+i+17}]$, where the indices of $a_{2j+i+16}$ and $a_{2j+i+17}$ are reduced (mod 6) to the range $[19, 24]$.

- For $i = 0, 1, 2, 3$ $[\chi_{10}, a_{6i+6}, a_{6i+1} \bowtie a_{6i+3}]$, $[\chi_{10}, a_{6i+5}, a_{6i+2} \bowtie a_{6i+6}]$, $[\chi_{10}, a_{6i+3}, a_{6i+4} \bowtie a_{6i+5}]$.

- $[\chi_{11}, a_6, a_3 \bowtie \chi_{14}]$, $[\chi_{11}, a_2, a_4 \bowtie \chi_{14}]$, $[\chi_{11}, a_5, a_1 \bowtie \chi_{14}]$, $[\chi_{11}, a_{12}, a_9 \bowtie \chi_{14}]$, $[\chi_{11}, a_8, a_{10} \bowtie \chi_{14}]$, $[\chi_{11}, a_{11}, a_7 \bowtie \chi_{14}]$, $[\chi_{11}, a_{18}, a_{15} \bowtie \chi_{14}]$, $[\chi_{11}, a_{14}, a_{16} \bowtie \chi_{14}]$, $[\chi_{11}, a_{17}, a_{13} \bowtie \chi_{14}]$, $[\chi_{11}, a_{24}, a_{21} \bowtie \chi_{13}]$, $[\chi_{11}, a_{22}, a_{20} \bowtie \chi_{13}]$, $[\chi_{11}, a_{19}, a_{23} \bowtie \chi_{14}]$, $[\chi_{12}, a_4, a_6 \bowtie \chi_{14}]$, $[\chi_{12}, a_3, a_5 \bowtie \chi_{14}]$, $[\chi_{12}, a_1, a_2 \bowtie \chi_{14}]$, $[\chi_{12}, a_{10}, a_{12} \bowtie \chi_{14}]$, $[\chi_{12}, a_9, a_{11} \bowtie \chi_{14}]$, $[\chi_{12}, a_7, a_8 \bowtie \chi_{14}]$, $[\chi_{12}, a_{16}, a_{18} \bowtie \chi_{14}]$, $[\chi_{12}, a_{15}, a_{17} \bowtie \chi_{14}]$, $[\chi_{12}, a_{13}, a_{14} \bowtie \chi_{14}]$, $[\chi_{12}, a_{24}, a_{22} \bowtie \chi_{14}]$, $[\chi_{12}, a_{23}, a_{21} \bowtie \chi_{14}]$, $[\chi_{12}, a_{19}, a_{20} \bowtie a_{21}]$.

Let $\mathcal{D}_2$ be the set of the following blocks:

- The kites of $\mathcal{D}_3$ where we remove the following ones: $[\chi_1, a_{12}, a_{24} \bowtie a_1]$, $[\chi_2, a_2, a_{24} \bowtie a_3]$, $[\chi_3, a_4, a_{24} \bowtie a_5]$, $[\chi_4, a_6, a_{24} \bowtie a_7]$, $[\chi_5, a_8, a_{24} \bowtie a_9]$, $[\chi_6, a_{10}, a_{24} \bowtie a_{11}]$, $[\chi_8, a_{24}, a_{16} \bowtie a_{19}]$, $[\chi_8, a_{23}, a_{15} \bowtie a_{24}]$, $[\chi_8, a_{20}, a_{18} \bowtie a_{21}]$.

- $[\chi_1, a_{12}, a_{24} \bowtie 2]$, $[\chi_2, a_2, a_{24} \bowtie 3]$, $[\chi_3, a_4, a_{24} \bowtie 4]$, $[\chi_4, a_6, a_{24} \bowtie 5]$, $[\chi_5, a_8, a_{24} \bowtie 9]$, $[\chi_6, a_{10}, a_{24} \bowtie 11]$, $[\chi_8, a_{16}, a_{24} \bowtie 12]$, $[\chi_8, a_{15}, a_{23} \bowtie 11]$, $[\chi_8, a_{18}, a_{20} \bowtie \chi_{14}]$, $[\chi_{13}, a_6, a_5 \bowtie a_{24}]$, $[\chi_{13}, a_4, a_1 \bowtie a_{24}]$, $[\chi_{13}, a_2, a_3 \bowtie a_{24}]$, $[\chi_{13}, a_{12}, a_{11} \bowtie a_{24}]$, $[\chi_{13}, a_{10}, a_7 \bowtie a_{24}]$, $[\chi_{13}, a_8, a_9 \bowtie a_{24}]$, $[\chi_{13}, a_{17}, a_{18} \bowtie a_{21}]$, $[\chi_{13}, a_{13}, a_{16} \bowtie a_{19}]$, $[\chi_{13}, a_{14}, a_{15} \bowtie a_{24}]$, $[\chi_{13}, a_{23}, a_{24} \bowtie \chi_{14}]$, $[\chi_{13}, a_{22}, a_{19} \bowtie \chi_{14}]$.

It is easy to check that $\mathcal{D} \setminus \mathcal{A} = \mathcal{D}_1 \cup \mathcal{D}_2$.

Case $\alpha = 10$ and $\beta = 15$. Let $X = \{\infty_1, \infty_2, \ldots, \infty_{10}\}$, $Y = \{\chi_1, \chi_2, \ldots, \chi_{15}\}$. The set $\mathcal{A}$ is given by the following kites:

$[\chi_1, \infty_2, \infty_1 \bowtie \chi_{10}]$, $[\chi_1, \infty_{10}, \infty_3 \bowtie \chi_{12}]$, $[\chi_1, \infty_9, \infty_4 \bowtie \chi_{14}]$, $[\chi_1, \infty_8, \infty_5 \bowtie \chi_{13}]$,
$[\chi_1, \infty_7, \infty_6 \bowtie \chi_{15}]$, $[\chi_2, \infty_3, \infty_1 \bowtie \chi_{11}]$, $[\chi_2, \infty_4, \infty_2 \bowtie \chi_{10}]$, $[\chi_2, \infty_{10}, \infty_5 \bowtie \chi_{14}]$,
$[\chi_2, \infty_9, \infty_6 \bowtie \chi_{10}]$, $[\chi_2, \infty_8, \infty_7 \bowtie \chi_{10}]$, $[\chi_3, \infty_4, \infty_1 \bowtie \chi_{12}]$, $[\chi_3, \infty_5, \infty_3 \bowtie \chi_{13}]$,
$[\chi_3, \infty_6, \infty_2 \bowtie \chi_{15}]$, $[\chi_3, \infty_{10}, \infty_7 \bowtie \chi_{11}]$, $[\chi_3, \infty_9, \infty_8 \bowtie \chi_{14}]$, $[\chi_4, \infty_5, \infty_1 \bowtie \chi_{13}]$,
$[\chi_4, \infty_6, \infty_4 \bowtie \chi_{15}]$, $[\chi_4, \infty_7, \infty_3 \bowtie \chi_{14}]$, $[\chi_4, \infty_8, \infty_2 \bowtie \chi_{12}]$, $[\chi_4, \infty_{10}, \infty_9 \bowtie \chi_{12}]$,
$[\chi_5, \infty_6, \infty_1 \bowtie \chi_{14}]$, $[\chi_5, \infty_7, \infty_5 \bowtie \chi_{10}]$, $[\chi_5, \infty_4, \infty_8 \bowtie \chi_{15}]$, $[\chi_5, \infty_9, \infty_3 \bowtie \chi_{15}]$,
$[\chi_5, \infty_{10}, \infty_2 \bowtie \chi_{13}]$, $[\chi_6, \infty_7, \infty_1 \bowtie \chi_{15}]$, $[\chi_6, \infty_8, \infty_6 \bowtie \chi_{11}]$, $[\chi_6, \infty_9, \infty_5 \bowtie \chi_{11}]$,
$[\chi_6, \infty_{10}, \infty_4 \bowtie \chi_{10}]$, $[\chi_6, \infty_3, \infty_2 \bowtie \chi_{14}]$, $[\chi_7, \infty_1, \infty_8 \bowtie \chi_{10}]$, $[\chi_7, \infty_9, \infty_7 \bowtie \chi_{12}]$,
$[\chi_7, \infty_{10}, \infty_6 \bowtie \chi_{12}]$, $[\chi_7, \infty_5, \infty_2 \bowtie \chi_{11}]$, $[\chi_7, \infty_3, \infty_4 \bowtie \chi_{11}]$, $[\chi_8, \infty_1, \infty_9 \bowtie \chi_{13}]$,
$[\chi_8, \infty_{10}, \infty_8 \bowtie \chi_{11}]$, $[\chi_8, \infty_2, \infty_7 \bowtie \chi_{13}]$, $[\chi_8, \infty_6, \infty_3 \bowtie \chi_{10}]$, $[\chi_8, \infty_5, \infty_4 \bowtie \chi_{12}]$,
$[\chi_9, \infty_1, \infty_{10} \bowtie \chi_{10}]$, $[\chi_9, \infty_2, \infty_9 \bowtie \chi_{14}]$, $[\chi_9, \infty_8, \infty_3 \bowtie \chi_{11}]$, $[\chi_9, \infty_7, \infty_4 \bowtie \chi_{13}]$,
$[\chi_9, \infty_6, \infty_5 \bowtie \chi_{12}]$, $[\chi_1, \chi_3, \chi_9 \bowtie \chi_4]$, $[\chi_{10}, \chi_4, \chi_2 \bowtie \chi_{12}]$, $[\chi_{11}, \chi_5, \chi_3 \bowtie \chi_{13}]$,
$[\chi_{12}, \chi_6, \chi_4 \bowtie \chi_{14}]$, $[\chi_5, \chi_7, \chi_{13} \bowtie \infty_6]$, $[\chi_6, \chi_8, \chi_{14} \bowtie \infty_6]$, $[\chi_7, \chi_9, \chi_{15} \bowtie \infty_5]$,
$[\chi_1, \chi_8, \chi_{10} \bowtie \infty_9]$, $[\chi_2, \chi_9, \chi_{11} \bowtie \infty_9]$, $[\chi_{10}, \chi_3, \chi_{12} \bowtie \infty_8]$, $[\chi_4, \chi_{11}, \chi_{13} \bowtie \infty_8]$,
$[\chi_5, \chi_{12}, \chi_{14} \bowtie \infty_7]$, $[\chi_6, \chi_{13}, \chi_{15} \bowtie \infty_7]$, $[\chi_7, \chi_{14}, \chi_1 \bowtie \chi_{11}]$, $[\chi_8, \chi_2, \chi_{15} \bowtie \chi_5]$,
$[\chi_1, \chi_2, \chi_5 \bowtie \chi_{10}]$, $[\chi_2, \chi_3, \chi_6 \bowtie \chi_{11}]$, $[\chi_3, \chi_4, \chi_7 \bowtie \chi_{12}]$, $[\chi_4, \chi_5, \chi_8 \bowtie \chi_{13}]$,
$[\chi_5, \chi_6, \chi_9 \bowtie \chi_{14}]$, $[\chi_6, \chi_7, \chi_{10} \bowtie \chi_{15}]$, $[\chi_7, \chi_8, \chi_{11} \bowtie \infty_{10}]$, $[\chi_8, \chi_9, \chi_{12} \bowtie \infty_{10}]$,
$[\chi_9, \chi_{10}, \chi_{13} \bowtie \infty_{10}]$, $[\chi_{10}, \chi_{11}, \chi_{14} \bowtie \infty_{10}]$, $[\chi_{11}, \chi_{12}, \chi_{15} \bowtie \infty_9]$, $[\chi_{12}, \chi_{13}, \chi_1 \bowtie \chi_6]$,
$[\chi_{13}, \chi_{14}, \chi_2 \bowtie \chi_7]$, $[\chi_{14}, \chi_{15}, \chi_3 \bowtie \chi_8]$, $[\chi_4, \chi_1, \chi_{15} \bowtie \infty_{10}]$.

Let $\mathcal{D}_1$ be the set of the following kites:

$[a_1, 1, \infty_1 \bowtie a_{17}]$, $[a_1, 9, \infty_2 \bowtie a_9]$, $[a_1, 8, \infty_5 \bowtie a_{17}]$, $[a_1, 16, \infty_6 \bowtie a_4]$, $[a_1, 7, \infty_8 \bowtie a_{17}]$,
$[a_1, 15, \infty_9 \bowtie a_4]$, $[a_2, 2, \infty_1 \bowtie a_{18}]$, $[a_2, 10, \infty_2 \bowtie a_{10}]$, $[a_2, 1, \infty_4 \bowtie a_1]$, $[a_2, 9, \infty_5 \bowtie a_{10}]$,
$[a_2, 8, \infty_8 \bowtie a_{18}]$, $[a_2, 16, \infty_9 \bowtie a_5]$, $[a_3, 3, \infty_1 \bowtie a_{19}]$, $[a_3, 11, \infty_2 \bowtie a_{11}]$, $[a_3, 2, \infty_4 \bowtie a_{18}]$,
$[a_3, 10, \infty_5 \bowtie a_{11}]$, $[a_3, 1, \infty_7 \bowtie a_1]$, $[a_3, 9, \infty_8 \bowtie a_{11}]$, $[a_4, 4, \infty_1 \bowtie a_{20}]$, $[a_4, 12, \infty_2 \bowtie a_{12}]$,
$[a_4, 3, \infty_4 \bowtie a_{19}]$, $[a_4, 11, \infty_5 \bowtie a_{12}]$, $[a_4, 2, \infty_7 \bowtie a_2]$, $[a_4, 10, \infty_8 \bowtie a_{12}]$, $[a_4, 1, \infty_{10} \bowtie a_1]$,
$[a_5, 5, \infty_1 \bowtie a_{21}]$, $[a_5, 13, \infty_2 \bowtie a_{13}]$, $[a_5, 4, \infty_4 \bowtie a_{20}]$, $[a_5, 12, \infty_5 \bowtie a_{13}]$, $[a_5, 3, \infty_7 \bowtie a_{19}]$,
$[a_5, 11, \infty_8 \bowtie a_{13}]$, $[a_5, 2, \infty_{10} \bowtie a_2]$, $[a_5, 16, 1 \bowtie a_7]$, $[a_6, 6, \infty_1 \bowtie a_{22}]$, $[a_6, 14, \infty_2 \bowtie a_{14}]$,
$[a_6, 5, \infty_4 \bowtie a_{21}]$, $[a_6, 13, \infty_5 \bowtie a_{14}]$, $[a_6, 4, \infty_7 \bowtie a_{20}]$, $[a_6, 12, \infty_8 \bowtie a_{14}]$, $[a_6, 3, \infty_{10} \bowtie a_3]$,

$[a_7, \infty_1, 7 \bowtie a_2]$, $[a_7, 15, \infty_2 \bowtie a_{15}]$, $[a_7, 6, \infty_4 \bowtie a_{22}]$, $[a_7, 14, \infty_5 \bowtie a_{15}]$, $[a_7, 5, \infty_7 \bowtie a_{21}]$, $[a_7, 13, \infty_8 \bowtie a_{15}]$, $[a_7, 4, \infty_{10} \bowtie a_{21}]$, $[a_8, 8, \infty_1 \bowtie a_{23}]$, $[a_8, 16, \infty_2 \bowtie a_{16}]$, $[a_8, 7, \infty_4 \bowtie a_{23}]$, $[a_8, 15, \infty_5 \bowtie a_{16}]$, $[a_8, 6, \infty_7 \bowtie a_{22}]$, $[a_8, 14, \infty_8 \bowtie a_{16}]$, $[a_8, 5, \infty_{10} \bowtie a_{23}]$, $[a_9, 9, \infty_1 \bowtie a_{24}]$, $[a_9, 1, \infty_3 \bowtie a_1]$, $[a_9, 8, \infty_4 \bowtie a_{24}]$, $[a_9, \infty_5, 16 \bowtie a_4]$, $[a_9, 7, \infty_7 \bowtie a_{23}]$, $[a_9, \infty_8, 15 \bowtie a_2]$, $[a_9, 6, \infty_{10} \bowtie a_{22}]$, $[a_{10}, \infty_1, 10 \bowtie a_1]$, $[a_{10}, 2, \infty_3 \bowtie a_2]$, $[a_{10}, \infty_4, 9 \bowtie a_4]$, $[a_{10}, 1, \infty_6 \bowtie a_2]$, $[a_{10}, 8, \infty_7 \bowtie a_{24}]$, $[a_{10}, \infty_8, 16 \bowtie a_3]$, $[a_{10}, 7, \infty_{10} \bowtie a_{24}]$, $[a_{11}, \infty_1, 11 \bowtie a_1]$, $[a_{11}, 3, \infty_3 \bowtie a_3]$, $[a_{11}, \infty_4, 10 \bowtie a_5]$, $[a_{11}, 2, \infty_6 \bowtie a_3]$, $[a_{11}, \infty_7, 9 \bowtie a_5]$, $[a_{11}, 1, \infty_9 \bowtie a_3]$, $[a_{11}, \infty_{10}, 8 \bowtie a_3]$, $[a_{12}, \infty_1, 12 \bowtie a_1]$, $[a_{12}, 4, \infty_3 \bowtie a_4]$, $[a_{12}, \infty_4, 11 \bowtie a_2]$, $[a_{12}, 3, \infty_6 \bowtie a_5]$, $[a_{12}, \infty_7, 10 \bowtie a_6]$, $[a_{12}, 2, \infty_9 \bowtie a_6]$, $[a_{12}, 9, \infty_{10} \bowtie a_{20}]$, $[a_{13}, \infty_1, 13 \bowtie a_{24}]$, $[a_{13}, 5, \infty_3 \bowtie a_5]$, $[a_{13}, \infty_4, 12 \bowtie a_2]$, $[a_{13}, 4, \infty_6 \bowtie a_6]$, $[a_{13}, \infty_7, 11 \bowtie a_6]$, $[a_{13}, 3, \infty_9 \bowtie a_7]$, $[a_{13}, \infty_{10}, 10 \bowtie a_7]$, $[a_{14}, \infty_1, 14 \bowtie a_1]$, $[a_{14}, 6, \infty_3 \bowtie a_6]$, $[a_{14}, \infty_4, 13 \bowtie a_2]$, $[a_{14}, 5, \infty_6 \bowtie a_7]$, $[a_{14}, \infty_7, 12 \bowtie a_3]$, $[a_{14}, 4, \infty_9 \bowtie a_8]$, $[a_{14}, \infty_{10}, 11 \bowtie a_7]$, $[a_{15}, \infty_1, 15 \bowtie a_3]$, $[a_{15}, 7, \infty_3 \bowtie a_7]$, $[a_{15}, \infty_4, 14 \bowtie a_2]$, $[a_{15}, 6, \infty_6 \bowtie a_8]$, $[a_{15}, \infty_7, 13 \bowtie a_3]$, $[a_{15}, 5, \infty_9 \bowtie a_9]$, $[a_{15}, \infty_{10}, 12 \bowtie a_7]$, $[a_{16}, \infty_1, 16 \bowtie a_{23}]$, $[a_{16}, 8, \infty_3 \bowtie a_8]$, $[a_{16}, \infty_4, 15 \bowtie a_4]$, $[a_{16}, 7, \infty_6 \bowtie a_9]$, $[a_{16}, \infty_7, 14 \bowtie a_3]$, $[a_{16}, 6, \infty_9 \bowtie a_{10}]$, $[a_{16}, \infty_{10}, 13 \bowtie a_4]$, $[a_{17}, \infty_2, 1 \bowtie a_{15}]$, $[a_{17}, \infty_3, 9 \bowtie a_6]$, $[a_{17}, \infty_4, 16 \bowtie a_6]$, $[a_{17}, \infty_6, 8 \bowtie a_4]$, $[a_{17}, \infty_7, 15 \bowtie a_5]$, $[a_{17}, \infty_9, 7 \bowtie a_3]$, $[a_{17}, \infty_{10}, 14 \bowtie a_4]$, $[a_{18}, \infty_2, 2 \bowtie a_{24}]$, $[a_{18}, \infty_3, 10 \bowtie a_8]$, $[a_{18}, \infty_5, 1 \bowtie a_6]$, $[a_{18}, \infty_6, 9 \bowtie a_8]$, $[a_{18}, \infty_7, 16 \bowtie a_7]$, $[a_{18}, \infty_9, 8 \bowtie a_5]$, $[a_{18}, \infty_{10}, 15 \bowtie a_6]$, $[a_{19}, \infty_2, 3 \bowtie a_1]$, $[a_{19}, \infty_3, 11 \bowtie a_8]$, $[a_{19}, \infty_5, 2 \bowtie a_6]$, $[a_{19}, \infty_6, 10 \bowtie a_9]$, $[a_{19}, \infty_8, 1 \bowtie a_{14}]$, $[a_{19}, \infty_9, 9 \bowtie a_{13}]$, $[a_{19}, \infty_{10}, 16 \bowtie a_{11}]$, $[a_{20}, \infty_2, 4 \bowtie a_1]$, $[a_{20}, \infty_3, 12 \bowtie a_8]$, $[a_{20}, \infty_5, 3 \bowtie a_2]$, $[a_{20}, \infty_6, 11 \bowtie a_9]$, $[a_{20}, \infty_8, 2 \bowtie a_7]$, $[a_{20}, \infty_9, 10 \bowtie a_{14}]$, $[a_{21}, \infty_2, 5 \bowtie a_1]$, $[a_{21}, \infty_3, 13 \bowtie a_8]$, $[a_{21}, \infty_5, 4 \bowtie a_2]$, $[a_{21}, \infty_6, 12 \bowtie a_9]$, $[a_{21}, \infty_8, 3 \bowtie a_7]$, $[a_{21}, \infty_9, 11 \bowtie a_{10}]$, $[a_{22}, \infty_2, 6 \bowtie a_1]$, $[a_{22}, \infty_3, 14 \bowtie a_5]$, $[a_{22}, \infty_5, 5 \bowtie a_2]$, $[a_{22}, \infty_6, 13 \bowtie a_9]$, $[a_{22}, \infty_8, 4 \bowtie a_3]$, $[a_{22}, \infty_9, 12 \bowtie a_{10}]$, $[a_{23}, \infty_2, 7 \bowtie a_4]$, $[a_{23}, \infty_3, 15 \bowtie a_{10}]$, $[a_{23}, \infty_5, 6 \bowtie a_2]$, $[a_{23}, \infty_6, 14 \bowtie a_9]$, $[a_{23}, \infty_8, 5 \bowtie a_3]$, $[a_{23}, \infty_9, 13 \bowtie a_{10}]$, $[a_{24}, \infty_2, 8 \bowtie a_6]$, $[a_{24}, \infty_3, 16 \bowtie a_{12}]$, $[a_{24}, \infty_5, 7 \bowtie a_5]$, $[a_{24}, \infty_6, 15 \bowtie a_{11}]$, $[a_{24}, \infty_8, 6 \bowtie a_3]$, $[a_{24}, \infty_9, 14 \bowtie a_{11}]$, $[\chi_1, 16, 2 \bowtie a_8]$, $[\chi_1, 1, 3 \bowtie a_8]$, $[\chi_1, 15, 4 \bowtie a_8]$, $[\chi_1, 14, 5 \bowtie a_4]$, $[\chi_1, 13, 6 \bowtie a_4]$, $[\chi_1, 12, 7 \bowtie a_6]$, $[\chi_1, 11, 8 \bowtie a_7]$, $[\chi_1, 10, 9 \bowtie a_{14}]$, $[\chi_2, 16, 3 \bowtie a_9]$, $[\chi_2, 4, 2 \bowtie a_9]$, $[\chi_2, 5, 1 \bowtie a_8]$, $[\chi_2, 15, 6 \bowtie a_5]$, $[\chi_2, 7, 14 \bowtie a_{10}]$, $[\chi_2, 13, 8 \bowtie a_{12}]$, $[\chi_2, 9, 12 \bowtie a_{11}]$, $[\chi_2, 10, 11 \bowtie a_{15}]$, $[\chi_3, 16, 4 \bowtie a_9]$, $[\chi_3, 3, 5 \bowtie a_9]$, $[\chi_3, 2, 6 \bowtie a_{10}]$, $[\chi_3, 7, 1 \bowtie a_{12}]$, $[\chi_3, 8, 15 \bowtie a_{12}]$, $[\chi_3, 9, 14 \bowtie a_{12}]$, $[\chi_3, 10, 13 \bowtie a_{11}]$, $[\chi_3, 12, 11 \bowtie a_{16}]$, $[\chi_4, 16, 5 \bowtie a_{10}]$, $[\chi_4, 6, 4 \bowtie a_{10}]$, $[\chi_4, 7, 3 \bowtie a_{10}]$, $[\chi_4, 8, 2 \bowtie a_{13}]$, $[\chi_4, 9, 1 \bowtie a_{13}]$, $[\chi_4, 10, 15 \bowtie a_{13}]$, $[\chi_4, 11, 14 \bowtie a_{13}]$, $[\chi_4, 13, 12 \bowtie a_{17}]$, $[\chi_5, 16, 6 \bowtie a_{11}]$, $[\chi_5, 7, 5 \bowtie a_{11}]$, $[\chi_5, 8, 4 \bowtie a_{11}]$, $[\chi_5, 9, 3 \bowtie a_{14}]$, $[\chi_5, 10, 2 \bowtie a_{14}]$, $[\chi_5, 11, 1 \bowtie a_{24}]$, $[\chi_5, 12, 15 \bowtie a_{14}]$, $[\chi_5, 14, 13 \bowtie a_{12}]$, $[\chi_6, 16, 7 \bowtie a_{11}]$, $[\chi_6, 8, 6 \bowtie a_{12}]$, $[\chi_6, 9, 5 \bowtie a_{12}]$, $[\chi_6, 10, 4 \bowtie a_{15}]$, $[\chi_6, 11, 3 \bowtie a_{15}]$, $[\chi_6, 12, 2 \bowtie a_{15}]$, $[\chi_6, 1, 13 \bowtie a_1]$, $[\chi_6, 15, 14 \bowtie a_{18}]$, $[\chi_7, 8, 16 \bowtie a_{13}]$, $[\chi_7, 9, 7 \bowtie a_{12}]$, $[\chi_7, 10, 6 \bowtie a_{13}]$, $[\chi_7, 11, 5 \bowtie a_{16}]$, $[\chi_7, 12, 4 \bowtie a_{16}]$, $[\chi_7, 13, 3 \bowtie a_{16}]$, $[\chi_7, 14, 2 \bowtie a_{16}]$, $[\chi_7, 15, 1 \bowtie a_{16}]$, $[\chi_8, 9, 16 \bowtie a_{14}]$, $[\chi_8, 10, 8 \bowtie a_{14}]$, $[\chi_8, 11, 7 \bowtie a_{13}]$, $[\chi_8, 6, 12 \bowtie a_{24}]$, $[\chi_8, 13, 5 \bowtie a_{17}]$, $[\chi_8, 14, 4 \bowtie a_{17}]$, $[\chi_8, 15, 3 \bowtie a_{17}]$, $[\chi_8, 1, 2 \bowtie a_{17}]$, $[\chi_9, 16, 10 \bowtie a_{23}]$, $[\chi_9, 11, 9 \bowtie a_{15}]$, $[\chi_9, 12, 8 \bowtie a_{15}]$, $[\chi_9, 13, 7 \bowtie a_{14}]$, $[\chi_9, 14, 6 \bowtie a_{17}]$, $[\chi_9, 5, 15 \bowtie a_{19}]$, $[\chi_9, 1, 4 \bowtie a_{18}]$, $[\chi_9, 2, 3 \bowtie a_{18}]$, $[\chi_{10}, 11, 16 \bowtie a_{15}]$, $[\chi_{10}, 12, 10 \bowtie a_{16}]$, $[\chi_{10}, 13, 9 \bowtie a_{23}]$, $[\chi_{10}, 14, 8 \bowtie a_{19}]$, $[\chi_{10}, 15, 7 \bowtie a_{18}]$, $[\chi_{10}, 1, 6 \bowtie a_{18}]$, $[\chi_{10}, 2, 5 \bowtie a_{18}]$, $[\chi_{10}, 4, 3 \bowtie a_{22}]$, $[\chi_{11}, 16, 12 \bowtie a_{19}]$, $[\chi_{11}, 11, 13 \bowtie a_{19}]$, $[\chi_{11}, 10, 14 \bowtie a_{19}]$, $[\chi_{11}, 15, 9 \bowtie a_7]$, $[\chi_{11}, 8, 1 \bowtie a_{20}]$, $[\chi_{11}, 2, 7 \bowtie a_{19}]$, $[\chi_{11}, 3, 6 \bowtie a_{19}]$, $[\chi_{11}, 5, 4 \bowtie a_{19}]$, $[\chi_{12}, 16, 13 \bowtie a_{20}]$, $[\chi_{12}, 12, 14 \bowtie a_{20}]$, $[\chi_{12}, 11, 15 \bowtie a_{20}]$, $[\chi_{12}, 10, 1 \bowtie a_{21}]$, $[\chi_{12}, 2, 9 \bowtie a_{21}]$, $[\chi_{12}, 3, 8 \bowtie a_{21}]$, $[\chi_{12}, 4, 7 \bowtie a_{21}]$, $[\chi_{12}, 6, 5 \bowtie a_{19}]$, $[\chi_{13}, 14, 16 \bowtie a_{20}]$, $[\chi_{13}, 13, 15 \bowtie a_{21}]$, $[\chi_{13}, 12, 1 \bowtie a_{22}]$, $[\chi_{13}, 11, 2 \bowtie a_{21}]$, $[\chi_{13}, 3, 10 \bowtie a_{21}]$,

$[\chi_{13}, 4, 9 \bowtie a_{20}]$, $[\chi_{13}, 8, 5 \bowtie a_{20}]$, $[\chi_{13}, 6, 7 \bowtie a_{22}]$, $[\chi_{14}, 15, 16 \bowtie a_{21}]$, $[\chi_{14}, 1, 14 \bowtie a_{21}]$, $[\chi_{14}, 13, 2 \bowtie a_{22}]$, $[\chi_{14}, 12, 3 \bowtie a_{24}]$, $[\chi_{14}, 4, 11 \bowtie a_{22}]$, $[\chi_{14}, 5, 10 \bowtie a_{22}]$, $[\chi_{14}, 9, 6 \bowtie a_{21}]$, $[\chi_{14}, 7, 8 \bowtie a_{22}]$, $[\chi_{15}, 16, 1 \bowtie a_{23}]$, $[\chi_{15}, 15, 2 \bowtie a_{23}]$, $[\chi_{15}, 14, 3 \bowtie a_{23}]$, $[\chi_{15}, 13, 4 \bowtie a_{24}]$, $[\chi_{15}, 12, 5 \bowtie a_{24}]$, $[\chi_{15}, 11, 6 \bowtie a_{20}]$, $[\chi_{15}, 10, 7 \bowtie a_{20}]$, $[\chi_{15}, 9, 8 \bowtie a_{20}]$.

Note that the blocks in $\mathcal{D}_1$ cover all edges of K_{65} on $X \cup Y \cup Z \cup T$ except all edges of K_{24} on T, $K_{15,24}$ on $Y \cup T$, and the following ones: $\{a_1, 2\}$, $\{a_{13}, 8\}$, $\{a_{15}, 10\}$, $\{a_{16}, 9\}$, $\{a_{16}, 12\}$, $\{a_{17}, 10\}$, $\{a_{17}, 11\}$, $\{a_{17}, 13\}$, $\{a_{18}, 11\}$, $\{a_{18}, 12\}$, $\{a_{18}, 13\}$, $\{a_{22}, 15\}$, $\{a_{22}, 16\}$, $\{a_{23}, 4\}$, $\{a_{23}, 8\}$, $\{a_{23}, 11\}$, $\{a_{23}, 12\}$, $\{a_{24}, 9\}$, $\{a_{24}, 10\}$, $\{a_{24}, 11\}$. Using these missing edges we construct the following set $\mathcal{D}_2$ of kites:

- For $i = 1, \ldots, 12$, $[\chi_1, a_{i+1}, a_{i+12} \bowtie \chi_{15}]$, where the indices of a_{1+i} are reduced (mod 12) to the range $[1, 12]$. Replace the blocks $[\chi_1, a_6, a_{17} \bowtie \chi_{15}]$ and $[\chi_1, a_{12}, a_{23} \bowtie \chi_{15}]$ with $[\chi_1, a_6, a_{17} \bowtie a_5]$, $[\chi_1, a_{12}, a_{23} \bowtie a_{11}]$.

- For $i = 1, \ldots, 12$, $[\chi_2, a_{i+3}, a_{i+12} \bowtie a_{i+2}]$, where the indices of a_{i+2} and a_{i+3} are reduced (mod 12) to the range $[1, 12]$. Replace the blocks $[\chi_2, a_8, a_{17} \bowtie a_7]$ and $[\chi_2, a_2, a_{23} \bowtie a_1]$ with $[\chi_2, a_8, a_{17} \bowtie \chi_{15}]$, $[\chi_2, a_2, a_{23} \bowtie \chi_{15}]$.

- For $i = 1, \ldots, 12$, $[\chi_3, a_{i+4}, a_{i+12} \bowtie a_{i+5}]$, where the indices of a_{i+4} and a_{i+5} are reduced (mod 12) to the range $[1, 12]$.

- For $i = 1, \ldots, 12$, $[\chi_4, a_{i+7}, a_{i+12} \bowtie a_{i+6}]$, where the indices of a_{i+6} and a_{i+7} are reduced (mod 12) to the range $[1, 12]$. Replace the block $[\chi_4, a_6, a_{23} \bowtie a_5]$ with $[\chi_4, a_6, a_{23} \bowtie 8]$.

- For $i = 1, \ldots, 12$, $[\chi_5, a_{i+9}, a_{i+12} \bowtie a_{i+8}]$, where the indices of a_{i+8} and a_{i+9} are reduced (mod 12) to the range $[1, 12]$. Replace the blocks $[\chi_5, a_1, a_{16} \bowtie a_{12}]$ and $[\chi_5, a_8, a_{23} \bowtie a_7]$ with $[\chi_5, a_1, a_{16} \bowtie 12]$, $[\chi_5, a_8, a_{23} \bowtie 4]$.

- For $i = 1, \ldots, 12$, $[\chi_6, a_{i+11}, a_{i+12} \bowtie a_{i+10}]$, where the indices of a_{i+10} and a_{i+11} are reduced (mod 12) to the range $[1, 12]$. Replace the blocks $[\chi_6, a_4, a_{17} \bowtie a_3]$ and $[\chi_6, a_9, a_{22} \bowtie a_8]$ with $[\chi_6, a_4, a_{17} \bowtie 10]$, $[\chi_6, a_9, a_{22} \bowtie 16]$.

- For $i = 1, \ldots, 6$, $[\chi_7, a_{i+7}, a_i \bowtie a_{i+6}]$, $[\chi_8, a_{i+8}, a_i \bowtie a_{i+9}]$, $[\chi_9, a_{i+10}, a_i \bowtie a_{i+11}]$, where the indices of $a_{i+6}, a_{i+7}, \ldots, a_{i+11}$ are reduced (mod 6) to the range $[7, 12]$.

- For $i = 1, \ldots, 6$, $[\chi_7, a_{i+19}, a_{i+12} \bowtie a_{i+18}]$, $[\chi_8, a_{i+20}, a_{i+12} \bowtie a_{i+21}]$, $[\chi_9, a_{i+22}, a_{i+12} \bowtie a_{i+23}]$, where the indices of $a_{i+18}, a_{i+19}, \ldots, a_{i+23}$ are reduced (mod 6) to the range $[19, 24]$. Replace the block $[\chi_7, a_{22}, a_{15} \bowtie a_{21}]$ with $[\chi_7, a_{22}, a_{15} \bowtie 10]$.

- For $i = 0, 1, 2, 3$, $[\chi_{11}, a_{6i+3}, a_{6i+6} \bowtie \chi_{14}]$, $[\chi_{11}, a_{6i+4}, a_{6i+2} \bowtie \chi_{14}]$, $[\chi_{11}, a_{6i+1}, a_{6i+5} \bowtie \chi_{14}]$, $[\chi_{12}, a_{6i+6}, a_{6i+4} \bowtie \chi_{14}]$, $[\chi_{12}, a_{6i+5}, a_{6i+3} \bowtie \chi_{14}]$, $[\chi_{12}, a_{6i+2}, a_{6i+1} \bowtie \chi_{14}]$.

- $[\chi_{10}, a_6, a_1 \bowtie a_{13}]$, $[\chi_{10}, a_5, a_2 \bowtie a_{14}]$, $[\chi_{10}, a_4, a_3 \bowtie a_{15}]$, $[\chi_{10}, a_{12}, a_7 \bowtie a_{19}]$, $[\chi_{10}, a_{11}, a_8 \bowtie a_{20}]$, $[\chi_{10}, a_{10}, a_9 \bowtie a_{21}]$, $[\chi_{10}, a_{13}, a_{18} \bowtie 12]$, $[\chi_{10}, a_{17}, a_{14} \bowtie a_{15}]$, $[\chi_{10}, a_{15}, a_{16} \bowtie a_{13}]$, $[\chi_{10}, a_{19}, a_{24} \bowtie 10]$, $[\chi_{10}, a_{23}, a_{20} \bowtie a_{21}]$, $[\chi_{10}, a_{21}, a_{22} \bowtie a_{19}]$, $[\chi_{13}, a_6, a_5 \bowtie a_{23}]$, $[\chi_{13}, a_4, a_1 \bowtie 2]$, $[\chi_{13}, a_2, a_3 \bowtie a_{17}]$, $[\chi_{13}, a_{11}, a_{12} \bowtie a_{16}]$, $[\chi_{13}, a_{10}, a_7 \bowtie a_{23}]$,

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$[\chi_{13}, a_9, a_8 \bowtie a_{22}]$, $[\chi_{13}, a_{17}, a_{16} \bowtie 9]$, $[\chi_{13}, a_{14}, a_{18} \bowtie 11]$, $[\chi_{13}, a_{15}, a_{13} \bowtie 8]$, $[\chi_{13}, a_{20}, a_{24} \bowtie 9]$, $[\chi_{13}, a_{23}, a_{22} \bowtie 15]$, $[\chi_{13}, a_{19}, a_{21} \bowtie a_{15}]$, $[\chi_{15}, a_2, a_6 \bowtie a_{18}]$, $[\chi_{15}, a_5, a_4 \bowtie a_{16}]$, $[\chi_{15}, a_3, a_1 \bowtie a_{23}]$, $[\chi_{15}, a_8, a_{12} \bowtie a_{24}]$, $[\chi_{15}, a_{11}, a_{10} \bowtie a_{22}]$, $[\chi_{15}, a_9, a_7 \bowtie a_{17}]$, $[11, a_{24}, a_{23} \bowtie 12]$, $[13, a_{18}, a_{17} \bowtie 11]$.

It is easy to check that $\mathcal{D} \setminus \mathcal{A} = \mathcal{D}_1 \cup \mathcal{D}_2$.

Case $\alpha = 13$ and $\beta = 19$. Let $X = \{\infty_1, \infty_2, \ldots, \infty_{13}\}$, $Y = \{\chi_1, \chi_2, \ldots, \chi_{19}\}$. The set $\mathcal{A}$ is given by the following kites:

$[\chi_1, \infty_1, \infty_{12} \bowtie \chi_{13}]$, $[\chi_1, \infty_{11}, \infty_2 \bowtie \chi_{14}]$, $[\chi_1, \infty_{10}, \infty_3 \bowtie \chi_{14}]$, $[\chi_1, \infty_9, \infty_4 \bowtie \chi_{14}]$, $[\chi_1, \infty_8, \infty_5 \bowtie \chi_{14}]$, $[\chi_1, \infty_7, \infty_6 \bowtie \chi_{15}]$, $[\chi_2, \infty_{13}, \infty_2 \bowtie \chi_{15}]$, $[\chi_2, \infty_{12}, \infty_3 \bowtie \chi_{15}]$, $[\chi_2, \infty_4, \infty_{11} \bowtie \chi_{19}]$, $[\chi_2, \infty_{10}, \infty_5 \bowtie \chi_{16}]$, $[\chi_2, \infty_9, \infty_6 \bowtie \chi_{16}]$, $[\chi_2, \infty_8, \infty_7 \bowtie \chi_{15}]$, $[\chi_3, \infty_1, \infty_3 \bowtie \chi_{16}]$, $[\chi_3, \infty_{13}, \infty_4 \bowtie \chi_{16}]$, $[\chi_3, \infty_{12}, \infty_5 \bowtie \chi_{17}]$, $[\chi_3, \infty_{11}, \infty_6 \bowtie \chi_{17}]$, $[\chi_3, \infty_7, \infty_{10} \bowtie \chi_{14}]$, $[\chi_3, \infty_9, \infty_8 \bowtie \chi_{14}]$, $[\chi_4, \infty_2, \infty_4 \bowtie \chi_{17}]$, $[\chi_4, \infty_5, \infty_1 \bowtie \chi_{17}]$, $[\chi_4, \infty_6, \infty_{13} \bowtie \chi_{14}]$, $[\chi_4, \infty_7, \infty_{12} \bowtie \chi_{14}]$, $[\chi_4, \infty_{11}, \infty_8 \bowtie \chi_{15}]$, $[\chi_4, \infty_{10}, \infty_9 \bowtie \chi_{18}]$, $[\chi_5, \infty_5, \infty_3 \bowtie \chi_{17}]$, $[\chi_5, \infty_6, \infty_2 \bowtie \chi_{18}]$, $[\chi_5, \infty_1, \infty_7 \bowtie \chi_{17}]$, $[\chi_5, \infty_{13}, \infty_8 \bowtie \chi_{16}]$, $[\chi_5, \infty_{12}, \infty_9 \bowtie \chi_{19}]$, $[\chi_5, \infty_{11}, \infty_{10} \bowtie \chi_{18}]$, $[\chi_6, \infty_6, \infty_4 \bowtie \chi_{18}]$, $[\chi_6, \infty_7, \infty_3 \bowtie \chi_{18}]$, $[\chi_6, \infty_2, \infty_8 \bowtie \chi_{17}]$, $[\chi_6, \infty_9, \infty_1 \bowtie \chi_{16}]$, $[\chi_6, \infty_{13}, \infty_{10} \bowtie \chi_{19}]$, $[\chi_6, \infty_{12}, \infty_{11} \bowtie \chi_{12}]$, $[\chi_7, \infty_7, \infty_5 \bowtie \chi_{18}]$, $[\chi_7, \infty_8, \infty_4 \bowtie \chi_{19}]$, $[\chi_7, \infty_3, \infty_9 \bowtie \chi_{15}]$, $[\chi_7, \infty_2, \infty_{10} \bowtie \chi_{15}]$, $[\chi_7, \infty_1, \infty_{11} \bowtie \chi_{15}]$, $[\chi_7, \infty_{12}, \infty_{13} \bowtie \chi_{15}]$, $[\chi_8, \infty_6, \infty_8 \bowtie \chi_{18}]$, $[\chi_8, \infty_5, \infty_9 \bowtie \chi_{17}]$, $[\chi_8, \infty_4, \infty_{10} \bowtie \chi_{16}]$, $[\chi_8, \infty_3, \infty_{11} \bowtie \chi_{17}]$, $[\chi_8, \infty_2, \infty_{12} \bowtie \chi_{15}]$, $[\chi_8, \infty_1, \infty_{13} \bowtie \chi_{16}]$, $[\chi_9, \infty_7, \infty_9 \bowtie \chi_{16}]$, $[\chi_9, \infty_6, \infty_{10} \bowtie \chi_{17}]$, $[\chi_9, \infty_5, \infty_{11} \bowtie \chi_{14}]$, $[\chi_9, \infty_4, \infty_{12} \bowtie \chi_{16}]$, $[\chi_9, \infty_3, \infty_{13} \bowtie \chi_1]$, $[\chi_9, \infty_2, \infty_1 \bowtie \chi_{14}]$, $[\chi_{10}, \infty_{10}, \infty_8 \bowtie \chi_{19}]$, $[\chi_{10}, \infty_7, \infty_{11} \bowtie \chi_{16}]$, $[\chi_{10}, \infty_6, \infty_{12} \bowtie \chi_{17}]$, $[\chi_{10}, \infty_5, \infty_{13} \bowtie \chi_{18}]$, $[\chi_{10}, \infty_4, \infty_1 \bowtie \chi_2]$, $[\chi_{10}, \infty_3, \infty_2 \bowtie \chi_3]$, $[\chi_{11}, \infty_9, \infty_{11} \bowtie \chi_{18}]$, $[\chi_{11}, \infty_8, \infty_{12} \bowtie \chi_{18}]$, $[\chi_{11}, \infty_7, \infty_{13} \bowtie \chi_{19}]$, $[\chi_{11}, \infty_6, \infty_1 \bowtie \chi_{15}]$, $[\chi_{11}, \infty_5, \infty_2 \bowtie \chi_{19}]$, $[\chi_{11}, \infty_4, \infty_3 \bowtie \chi_{19}]$, $[\chi_{12}, \infty_{10}, \infty_{12} \bowtie \chi_{19}]$, $[\chi_{12}, \infty_{13}, \infty_9 \bowtie \chi_{10}]$, $[\chi_{12}, \infty_8, \infty_1 \bowtie \chi_{19}]$, $[\chi_{12}, \infty_2, \infty_7 \bowtie \chi_8]$, $[\chi_{12}, \infty_6, \infty_3 \bowtie \chi_4]$, $[\chi_{12}, \infty_5, \infty_4 \bowtie \chi_{15}]$, $[\chi_{13}, \infty_{11}, \infty_{13} \bowtie \chi_{17}]$, $[\chi_{13}, \infty_1, \infty_{10} \bowtie \chi_{11}]$, $[\chi_{13}, \infty_2, \infty_9 \bowtie \chi_{14}]$, $[\chi_{13}, \infty_3, \infty_8 \bowtie \chi_9]$, $[\chi_{13}, \infty_7, \infty_4 \bowtie \chi_5]$, $[\chi_{13}, \infty_6, \infty_5 \bowtie \chi_6]$, $[\chi_5, \chi_{12}, \chi_{14} \bowtie \infty_7]$, $[\chi_6, \chi_{13}, \chi_{15} \bowtie \infty_5]$, $[\chi_7, \chi_{14}, \chi_{16} \bowtie \infty_2]$, $[\chi_8, \chi_{15}, \chi_{17} \bowtie \infty_2]$, $[\chi_9, \chi_{16}, \chi_{18} \bowtie \infty_1]$, $[\chi_{10}, \chi_{17}, \chi_{19} \bowtie \infty_5]$, $[\chi_{11}, \chi_1, \chi_{18} \bowtie \infty_6]$, $[\chi_{12}, \chi_2, \chi_{19} \bowtie \infty_6]$, $[\chi_6, \chi_4, \chi_{16} \bowtie \infty_7]$, $[\chi_{17}, \chi_5, \chi_7 \bowtie \infty_6]$, $[\chi_8, \chi_6, \chi_{18} \bowtie \infty_7]$, $[\chi_9, \chi_7, \chi_{19} \bowtie \infty_7]$, $[\chi_4, \chi_2, \chi_{14} \bowtie \infty_6]$, $[\chi_{12}, \chi_7, \chi_1 \bowtie \chi_4]$, $[\chi_{13}, \chi_8, \chi_2 \bowtie \chi_{18}]$, $[\chi_{14}, \chi_9, \chi_3 \bowtie \chi_{19}]$, $[\chi_{15}, \chi_{10}, \chi_4 \bowtie \chi_{19}]$, $[\chi_5, \chi_{11}, \chi_{16} \bowtie \chi_{17}]$, $[\chi_6, \chi_{12}, \chi_{17} \bowtie \chi_1]$, $[\chi_7, \chi_{13}, \chi_{18} \bowtie \chi_{19}]$, $[\chi_8, \chi_{14}, \chi_{19} \bowtie \chi_1]$, $[\chi_9, \chi_{15}, \chi_1 \bowtie \chi_{16}]$, $[\chi_{10}, \chi_{16}, \chi_2 \bowtie \chi_{17}]$, $[\chi_{11}, \chi_3, \chi_{17} \bowtie \chi_{18}]$, $[\chi_{12}, \chi_4, \chi_{18} \bowtie \chi_3]$, $[\chi_{13}, \chi_5, \chi_{19} \bowtie \chi_{16}]$, $[\chi_6, \chi_1, \chi_{14} \bowtie \chi_{15}]$, $[\chi_7, \chi_2, \chi_{15} \bowtie \chi_{19}]$, $[\chi_8, \chi_3, \chi_{16} \bowtie \chi_{15}]$, $[\chi_9, \chi_4, \chi_{17} \bowtie \chi_{14}]$, $[\chi_{10}, \chi_5, \chi_{18} \bowtie \chi_{15}]$, $[\chi_{19}, \chi_6, \chi_{11} \bowtie \chi_{12}]$, $[\chi_3, \chi_1, \chi_{13} \bowtie \chi_{17}]$, $[\chi_5, \chi_3, \chi_{15} \bowtie \chi_{12}]$, $[\chi_{10}, \chi_8, \chi_1 \bowtie \chi_2]$, $[\chi_2, \chi_9, \chi_{11} \bowtie \chi_{15}]$, $[\chi_3, \chi_{10}, \chi_{12} \bowtie \chi_{13}]$, $[\chi_4, \chi_{11}, \chi_{13} \bowtie \chi_{14}]$, $[\chi_6, \chi_3, \chi_2 \bowtie \chi_5]$, $[\chi_3, \chi_4, \chi_7 \bowtie \chi_6]$, $[\chi_4, \chi_8, \chi_5 \bowtie \chi_1]$, $[\chi_5, \chi_9, \chi_6 \bowtie \chi_{10}]$, $[\chi_{11}, \chi_8, \chi_7 \bowtie \chi_{10}]$, $[\chi_8, \chi_9, \chi_{12} \bowtie \chi_{16}]$, $[\chi_9, \chi_{10}, \chi_{13} \bowtie \chi_{16}]$, $[\chi_{10}, \chi_{11}, \chi_{14} \bowtie \chi_{18}]$.

The set $\mathcal{D} \setminus \mathcal{A}$ is given by the following kites:

$[a_1, \infty_1, 1 \bowtie \chi_{16}]$, $[a_1, 9, \infty_2 \bowtie a_9]$, $[a_1, 8, \infty_5 \bowtie a_{10}]$, $[a_1, \infty_6, 16 \bowtie \chi_{16}]$, $[a_1, \infty_8, 7 \bowtie a_3]$, $[a_1, 15, \infty_9 \bowtie a_3]$, $[a_1, \infty_{11}, 6 \bowtie \chi_{16}]$, $[a_1, 14, \infty_{12} \bowtie a_4]$, $[a_2, 2, \infty_1 \bowtie a_{18}]$, $[a_2, 10, \infty_2 \bowtie a_{10}]$, $[a_2, 1, \infty_4 \bowtie a_1]$, $[a_2, \infty_5, 9 \bowtie \chi_{16}]$, $[a_2, 8, \infty_8 \bowtie a_{18}]$, $[a_2, 16, \infty_9 \bowtie a_4]$, $[a_2, \infty_{11}, 7 \bowtie \chi_{16}]$, $[a_2, \infty_{12}, 15 \bowtie \chi_{16}]$, $[a_3, 3, \infty_1 \bowtie a_{19}]$, $[a_3, 11, \infty_2 \bowtie a_{11}]$, $[a_3, \infty_4, 2 \bowtie a_{22}]$, $[a_3, \infty_5, 10 \bowtie \chi_{16}]$, $[a_3, \infty_7, 1 \bowtie \chi_{17}]$, $[a_3, 9, \infty_8 \bowtie a_{11}]$, $[a_3, \infty_{11}, 8 \bowtie \chi_{16}]$, $[a_3, 16, \infty_{12} \bowtie a_5]$, $[a_4, \infty_1, 4 \bowtie \chi_{16}]$, $[a_4, 12, \infty_2 \bowtie a_{12}]$, $[a_4, \infty_4, 3 \bowtie \chi_{16}]$, $[a_4, 11, \infty_5 \bowtie a_{11}]$, $[a_4, 2, \infty_7 \bowtie a_2]$, $[a_4, 10, \infty_8 \bowtie a_{12}]$,

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$[a_4, \infty_{10}, 1 \bowtie \chi_{18}]$, $[a_4, \infty_{11}, 9 \bowtie a_{21}]$, $[a_5, 5, \infty_1 \bowtie a_{20}]$, $[a_5, \infty_2, 13 \bowtie \chi_{16}]$, $[a_5, \infty_4, 4 \bowtie a_2]$,
$[a_5, \infty_5, 12 \bowtie \chi_{16}]$, $[a_5, \infty_7, 3 \bowtie a_{24}]$, $[a_5, 11, \infty_8 \bowtie a_{15}]$, $[a_5, \infty_{10}, 2 \bowtie \chi_{16}]$, $[a_5, 10, \infty_{11} \bowtie a_{12}]$,
$[a_5, 1, \infty_{13} \bowtie a_1]$, $[a_6, \infty_1, 6 \bowtie \chi_{17}]$, $[a_6, 14, \infty_2 \bowtie a_{13}]$, $[a_6, 5, \infty_4 \bowtie a_{19}]$, $[a_6, \infty_5, 13 \bowtie \chi_{17}]$,
$[a_6, \infty_7, 4 \bowtie a_{24}]$, $[a_6, 12, \infty_8 \bowtie a_{13}]$, $[a_6, 3, \infty_{10} \bowtie a_3]$, $[a_6, 11, \infty_{11} \bowtie a_{13}]$, $[a_6, 2, \infty_{13} \bowtie a_2]$,
$[a_7, \infty_1, 7 \bowtie \chi_{17}]$, $[a_7, \infty_2, 15 \bowtie \chi_{17}]$, $[a_7, 6, \infty_4 \bowtie a_{24}]$, $[a_7, \infty_5, 14 \bowtie \chi_{16}]$, $[a_7, 5, \infty_7 \bowtie a_{20}]$,
$[a_7, \infty_8, 13 \bowtie \chi_{18}]$, $[a_7, 4, \infty_{10} \bowtie a_1]$, $[a_7, 12, \infty_{11} \bowtie a_{18}]$, $[a_7, \infty_{13}, 3 \bowtie \chi_{17}]$, $[a_8, 8, \infty_1 \bowtie a_{21}]$,
$[a_8, 16, \infty_2 \bowtie a_{14}]$, $[a_8, \infty_4, 7 \bowtie \chi_{18}]$, $[a_8, 15, \infty_5 \bowtie a_{12}]$, $[a_8, 6, \infty_7 \bowtie a_{22}]$, $[a_8, \infty_8, 14 \bowtie a_{19}]$,
$[a_8, \infty_{10}, 5 \bowtie \chi_{16}]$, $[a_8, 13, \infty_{11} \bowtie a_{14}]$, $[a_8, 4, \infty_{13} \bowtie a_3]$, $[a_9, 9, \infty_1 \bowtie a_{22}]$, $[a_9, 1, \infty_3 \bowtie a_2]$,
$[a_9, 8, \infty_4 \bowtie a_{20}]$, $[a_9, \infty_5, 16 \bowtie \chi_{17}]$, $[a_9, \infty_7, 7 \bowtie \chi_{19}]$, $[a_9, \infty_8, 15 \bowtie a_3]$, $[a_9, \infty_{10}, 6 \bowtie a_{11}]$,
$[a_9, \infty_{11}, 14 \bowtie a_3]$, $[a_9, \infty_{13}, 5 \bowtie \chi_{17}]$, $[a_{10}, 10, \infty_1 \bowtie a_{23}]$, $[a_{10}, \infty_3, 2 \bowtie \chi_{17}]$, $[a_{10}, 9, \infty_4 \bowtie a_{21}]$,
$[a_{10}, \infty_6, 1 \bowtie \chi_{19}]$, $[a_{10}, \infty_7, 8 \bowtie a_5]$, $[a_{10}, 16, \infty_8 \bowtie a_{16}]$, $[a_{10}, \infty_{10}, 7 \bowtie a_{14}]$, $[a_{10}, 15, \infty_{11} \bowtie a_{15}]$,
$[a_{10}, 6, \infty_{13} \bowtie a_4]$, $[a_{11}, 11, \infty_1 \bowtie a_{24}]$, $[a_{11}, 3, \infty_3 \bowtie a_3]$, $[a_{11}, 10, \infty_4 \bowtie a_{22}]$, $[a_{11}, \infty_6, 2 \bowtie \chi_{18}]$,
$[a_{11}, \infty_7, 9 \bowtie a_5]$, $[a_{11}, 1, \infty_9 \bowtie a_5]$, $[a_{11}, \infty_{10}, 8 \bowtie \chi_{18}]$, $[a_{11}, \infty_{11}, 16 \bowtie a_4]$, $[a_{11}, 7, \infty_{13} \bowtie a_{21}]$,
$[a_{12}, \infty_1, 12 \bowtie \chi_{19}]$, $[a_{12}, \infty_3, 4 \bowtie a_1]$, $[a_{12}, 11, \infty_4, \bowtie a_{23}]$, $[a_{12}, 3, \infty_6 \bowtie a_2]$, $[a_{12}, \infty_7, 10 \bowtie a_1]$,
$[a_{12}, \infty_9, 2 \bowtie \chi_{19}]$, $[a_{12}, 9, \infty_{10} \bowtie a_{21}]$, $[a_{12}, \infty_{12}, 1 \bowtie a_6]$, $[a_{12}, \infty_{13}, 8 \bowtie \chi_{19}]$, $[a_{13}, \infty_1, 13 \bowtie a_2]$,
$[a_{13}, \infty_3, 5 \bowtie a_1]$, $[a_{13}, \infty_4, 12 \bowtie a_2]$, $[a_{13}, 4, \infty_6 \bowtie a_3]$, $[a_{13}, \infty_7, 11 \bowtie a_9]$, $[a_{13}, \infty_9, 3 \bowtie \chi_{18}]$,
$[a_{13}, \infty_{10}, 10 \bowtie a_{16}]$, $[a_{13}, \infty_{12}, 2 \bowtie a_{24}]$, $[a_{13}, 9, \infty_{13} \bowtie a_{24}]$, $[a_{14}, \infty_1, 14 \bowtie \chi_{17}]$, $[a_{14}, 6, \infty_3 \bowtie$
$a_4]$, $[a_{14}, \infty_4, 13 \bowtie a_3]$, $[a_{14}, 5, \infty_6 \bowtie a_4]$, $[a_{14}, \infty_7, 12 \bowtie a_{18}]$, $[a_{14}, 4, \infty_9 \bowtie a_6]$, $[a_{14}, 11, \infty_{10} \bowtie$
$a_{22}]$, $[a_{14}, \infty_{12}, 3 \bowtie \chi_{19}]$, $[a_{14}, 10, \infty_{13} \bowtie a_{22}]$, $[a_{15}, 15, \infty_1 \bowtie a_{17}]$, $[a_{15}, 7, \infty_3 \bowtie a_5]$, $[a_{15}, 14, \infty_4 \bowtie$
$a_{18}]$, $[a_{15}, 6, \infty_6 \bowtie a_5]$, $[a_{15}, 13, \infty_7 \bowtie a_{19}]$, $[a_{15}, 5, \infty_9 \bowtie a_7]$, $[a_{15}, \infty_{10}, 12 \bowtie \chi_{17}]$, $[a_{15}, 4, \infty_{12} \bowtie$
$a_{11}]$, $[a_{15}, \infty_{13}, 11 \bowtie \chi_{16}]$, $[a_{16}, \infty_1, 16 \bowtie \chi_{18}]$, $[a_{16}, 8, \infty_3 \bowtie a_6]$, $[a_{16}, \infty_4, 15 \bowtie \chi_{18}]$, $[a_{16}, 7, \infty_6 \bowtie$
$a_6]$, $[a_{16}, 14, \infty_7 \bowtie a_{21}]$, $[a_{16}, 6, \infty_9, \bowtie a_8]$, $[a_{16}, 13, \infty_{10}, \bowtie a_{23}]$, $[a_{16}, 5, \infty_{12} \bowtie a_6]$, $[a_{16}, \infty_{13}, 12 \bowtie$
$\chi_{18}]$, $[a_{17}, \infty_2, 1 \bowtie a_7]$, $[a_{17}, \infty_3, 9 \bowtie \chi_{17}]$, $[a_{17}, \infty_4, 16 \bowtie \chi_{19}]$, $[a_{17}, 8, \infty_6 \bowtie a_7]$, $[a_{17}, 15, \infty_7 \bowtie$
$a_{23}]$, $[a_{17}, 7, \infty_9 \bowtie a_9]$, $[a_{17}, \infty_{10}, 14 \bowtie a_2]$, $[a_{17}, \infty_{12}, 6 \bowtie a_{20}]$, $[a_{17}, \infty_{13}, 13 \bowtie a_4]$, $[a_{18}, 2, \infty_2 \bowtie$
$a_{16}]$, $[a_{18}, \infty_3, 10 \bowtie a_{23}]$, $[a_{18}, 1, \infty_5 \bowtie a_{13}]$, $[a_{18}, \infty_6, 9 \bowtie \chi_{18}]$, $[a_{18}, 16, \infty_7 \bowtie a_{24}]$, $[a_{18}, \infty_9, 8 \bowtie$
$a_{14}]$, $[a_{18}, \infty_{10}, 15 \bowtie \chi_{19}]$, $[a_{18}, 7, \infty_{12} \bowtie a_7]$, $[a_{18}, 14, \infty_{13} \bowtie a_{23}]$, $[a_{19}, 3, \infty_2 \bowtie a_{15}]$, $[a_{19}, \infty_3, 11 \bowtie$
$\chi_{17}]$, $[a_{19}, 2, \infty_5 \bowtie a_{14}]$, $[a_{19}, \infty_6, 10 \bowtie \chi_{17}]$, $[a_{19}, 1, \infty_8 \bowtie a_{14}]$, $[a_{19}, \infty_9, 9 \bowtie \chi_{19}]$, $[a_{19}, 16, \infty_{10} \bowtie$
$a_{24}]$, $[a_{19}, 8, \infty_{12} \bowtie a_8]$, $[a_{19}, \infty_{13}, 15 \bowtie a_4]$, $[a_{20}, \infty_2, 4 \bowtie \chi_{17}]$, $[a_{20}, 12, \infty_3 \bowtie a_7]$, $[a_{20}, 3, \infty_5 \bowtie$
$a_{15}]$, $[a_{20}, \infty_6, 11 \bowtie \chi_{19}]$, $[a_{20}, \infty_8, 2 \bowtie a_{23}]$, $[a_{20}, \infty_9, 10 \bowtie \chi_{18}]$, $[a_{20}, 1, \infty_{11} \bowtie a_{16}]$, $[a_{20}, 9, \infty_{12} \bowtie$
$a_9]$, $[a_{20}, \infty_{13}, 16 \bowtie a_5]$, $[a_{21}, \infty_2, 5 \bowtie \chi_{18}]$, $[a_{21}, 13, \infty_3 \bowtie a_1]$, $[a_{21}, \infty_5, 4 \bowtie \chi_{18}]$, $[a_{21}, 12, \infty_6 \bowtie$
$a_8]$, $[a_{21}, 3, \infty_8 \bowtie a_{17}]$, $[a_{21}, \infty_9, 11 \bowtie a_1]$, $[a_{21}, 2, \infty_{11} \bowtie a_{17}]$, $[a_{21}, \infty_{12}, 10 \bowtie \chi_{19}]$, $[a_{22}, \infty_2, 6 \bowtie$
$\chi_{18}]$, $[a_{22}, 14, \infty_3 \bowtie a_8]$, $[a_{22}, 5, \infty_5 \bowtie a_{16}]$, $[a_{22}, \infty_6, 13 \bowtie \chi_{19}]$, $[a_{22}, \infty_8, 4 \bowtie \chi_{19}]$, $[a_{22}, \infty_9, 12 \bowtie$
$a_1]$, $[a_{22}, 3, \infty_{11} \bowtie a_{19}]$, $[a_{22}, \infty_{12}, 11 \bowtie \chi_{18}]$, $[a_{23}, \infty_2, 7 \bowtie a_{21}]$, $[a_{23}, \infty_3, 15 \bowtie a_5]$, $[a_{23}, \infty_5, 6 \bowtie$
$a_{21}]$, $[a_{23}, \infty_6, 14 \bowtie \chi_{18}]$, $[a_{23}, \infty_8, 5 \bowtie \chi_{19}]$, $[a_{23}, 13, \infty_9 \bowtie a_{10}]$, $[a_{23}, \infty_{11}, 4 \bowtie a_{18}]$, $[a_{23}, 12, \infty_{12} \bowtie$
$a_{10}]$, $[a_{24}, \infty_2, 8 \bowtie \chi_{17}]$, $[a_{24}, \infty_3, 16 \bowtie a_6]$, $[a_{24}, 7, \infty_5 \bowtie a_{17}]$, $[a_{24}, 15, \infty_6 \bowtie a_9]$, $[a_{24}, \infty_8, 6 \bowtie$
$\chi_{19}]$, $[a_{24}, \infty_9, 14 \bowtie \chi_{19}]$, $[a_{24}, \infty_{11}, 5 \bowtie a_2]$, $[a_{24}, \infty_{12}, 13 \bowtie a_1]$, $[\chi_1, 16, 2 \bowtie a_1]$, $[\chi_1, 1, 3 \bowtie a_8]$,
$[\chi_1, 4, 15 \bowtie a_6]$, $[\chi_1, 14, 5 \bowtie a_{10}]$, $[\chi_1, 13, 6 \bowtie a_2]$, $[\chi_1, 7, 12 \bowtie a_8]$, $[\chi_1, 8, 11 \bowtie a_8]$, $[\chi_1, 10, 9 \bowtie a_6]$,
$[\chi_2, 1, 16 \bowtie a_7]$, $[\chi_2, 15, 2 \bowtie a_8]$, $[\chi_2, 14, 3 \bowtie a_2]$, $[\chi_2, 13, 4 \bowtie a_3]$, $[\chi_2, 12, 5 \bowtie a_3]$, $[\chi_2, 6, 11 \bowtie a_2]$,
$[\chi_2, 10, 7 \bowtie a_6]$, $[\chi_2, 8, 9 \bowtie a_{23}]$, $[\chi_3, 3, 16 \bowtie a_{23}]$, $[\chi_3, 4, 2 \bowtie a_7]$, $[\chi_3, 5, 1 \bowtie a_{15}]$, $[\chi_3, 15, 6 \bowtie$
$a_3]$, $[\chi_3, 14, 7 \bowtie a_{20}]$, $[\chi_3, 13, 8 \bowtie a_4]$, $[\chi_3, 12, 9 \bowtie a_{22}]$, $[\chi_3, 11, 10 \bowtie a_6]$, $[\chi_4, 16, 4 \bowtie a_9]$,
$[\chi_4, 5, 3 \bowtie a_9]$, $[\chi_4, 2, 6 \bowtie a_4]$, $[\chi_4, 1, 7 \bowtie a_5]$, $[\chi_4, 15, 8 \bowtie a_6]$, $[\chi_4, 9, 14 \bowtie a_4]$, $[\chi_4, 13, 10 \bowtie a_8]$,
$[\chi_4, 12, 11 \bowtie a_7]$, $[\chi_5, 16, 5 \bowtie a_{11}]$, $[\chi_5, 4, 6 \bowtie a_5]$, $[\chi_5, 3, 7 \bowtie a_{12}]$, $[\chi_5, 2, 8 \bowtie a_7]$, $[\chi_5, 1, 9 \bowtie$
$a_7]$, $[\chi_5, 15, 10 \bowtie a_7]$, $[\chi_5, 14, 11 \bowtie a_{23}]$, $[\chi_5, 13, 12 \bowtie a_9]$, $[\chi_6, 6, 16 \bowtie a_{22}]$, $[\chi_6, 7, 5 \bowtie a_{12}]$,

$[\chi_6, 4, 8 \bowtie a_{22}]$, $[\chi_6, 3, 9 \bowtie a_8]$, $[\chi_6, 10, 2 \bowtie a_9]$, $[\chi_6, 11, 1 \bowtie a_{24}]$, $[\chi_6, 12, 15 \bowtie a_{11}]$, $[\chi_6, 14, 13 \bowtie a_9]$, $[\chi_7, 7, 16 \bowtie a_{21}]$, $[\chi_7, 6, 8 \bowtie a_{20}]$, $[\chi_7, 5, 9 \bowtie a_{14}]$, $[\chi_7, 4, 10 \bowtie a_9]$, $[\chi_7, 3, 11 \bowtie a_{10}]$, $[\chi_7, 2, 12 \bowtie a_{10}]$, $[\chi_7, 13, 1 \bowtie a_8]$, $[\chi_7, 15, 14 \bowtie a_{10}]$, $[\chi_8, 8, 16 \bowtie a_{12}]$, $[\chi_8, 7, 9 \bowtie a_{24}]$, $[\chi_8, 6, 10 \bowtie a_{24}]$, $[\chi_8, 5, 11 \bowtie a_{24}]$, $[\chi_8, 4, 12 \bowtie a_{11}]$, $[\chi_8, 3, 13 \bowtie a_{10}]$, $[\chi_8, 2, 14 \bowtie a_{11}]$, $[\chi_8, 1, 15 \bowtie a_{12}]$, $[\chi_9, 9, 16 \bowtie a_{13}]$, $[\chi_9, 10, 8 \bowtie a_{13}]$, $[\chi_9, 11, 7 \bowtie a_{13}]$, $[\chi_9, 12, 6 \bowtie a_{13}]$, $[\chi_9, 5, 13 \bowtie a_{11}]$, $[\chi_9, 14, 4 \bowtie a_{11}]$, $[\chi_9, 3, 15 \bowtie a_{13}]$, $[\chi_9, 2, 1 \bowtie a_{14}]$, $[\chi_{10}, 10, 16 \bowtie a_{14}]$, $[\chi_{10}, 11, 9 \bowtie a_{15}]$, $[\chi_{10}, 8, 12 \bowtie a_{24}]$, $[\chi_{10}, 7, 13 \bowtie a_{12}]$, $[\chi_{10}, 6, 14 \bowtie a_{12}]$, $[\chi_{10}, 15, 5 \bowtie a_{17}]$, $[\chi_{10}, 1, 4 \bowtie a_{19}]$, $[\chi_{10}, 3, 2 \bowtie a_{14}]$, $[\chi_{11}, 11, 16 \bowtie a_{15}]$, $[\chi_{11}, 10, 12 \bowtie a_3]$, $[\chi_{11}, 13, 9 \bowtie a_{16}]$, $[\chi_{11}, 8, 14 \bowtie a_{13}]$, $[\chi_{11}, 7, 15 \bowtie a_{22}]$, $[\chi_{11}, 6, 1 \bowtie a_{16}]$, $[\chi_{11}, 5, 2 \bowtie a_{15}]$, $[\chi_{11}, 4, 3 \bowtie a_{15}]$, $[\chi_{12}, 16, 12 \bowtie a_{17}]$, $[\chi_{12}, 11, 13 \bowtie a_{18}]$, $[\chi_{12}, 10, 14 \bowtie a_{20}]$, $[\chi_{12}, 9, 15 \bowtie a_{14}]$, $[\chi_{12}, 8, 1 \bowtie a_{13}]$, $[\chi_{12}, 7, 2 \bowtie a_{16}]$, $[\chi_{12}, 6, 3 \bowtie a_{16}]$, $[\chi_{12}, 5, 4 \bowtie a_{16}]$, $[\chi_{13}, 16, 13 \bowtie a_{19}]$, $[\chi_{13}, 12, 14 \bowtie a_{21}]$, $[\chi_{13}, 15, 11 \bowtie a_{16}]$, $[\chi_{13}, 10, 1 \bowtie a_{21}]$, $[\chi_{13}, 9, 2 \bowtie a_{17}]$, $[\chi_{13}, 8, 3 \bowtie a_{17}]$, $[\chi_{13}, 7, 4 \bowtie a_{17}]$, $[\chi_{13}, 6, 5 \bowtie a_{18}]$, $[\chi_{14}, 16, 14 \bowtie a_5]$, $[\chi_{14}, 13, 15 \bowtie a_{20}]$, $[\chi_{14}, 12, 1 \bowtie a_{22}]$, $[\chi_{14}, 2, 11 \bowtie a_{17}]$, $[\chi_{14}, 10, 3 \bowtie a_{18}]$, $[\chi_{14}, 9, 4 \bowtie a_{10}]$, $[\chi_{14}, 8, 5 \bowtie a_{20}]$, $[\chi_{14}, 7, 6 \bowtie a_{19}]$, $[\chi_{15}, 16, 15 \bowtie a_{21}]$, $[\chi_{15}, 14, 1 \bowtie a_{23}]$, $[\chi_{15}, 2, 13 \bowtie a_{20}]$, $[\chi_{15}, 3, 12 \bowtie a_{19}]$, $[\chi_{15}, 4, 11 \bowtie a_{18}]$, $[\chi_{15}, 10, 5 \bowtie a_{19}]$, $[\chi_{15}, 9, 6 \bowtie a_{18}]$, $[\chi_{15}, 8, 7 \bowtie a_{19}]$, $[\chi_1, a_1, a_{13} \bowtie \chi_2]$, $[\chi_1, a_2, a_{14} \bowtie \chi_2]$, $[\chi_1, a_3, a_{15} \bowtie \chi_2]$, $[\chi_1, a_4, a_{16} \bowtie \chi_2]$, $[\chi_1, a_5, a_{17} \bowtie \chi_2]$, $[\chi_1, a_6, a_{18} \bowtie \chi_2]$, $[\chi_1, a_7, a_{19} \bowtie \chi_2]$, $[\chi_1, a_8, a_{20} \bowtie \chi_2]$, $[\chi_1, a_9, a_{21} \bowtie \chi_2]$, $[\chi_1, a_{10}, a_{22} \bowtie \chi_2]$, $[\chi_1, a_{11}, a_{23} \bowtie \chi_2]$, $[\chi_1, a_{12}, a_{24} \bowtie \chi_2]$, $[\chi_3, a_{14}, a_1 \bowtie \chi_2]$, $[\chi_3, a_{15}, a_2 \bowtie \chi_2]$, $[\chi_3, a_{16}, a_3 \bowtie \chi_2]$, $[\chi_3, a_{17}, a_4 \bowtie \chi_2]$, $[\chi_3, a_{18}, a_5 \bowtie \chi_2]$, $[\chi_3, a_{19}, a_6 \bowtie \chi_2]$, $[\chi_3, a_{20}, a_7 \bowtie \chi_2]$, $[\chi_3, a_{21}, a_8 \bowtie \chi_2]$, $[\chi_3, a_{22}, a_9 \bowtie \chi_2]$, $[\chi_3, a_{23}, a_{10} \bowtie \chi_2]$, $[\chi_3, a_{24}, a_{11} \bowtie \chi_2]$, $[\chi_3, a_{13}, a_{12} \bowtie \chi_2]$, $[\chi_4, a_1, a_{15} \bowtie \chi_5]$, $[\chi_4, a_2, a_{16} \bowtie \chi_5]$, $[\chi_4, a_3, a_{17} \bowtie \chi_5]$, $[\chi_4, a_4, a_{18} \bowtie \chi_5]$, $[\chi_4, a_5, a_{19} \bowtie \chi_5]$, $[\chi_4, a_6, a_{20} \bowtie \chi_5]$, $[\chi_4, a_7, a_{21} \bowtie \chi_5]$, $[\chi_4, a_8, a_{22} \bowtie \chi_5]$, $[\chi_4, a_9, a_{23} \bowtie \chi_5]$, $[\chi_4, a_{10}, a_{24} \bowtie \chi_5]$, $[\chi_4, a_{11}, a_{13} \bowtie \chi_5]$, $[\chi_4, a_{12}, a_{14} \bowtie \chi_5]$, $[\chi_6, a_{16}, a_1 \bowtie \chi_5]$, $[\chi_6, a_{17}, a_2 \bowtie \chi_5]$, $[\chi_6, a_{18}, a_3 \bowtie \chi_5]$, $[\chi_6, a_{19}, a_4 \bowtie \chi_5]$, $[\chi_6, a_{20}, a_5 \bowtie \chi_5]$, $[\chi_6, a_{21}, a_6 \bowtie \chi_5]$, $[\chi_6, a_{22}, a_7 \bowtie \chi_5]$, $[\chi_6, a_{23}, a_8 \bowtie \chi_5]$, $[\chi_6, a_{24}, a_9 \bowtie \chi_5]$, $[\chi_6, a_{13}, a_{10} \bowtie \chi_5]$, $[\chi_6, a_{14}, a_{11} \bowtie \chi_5]$, $[\chi_6, a_{15}, a_{12} \bowtie \chi_5]$, $[\chi_7, a_1, a_{17} \bowtie \chi_8]$, $[\chi_7, a_2, a_{18} \bowtie \chi_8]$, $[\chi_7, a_3, a_{19} \bowtie \chi_8]$, $[\chi_7, a_4, a_{20} \bowtie \chi_8]$, $[\chi_7, a_5, a_{21} \bowtie \chi_8]$, $[\chi_7, a_6, a_{22} \bowtie \chi_8]$, $[\chi_7, a_7, a_{23} \bowtie \chi_8]$, $[\chi_7, a_8, a_{24} \bowtie \chi_8]$, $[\chi_7, a_9, a_{13} \bowtie \chi_8]$, $[\chi_7, a_{10}, a_{14} \bowtie \chi_8]$, $[\chi_7, a_{11}, a_{15} \bowtie \chi_8]$, $[\chi_7, a_{12}, a_{16} \bowtie \chi_8]$, $[\chi_9, a_{18}, a_1 \bowtie \chi_8]$, $[\chi_9, a_{19}, a_2 \bowtie \chi_8]$, $[\chi_9, a_{20}, a_3 \bowtie \chi_8]$, $[\chi_9, a_{21}, a_4 \bowtie \chi_8]$, $[\chi_9, a_{22}, a_5 \bowtie \chi_8]$, $[\chi_9, a_{23}, a_6 \bowtie \chi_8]$, $[\chi_9, a_{24}, a_7 \bowtie \chi_8]$, $[\chi_9, a_{13}, a_8 \bowtie \chi_8]$, $[\chi_9, a_{14}, a_9 \bowtie \chi_8]$, $[\chi_9, a_{15}, a_{10} \bowtie \chi_8]$, $[\chi_9, a_{16}, a_{11} \bowtie \chi_8]$, $[\chi_9, a_{17}, a_{12} \bowtie \chi_8]$, $[\chi_{10}, a_{20}, a_1 \bowtie a_{19}]$, $[\chi_{10}, a_{21}, a_2 \bowtie a_{20}]$, $[\chi_{10}, a_{22}, a_3 \bowtie a_{21}]$, $[\chi_{10}, a_{23}, a_4 \bowtie 5]$, $[\chi_{10}, a_{24}, a_5 \bowtie a_{23}]$, $[\chi_{10}, a_{13}, a_6 \bowtie a_{24}]$, $[\chi_{10}, a_{14}, a_7 \bowtie a_{13}]$, $[\chi_{10}, a_{15}, a_8 \bowtie a_{14}]$, $[\chi_{10}, a_{16}, a_9 \bowtie a_{15}]$, $[\chi_{10}, a_{17}, a_{10} \bowtie a_{16}]$, $[\chi_{10}, a_{18}, a_{11} \bowtie a_{17}]$, $[\chi_{10}, a_{19}, a_{12} \bowtie a_{18}]$, $[\chi_{11}, a_{22}, a_1 \bowtie a_{21}]$, $[\chi_{11}, a_{23}, a_2 \bowtie a_{22}]$, $[\chi_{11}, a_{24}, a_3 \bowtie a_{23}]$, $[\chi_{11}, a_{13}, a_4 \bowtie a_{24}]$, $[\chi_{11}, a_{14}, a_5 \bowtie a_{13}]$, $[\chi_{11}, a_{15}, a_6 \bowtie a_{14}]$, $[\chi_{11}, a_{16}, a_7 \bowtie a_{15}]$, $[\chi_{11}, a_{17}, a_8 \bowtie a_{16}]$, $[\chi_{11}, a_{18}, a_9 \bowtie a_{17}]$, $[\chi_{11}, a_{19}, a_{10} \bowtie a_{18}]$, $[\chi_{11}, a_{20}, a_{11} \bowtie a_{19}]$, $[\chi_{11}, a_{21}, a_{12} \bowtie a_{20}]$, $[\chi_{12}, a_{24}, a_1 \bowtie \infty_7]$, $[\chi_{12}, a_{13}, a_2 \bowtie a_{24}]$, $[\chi_{12}, a_{14}, a_3 \bowtie a_{13}]$, $[\chi_{12}, a_{15}, a_4 \bowtie a_{14}]$, $[\chi_{12}, a_{16}, a_5 \bowtie a_{15}]$, $[\chi_{12}, a_{17}, a_6 \bowtie a_{16}]$, $[\chi_{12}, a_{18}, a_7 \bowtie a_{17}]$, $[\chi_{12}, a_{19}, a_8 \bowtie a_{18}]$, $[\chi_{12}, a_{20}, a_9 \bowtie a_{19}]$, $[\chi_{12}, a_{21}, a_{10} \bowtie a_{20}]$, $[\chi_{12}, a_{22}, a_{11} \bowtie a_{21}]$, $[\chi_{12}, a_{23}, a_{12} \bowtie a_{22}]$, $[\chi_{13}, a_8, a_1 \bowtie a_7]$, $[\chi_{13}, a_9, a_2 \bowtie a_8]$, $[\chi_{13}, a_{10}, a_3 \bowtie a_9]$, $[\chi_{13}, a_{11}, a_4 \bowtie a_{10}]$, $[\chi_{13}, a_{12}, a_5 \bowtie a_{11}]$, $[\chi_{13}, a_7, a_6 \bowtie a_{12}]$, $[\chi_{13}, a_{20}, a_{13} \bowtie a_{19}]$, $[\chi_{13}, a_{21}, a_{14} \bowtie a_{20}]$, $[\chi_{13}, a_{22}, a_{15} \bowtie 10]$, $[\chi_{13}, a_{23}, a_{16} \bowtie a_{22}]$, $[\chi_{13}, a_{24}, a_{17} \bowtie a_{23}]$, $[\chi_{13}, a_{19}, a_{18} \bowtie a_{24}]$, $[\chi_{14}, a_{10}, a_1 \bowtie \chi_{15}]$, $[\chi_{14}, a_{11}, a_2 \bowtie \chi_{15}]$, $[\chi_{14}, a_{12}, a_3 \bowtie \chi_{15}]$, $[\chi_{14}, a_7, a_4 \bowtie \chi_{15}]$, $[\chi_{14}, a_8, a_5 \bowtie \chi_{15}]$, $[\chi_{14}, a_9, a_6 \bowtie \chi_{15}]$, $[\chi_{14}, a_{22}, a_{13} \bowtie \chi_{15}]$, $[\chi_{14}, a_{23}, a_{14} \bowtie \chi_{15}]$, $[\chi_{14}, a_{24}, a_{15} \bowtie \chi_{15}]$, $[\chi_{14}, a_{19}, a_{16} \bowtie \chi_{15}]$, $[\chi_{14}, a_{20}, a_{17} \bowtie \chi_{15}]$, $[\chi_{14}, a_{21}, a_{18} \bowtie \chi_{15}]$, $[\chi_{16}, a_1, a_9 \bowtie \chi_{15}]$, $[\chi_{16}, a_2, a_{10} \bowtie \chi_{15}]$, $[\chi_{16}, a_3, a_{11} \bowtie \chi_{15}]$, $[\chi_{16}, a_4, a_{12} \bowtie \chi_{15}]$, $[\chi_{16}, a_5, a_7 \bowtie \chi_{15}]$, $[\chi_{16}, a_6, a_8 \bowtie \chi_{15}]$, $[\chi_{16}, a_{13}, a_{21} \bowtie \chi_{15}]$, $[\chi_{16}, a_{14}, a_{22} \bowtie$

$\chi_{15}]$, $[\chi_{16}, a_{15}, a_{23} \bowtie \chi_{15}]$, $[\chi_{16}, a_{16}, a_{24} \bowtie \chi_{15}]$, $[\chi_{16}, a_{17}, a_{19} \bowtie \chi_{15}]$, $[\chi_{16}, a_{18}, a_{20} \bowtie \chi_{15}]$, $[\chi_{17}, a_{12}, a_1 \bowtie \chi_{18}]$, $[\chi_{17}, a_7, a_2 \bowtie \chi_{18}]$, $[\chi_{17}, a_8, a_3 \bowtie \chi_{18}]$, $[\chi_{17}, a_9, a_4 \bowtie \chi_{18}]$, $[\chi_{17}, a_{10}, a_5 \bowtie \chi_{18}]$, $[\chi_{17}, a_{11}, a_6 \bowtie \chi_{18}]$, $[\chi_{17}, a_{24}, a_{13} \bowtie \chi_{18}]$, $[\chi_{17}, a_{19}, a_{14} \bowtie \chi_{18}]$, $[\chi_{17}, a_{20}, a_{15} \bowtie \chi_{18}]$, $[\chi_{17}, a_{21}, a_{16} \bowtie \chi_{18}]$, $[\chi_{17}, a_{22}, a_{17} \bowtie \chi_{18}]$, $[\chi_{17}, a_{23}, a_{18} \bowtie \chi_{18}]$, $[\chi_{19}, a_1, a_{11} \bowtie \chi_{18}]$, $[\chi_{19}, a_2, a_{12} \bowtie \chi_{18}]$, $[\chi_{19}, a_3, a_7 \bowtie \chi_{18}]$, $[\chi_{19}, a_4, a_8 \bowtie \chi_{18}]$, $[\chi_{19}, a_5, a_9 \bowtie \chi_{18}]$, $[\chi_{19}, a_6, a_{10} \bowtie \chi_{18}]$, $[\chi_{19}, a_{13}, a_{23} \bowtie \chi_{18}]$, $[\chi_{19}, a_{14}, a_{24} \bowtie \chi_{18}]$, $[\chi_{19}, a_{15}, a_{19} \bowtie \chi_{18}]$, $[\chi_{19}, a_{16}, a_{20} \bowtie \chi_{18}]$, $[\chi_{19}, a_{17}, a_{21} \bowtie \chi_{18}]$, $[\chi_{19}, a_{18}, a_{22} \bowtie \chi_{18}]$, $[a_1, a_{23}, 3 \bowtie a_{10}]$, $[7, a_4, a_{22} \bowtie 10]$, $[a_{15}, a_{21}, 8 \bowtie a_{23}]$, $[a_5, a_4, a_2 \bowtie \infty_{10}]$, $[a_{11}, a_8, a_{12} \bowtie 6]$, $[a_{23}, a_{24}, a_{20} \bowtie \infty_{10}]$, $[a_{18}, a_{14}, a_{17} \bowtie 10]$, $[a_3, a_5, a_1 \bowtie a_2]$, $[a_1, a_6, a_4 \bowtie a_3]$, $[a_2, a_3, a_6 \bowtie a_5]$, $[a_9, a_{11}, a_7 \bowtie a_8]$, $[a_7, a_{10}, a_{12} \bowtie a_9]$, $[a_8, a_9, a_{10} \bowtie a_{11}]$, $[a_{15}, a_{17}, a_{13} \bowtie a_{14}]$, $[a_{13}, a_{16}, a_{18} \bowtie a_{15}]$, $[a_{14}, a_{15}, a_{16} \bowtie a_{17}]$, $[a_{21}, a_{23}, a_{19} \bowtie a_{20}]$, $[a_{19}, a_{22}, a_{24} \bowtie a_{21}]$, $[a_{20}, a_{21}, a_{22} \bowtie a_{23}]$.

Case $\alpha = 13$ and $\beta = 20$. Let $X = \{\infty_1, \infty_2, \ldots, \infty_{13}\}$, $Y = \{\chi_1, \chi_2, \ldots, \chi_{20}\}$. The set $\mathcal{A}$ is given by the following kites:

$[\chi_{13}, \infty_1, \infty_{12} \bowtie \chi_{12}]$, $[\chi_{13}, \infty_2, \infty_{11} \bowtie \chi_{14}]$, $[\chi_{13}, \infty_3, \infty_{10} \bowtie \chi_{15}]$, $[\chi_{13}, \infty_4, \infty_9 \bowtie \chi_{19}]$, $[\chi_{13}, \infty_5, \infty_8 \bowtie \chi_{17}]$, $[\chi_{13}, \infty_6, \infty_7 \bowtie \chi_{20}]$, $[\chi_1, \infty_2, \infty_{13} \bowtie \chi_{13}]$, $[\chi_1, \infty_3, \infty_{12} \bowtie \chi_{19}]$, $[\chi_1, \infty_4, \infty_{11} \bowtie \chi_{15}]$, $[\chi_1, \infty_5, \infty_{10} \bowtie \chi_{16}]$, $[\chi_1, \infty_6, \infty_9 \bowtie \chi_{17}]$, $[\chi_1, \infty_7, \infty_8 \bowtie \chi_{18}]$, $[\chi_2, \infty_3, \infty_1 \bowtie \chi_1]$, $[\chi_2, \infty_4, \infty_{13} \bowtie \chi_{14}]$, $[\chi_2, \infty_5, \infty_{12} \bowtie \chi_{15}]$, $[\chi_2, \infty_6, \infty_{11} \bowtie \chi_{16}]$, $[\chi_2, \infty_7, \infty_{10} \bowtie \chi_{19}]$, $[\chi_2, \infty_8, \infty_9 \bowtie \chi_{18}]$, $[\chi_3, \infty_4, \infty_2 \bowtie \chi_2]$, $[\chi_3, \infty_5, \infty_1 \bowtie \chi_{14}]$, $[\chi_3, \infty_6, \infty_{13} \bowtie \chi_{19}]$, $[\chi_3, \infty_7, \infty_{12} \bowtie \chi_{16}]$, $[\chi_3, \infty_8, \infty_{11} \bowtie \chi_{17}]$, $[\chi_3, \infty_9, \infty_{10} \bowtie \chi_{18}]$, $[\chi_4, \infty_5, \infty_3 \bowtie \chi_3]$, $[\chi_4, \infty_6, \infty_2 \bowtie \chi_{14}]$, $[\chi_4, \infty_7, \infty_1 \bowtie \chi_{15}]$, $[\chi_4, \infty_8, \infty_{13} \bowtie \chi_{16}]$, $[\chi_4, \infty_9, \infty_{12} \bowtie \chi_{20}]$, $[\chi_4, \infty_{10}, \infty_{11} \bowtie \chi_{19}]$, $[\chi_5, \infty_6, \infty_4 \bowtie \chi_4]$, $[\chi_5, \infty_7, \infty_3 \bowtie \chi_{14}]$, $[\chi_5, \infty_8, \infty_2 \bowtie \chi_{15}]$, $[\chi_5, \infty_9, \infty_1 \bowtie \chi_{16}]$, $[\chi_5, \infty_{10}, \infty_{13} \bowtie \chi_{17}]$, $[\chi_5, \infty_{11}, \infty_{12} \bowtie \chi_{18}]$, $[\chi_6, \infty_7, \infty_5 \bowtie \chi_5]$, $[\chi_6, \infty_8, \infty_4 \bowtie \chi_{14}]$, $[\chi_6, \infty_9, \infty_3 \bowtie \chi_{15}]$, $[\chi_6, \infty_{10}, \infty_2 \bowtie \chi_{16}]$, $[\chi_6, \infty_{11}, \infty_1 \bowtie \chi_{17}]$, $[\chi_6, \infty_{12}, \infty_{13} \bowtie \chi_{20}]$, $[\chi_7, \infty_8, \infty_6 \bowtie \chi_6]$, $[\chi_7, \infty_9, \infty_5 \bowtie \chi_{14}]$, $[\chi_7, \infty_{10}, \infty_4 \bowtie \chi_{15}]$, $[\chi_7, \infty_{11}, \infty_3 \bowtie \chi_{16}]$, $[\chi_7, \infty_{12}, \infty_2 \bowtie \chi_{17}]$, $[\chi_7, \infty_{13}, \infty_1 \bowtie \chi_{18}]$, $[\chi_8, \infty_9, \infty_7 \bowtie \chi_7]$, $[\chi_8, \infty_{10}, \infty_6 \bowtie \chi_{14}]$, $[\chi_8, \infty_{11}, \infty_5 \bowtie \chi_{15}]$, $[\chi_8, \infty_{12}, \infty_4 \bowtie \chi_{16}]$, $[\chi_8, \infty_{13}, \infty_3 \bowtie \chi_{17}]$, $[\chi_8, \infty_1, \infty_2 \bowtie \chi_{18}]$, $[\chi_9, \infty_{10}, \infty_8 \bowtie \chi_{20}]$, $[\chi_9, \infty_{11}, \infty_7 \bowtie \chi_{14}]$, $[\chi_9, \infty_{12}, \infty_6 \bowtie \chi_{15}]$, $[\chi_9, \infty_{13}, \infty_5 \bowtie \chi_{16}]$, $[\chi_9, \infty_1, \infty_4 \bowtie \chi_{17}]$, $[\chi_9, \infty_2, \infty_3 \bowtie \chi_{18}]$, $[\chi_{10}, \infty_{11}, \infty_9 \bowtie \chi_{20}]$, $[\chi_{10}, \infty_{12}, \infty_8 \bowtie \chi_{14}]$, $[\chi_{10}, \infty_{13}, \infty_7 \bowtie \chi_{15}]$, $[\chi_{10}, \infty_1, \infty_6 \bowtie \chi_{16}]$, $[\chi_{10}, \infty_2, \infty_5 \bowtie \chi_{17}]$, $[\chi_{10}, \infty_3, \infty_4 \bowtie \chi_{18}]$, $[\chi_{11}, \infty_{12}, \infty_{10} \bowtie \chi_{20}]$, $[\chi_{11}, \infty_{13}, \infty_9 \bowtie \chi_{14}]$, $[\chi_{11}, \infty_1, \infty_8 \bowtie \chi_{15}]$, $[\chi_{11}, \infty_2, \infty_7 \bowtie \chi_{16}]$, $[\chi_{11}, \infty_3, \infty_6 \bowtie \chi_{17}]$, $[\chi_{11}, \infty_4, \infty_5 \bowtie \chi_{18}]$, $[\chi_{12}, \infty_{13}, \infty_{11} \bowtie \chi_{20}]$, $[\chi_{12}, \infty_1, \infty_{10} \bowtie \chi_{14}]$, $[\chi_{12}, \infty_2, \infty_9 \bowtie \chi_{15}]$, $[\chi_{12}, \infty_3, \infty_8 \bowtie \chi_{16}]$, $[\chi_{12}, \infty_4, \infty_7 \bowtie \chi_{17}]$, $[\chi_{12}, \infty_5, \infty_6 \bowtie \chi_{18}]$, $[\chi_1, \chi_4, \chi_8 \bowtie \infty_8]$, $[\chi_2, \chi_9, \chi_5 \bowtie \chi_{17}]$, $[\chi_3, \chi_6, \chi_{10} \bowtie \infty_{10}]$, $[\chi_4, \chi_7, \chi_{11} \bowtie \infty_{11}]$, $[\chi_5, \chi_8, \chi_{12} \bowtie \chi_{20}]$, $[\chi_6, \chi_{13}, \chi_9 \bowtie \chi_{19}]$, $[\chi_7, \chi_{10}, \chi_{14} \bowtie \infty_{12}]$, $[\chi_8, \chi_{11}, \chi_{15} \bowtie \infty_{13}]$, $[\chi_9, \chi_{12}, \chi_{16} \bowtie \infty_9]$, $[\chi_{10}, \chi_{13}, \chi_{17} \bowtie \infty_{10}]$, $[\chi_{11}, \chi_{14}, \chi_{18} \bowtie \infty_{11}]$, $[\chi_{12}, \chi_{15}, \chi_{19} \bowtie \infty_3]$, $[\chi_{13}, \chi_{16}, \chi_{20} \bowtie \infty_3]$, $[\chi_{14}, \chi_{17}, \chi_1 \bowtie \chi_{11}]$, $[\chi_{15}, \chi_{18}, \chi_2 \bowtie \chi_4]$, $[\chi_3, \chi_{16}, \chi_{19} \bowtie \infty_2]$, $[\chi_4, \chi_{17}, \chi_{20} \bowtie \infty_2]$, $[\chi_1, \chi_5, \chi_{18} \bowtie \chi_{20}]$, $[\chi_2, \chi_6, \chi_{19} \bowtie \infty_1]$, $[\chi_3, \chi_7, \chi_{20} \bowtie \infty_1]$, $[\chi_1, \chi_{12}, \chi_6 \bowtie \chi_4]$, $[\chi_2, \chi_7, \chi_{13} \bowtie \chi_3]$, $[\chi_3, \chi_{14}, \chi_8 \bowtie \chi_{10}]$, $[\chi_4, \chi_{15}, \chi_9 \bowtie \chi_1]$, $[\chi_5, \chi_{16}, \chi_{10} \bowtie \chi_{18}]$, $[\chi_6, \chi_{11}, \chi_{17} \bowtie \infty_{12}]$, $[\chi_7, \chi_{12}, \chi_{18} \bowtie \infty_{13}]$, $[\chi_8, \chi_{13}, \chi_{19} \bowtie \infty_6]$, $[\chi_9, \chi_{14}, \chi_{20} \bowtie \infty_6]$, $[\chi_{10}, \chi_{15}, \chi_1 \bowtie \chi_{13}]$, $[\chi_{11}, \chi_{16}, \chi_2 \bowtie \chi_{14}]$, $[\chi_{12}, \chi_{17}, \chi_3 \bowtie \chi_{15}]$, $[\chi_4, \chi_{13}, \chi_{18} \bowtie \chi_6]$, $[\chi_5, \chi_{14}, \chi_{19} \bowtie \infty_5]$, $[\chi_6, \chi_{15}, \chi_{20} \bowtie \infty_5]$, $[\chi_{16}, \chi_1, \chi_7 \bowtie \chi_{15}]$, $[\chi_{17}, \chi_2, \chi_8 \bowtie \chi_{20}]$, $[\chi_{18}, \chi_3, \chi_9 \bowtie \chi_{17}]$, $[\chi_{10}, \chi_4, \chi_{19} \bowtie \infty_4]$, $[\chi_{11}, \chi_5, \chi_{20} \bowtie \infty_4]$, $[\chi_1, \chi_3, \chi_2 \bowtie \chi_{20}]$, $[\chi_4, \chi_5, \chi_3 \bowtie \chi_{11}]$, $[\chi_6, \chi_7, \chi_5 \bowtie \chi_{13}]$, $[\chi_8, \chi_9, \chi_7 \bowtie \chi_{19}]$, $[\chi_{10}, \chi_{11}, \chi_9 \bowtie \infty_9]$, $[\chi_{12}, \chi_{13}, \chi_{11} \bowtie \chi_{19}]$, $[\chi_{13}, \chi_{14}, \chi_{15} \bowtie \chi_5]$, $[\chi_{15}, \chi_{16}, \chi_{17} \bowtie \chi_7]$, $[\chi_{17}, \chi_{18}, \chi_{19} \bowtie \infty_7]$, $[\chi_1, \chi_{20}, \chi_{19} \bowtie \infty_8]$, $[\chi_2, \chi_{12}, \chi_{10} \bowtie \chi_{20}]$, $[\chi_{12}, \chi_{14}, \chi_4 \bowtie \chi_{16}]$, $[\chi_{14}, \chi_{16}, \chi_6 \bowtie \chi_8]$, $[\chi_8, \chi_{16}, \chi_{18} \bowtie \infty_7]$.

The set $\mathcal{D} \setminus \mathcal{A}$ is given by the following kites:

$[a_1, \infty_1, 1 \bowtie \chi_{16}]$, $[a_1, 9, \infty_2 \bowtie a_9]$, $[a_1, 8, \infty_5 \bowtie a_{10}]$, $[a_1, \infty_6, 16 \bowtie \chi_{16}]$, $[a_1, \infty_8, 7 \bowtie a_3]$, $[a_1, 15, \infty_9 \bowtie a_3]$, $[a_1, \infty_{11}, 6 \bowtie \chi_{16}]$, $[a_1, 14, \infty_{12} \bowtie a_{11}]$, $[a_2, 2, \infty_1 \bowtie a_{18}]$, $[a_2, 10, \infty_2 \bowtie a_{10}]$, $[a_2, \infty_4, 1 \bowtie \chi_{17}]$, $[a_2, \infty_5, 9 \bowtie \chi_{16}]$, $[a_2, 8, \infty_8 \bowtie a_{18}]$, $[a_2, 16, \infty_9 \bowtie a_4]$, $[a_2, \infty_{11}, 7 \bowtie \chi_{16}]$, $[a_2, \infty_{12}, 15 \bowtie \chi_{16}]$, $[a_3, 3, \infty_1 \bowtie a_{19}]$, $[a_3, 11, \infty_2 \bowtie a_{11}]$, $[a_3, 2, \infty_4 \bowtie a_1]$, $[a_3, \infty_5, 10 \bowtie \chi_{16}]$, $[a_3, \infty_7, 1 \bowtie \chi_{18}]$, $[a_3, 9, \infty_8 \bowtie a_{11}]$, $[a_3, \infty_{11}, 8 \bowtie \chi_{16}]$, $[a_3, 16, \infty_2 \bowtie a_5]$, $[a_4, \infty_1, 4 \bowtie \chi_{16}]$, $[a_4, 12, \infty_2 \bowtie a_{12}]$, $[a_4, \infty_4, 3 \bowtie \chi_{16}]$, $[a_4, 11, \infty_5 \bowtie a_{11}]$, $[a_4, \infty_7, 2 \bowtie \chi_{16}]$, $[a_4, 10, \infty_8 \bowtie a_{12}]$, $[a_4, \infty_{10}, 1 \bowtie \chi_{19}]$, $[\infty_{11}, 9, a_4 \bowtie \infty_{12}]$, $[a_5, 5, \infty_1 \bowtie a_{20}]$, $[a_5, \infty_2, 13 \bowtie \chi_{16}]$, $[a_5, 4, \infty_4 \bowtie a_{18}]$, $[a_5, \infty_5, 12 \bowtie \chi_{16}]$, $[a_5, 3, \infty_7 \bowtie a_1]$, $[a_5, \infty_8, 11 \bowtie \chi_{16}]$, $[a_5, \infty_{10}, 2 \bowtie \chi_{17}]$, $[a_5, 10, \infty_{11} \bowtie a_{12}]$, $[a_5, 1, \infty_{13} \bowtie a_1]$, $[a_6, \infty_1, 6 \bowtie \chi_{17}]$, $[a_6, 14, \infty_2 \bowtie a_{13}]$, $[a_6, 5, \infty_4 \bowtie a_{19}]$, $[a_6, \infty_5, 13 \bowtie \chi_{17}]$, $[a_6, 4, \infty_7 \bowtie a_{19}]$, $[a_6, 12, \infty_8 \bowtie a_{13}]$, $[a_6, \infty_{10}, 3 \bowtie \chi_{17}]$, $[a_6, 11, \infty_{11} \bowtie a_{13}]$, $[a_6, 2, \infty_{13} \bowtie a_2]$, $[a_7, \infty_1, 7, \bowtie \chi_{17}]$, $[a_7, \infty_2, 15 \bowtie \chi_{17}]$, $[a_7, 6, \infty_4 \bowtie a_{24}]$, $[a_7, \infty_5, 14 \bowtie a_{16}]$, $[a_7, 5, \infty_7 \bowtie a_2]$, $[a_7, \infty_8, 13 \bowtie \chi_{18}]$, $[a_7, 4, \infty_{10} \bowtie a_1]$, $[a_7, 12, \infty_{11} \bowtie a_{18}]$, $[a_7, \infty_{13}, 3 \bowtie \chi_{18}]$, $[a_8, 8, \infty_1 \bowtie a_{21}]$, $[a_8, 16, \infty_2 \bowtie a_{14}]$, $[a_8, \infty_4, 7 \bowtie \chi_{18}]$, $[a_8, 15, \infty_5 \bowtie a_{12}]$, $[a_8, 6, \infty_7 \bowtie a_{22}]$, $[a_8, 14, \infty_8 \bowtie a_{16}]$, $[a_8, \infty_{10}, 5 \bowtie \chi_{16}]$, $[a_8, 13, \infty_{11} \bowtie a_{14}]$, $[a_8, 4, \infty_{13} \bowtie a_3]$, $[a_9, 9, \infty_1 \bowtie a_{22}]$, $[a_9, 1, \infty_3 \bowtie a_2]$, $[a_9, \infty_4, 8 \bowtie \chi_{17}]$, $[a_9, \infty_5, 16 \bowtie \chi_{17}]$, $[a_9, \infty_7, 7 \bowtie \chi_{19}]$, $[a_9, \infty_8, 15 \bowtie a_3]$, $[a_9, 6, \infty_{10} \bowtie a_3]$, $[a_9, 14, \infty_{11} \bowtie a_{15}]$, $[a_9, \infty_{13}, 5 \bowtie \chi_{19}]$, $[a_{10}, 10, \infty_1 \bowtie a_{23}]$, $[a_{10}, \infty_3, 2 \bowtie \chi_{18}]$, $[a_{10}, 9, \infty_4 \bowtie a_{21}]$, $[a_{10}, \infty_6, 1 \bowtie \chi_{20}]$, $[a_{10}, 8, \infty_7 \bowtie a_{23}]$, $[a_{10}, \infty_8, 16 \bowtie \chi_{18}]$, $[a_{10}, 7, \infty_{10} \bowtie a_{20}]$, $[a_{10}, \infty_{11}, 15 \bowtie \chi_{18}]$, $[a_{10}, 6, \infty_{13} \bowtie a_4]$, $[a_{11}, 11, \infty_1 \bowtie a_{24}]$, $[a_{11}, 3, \infty_3, \bowtie a_3]$, $[a_{11}, 10, \infty_4 \bowtie a_{22}]$, $[a_{11}, \infty_6, 2 \bowtie \chi_{19}]$, $[a_{11}, \infty_7, 9 \bowtie a_5]$, $[a_{11}, 1, \infty_9 \bowtie a_5]$, $[a_{11}, \infty_{10}, 8 \bowtie \chi_{18}]$, $[a_{11}, \infty_{11}, 16 \bowtie a_4]$, $[a_{11}, 7, \infty_{13} \bowtie a_{21}]$, $[a_{12}, \infty_1, 12 \bowtie \chi_{17}]$, $[a_{12}, \infty_3, 4 \bowtie a_1]$, $[a_{12}, 11, \infty_4 \bowtie a_{23}]$, $[a_{12}, 3, \infty_6 \bowtie a_2]$, $[a_{12}, \infty_7, 10 \bowtie a_1]$, $[a_{12}, \infty_9, 2 \bowtie \chi_{20}]$, $[a_{12}, 9, \infty_{10} \bowtie a_{21}]$, $[a_{12}, \infty_{12}, 1 \bowtie a_6]$, $[a_{12}, \infty_{13}, 8 \bowtie \chi_{19}]$, $[a_{13}, \infty_1, 13 \bowtie a_2]$, $[a_{13}, \infty_3, 5 \bowtie a_1]$, $[a_{13}, \infty_4, 12 \bowtie a_2]$, $[a_{13}, 4, \infty_6 \bowtie a_3]$, $[a_{13}, 11, \infty_7 \bowtie a_{20}]$, $[a_{13}, \infty_9, 3 \bowtie \chi_{19}]$, $[a_{13}, 10, \infty_{10} \bowtie a_2]$, $[a_{13}, 2, \infty_{12} \bowtie a_6]$, $[a_{13}, \infty_{13}, 9 \bowtie \chi_{20}]$, $[a_{14}, \infty_1, 14 \bowtie \chi_{17}]$, $[a_{14}, 6, \infty_3 \bowtie a_4]$, $[a_{14}, \infty_4, 13 \bowtie a_3]$, $[a_{14}, 5, \infty_6 \bowtie a_4]$, $[a_{14}, \infty_7, 12 \bowtie \chi_{18}]$, $[a_{14}, 4, \infty_9 \bowtie a_6]$, $[a_{14}, 11, \infty_{10}, \bowtie a_{22}]$, $[a_{14}, \infty_{12}, 3 \bowtie \chi_{20}]$, $[a_{14}, 10, \infty_{13} \bowtie a_{22}]$, $[a_{15}, 15, \infty_1 \bowtie a_{17}]$, $[a_{15}, 7, \infty_3 \bowtie a_5]$, $[a_{15}, 14, \infty_4 \bowtie a_{20}]$, $[a_{15}, 6, \infty_6 \bowtie a_5]$, $[a_{15}, \infty_7, 13 \bowtie a_4]$, $[a_{15}, 5, \infty_9 \bowtie a_7]$, $[a_{15}, \infty_{10}, 12 \bowtie \chi_{19}]$, $[a_{15}, \infty_{12}, 4 \bowtie \chi_{17}]$, $[a_{15}, \infty_{13}, 11 \bowtie \chi_{17}]$, $[a_{16}, \infty_1, 16 \bowtie \chi_{19}]$, $[a_{16}, 8, \infty_3 \bowtie a_6]$, $[a_{16}, \infty_4, 15 \bowtie \chi_{19}]$, $[a_{16}, 7, \infty_6 \bowtie a_6]$, $[a_{16}, \infty_7, 14 \bowtie \chi_{18}]$, $[a_{16}, 6, \infty_9 \bowtie a_8]$, $[a_{16}, 13, \infty_{10} \bowtie a_{23}]$, $[a_{16}, \infty_{12}, 5 \bowtie \chi_{20}]$, $[a_{16}, \infty_{13}, 12 \bowtie \chi_{20}]$, $[a_{17}, \infty_2, 1 \bowtie a_7]$, $[a_{17}, \infty_3, 9 \bowtie \chi_{17}]$, $[a_{17}, \infty_4, 16 \bowtie \chi_{20}]$, $[a_{17}, 8, \infty_6 \bowtie a_7]$, $[a_{17}, 15, \infty_7 \bowtie a_{21}]$, $[a_{17}, 7, \infty_9, \bowtie a_9]$, $[a_{17}, \infty_{10}, 14, \bowtie a_2]$, $[a_{17}, \infty_{12}, 6 \bowtie \chi_{18}]$, $[a_{17}, 13, \infty_{13} \bowtie a_{23}]$, $[a_{18}, 2, \infty_2 \bowtie a_{16}]$, $[a_{18}, \infty_3, 10 \bowtie \chi_{17}]$, $[a_{18}, 1, \infty_5 \bowtie a_{13}]$, $[a_{18}, \infty_6, 9 \bowtie \chi_{18}]$, $[a_{18}, 16, \infty_7 \bowtie a_{24}]$, $[a_{18}, 8, \infty_9 \bowtie a_{10}]$, $[a_{18}, \infty_{10}, 15 \bowtie \chi_{20}]$, $[a_{18}, 7, \infty_{12} \bowtie a_7]$, $[a_{18}, 14, \infty_{13} \bowtie a_{24}]$, $[a_{19}, 3, \infty_2 \bowtie a_{15}]$, $[a_{19}, \infty_3, 11 \bowtie \chi_{18}]$, $[a_{19}, 2, \infty_5 \bowtie a_{14}]$, $[a_{19}, \infty_6, 10 \bowtie \chi_{18}]$, $[a_{19}, 1, \infty_8 \bowtie a_{14}]$, $[a_{19}, \infty_9, 9 \bowtie \chi_{19}]$, $[a_{19}, 16, \infty_{10} \bowtie a_{24}]$, $[a_{19}, 8, \infty_{12} \bowtie a_8]$, $[a_{19}, \infty_{13}, 15 \bowtie a_4]$, $[a_{20}, \infty_2, 4 \bowtie \chi_{18}]$, $[a_{20}, 12, \infty_3 \bowtie a_1]$, $[a_{20}, 3, \infty_5 \bowtie a_{15}]$, $[a_{20}, \infty_6, 11 \bowtie \chi_{19}]$, $[a_{20}, 2, \infty_8 \bowtie a_{15}]$, $[a_{20}, \infty_9, 10 \bowtie \chi_{19}]$, $[a_{20}, 1, \infty_{11} \bowtie a_{16}]$, $[a_{20}, 9, \infty_{12} \bowtie a_9]$, $[a_{20}, \infty_{13}, 16 \bowtie a_5]$, $[a_{21}, \infty_2, 5 \bowtie \chi_{17}]$, $[a_{21}, 13, \infty_3, \bowtie a_7]$, $[a_{21}, \infty_5, 4 \bowtie \chi_{19}]$, $[a_{21}, 12, \infty_6 \bowtie a_8]$, $[a_{21}, 3, \infty_8 \bowtie a_{17}]$, $[a_{21}, \infty_9, 11 \bowtie a_1]$, $[a_{21}, 2, \infty_{11} \bowtie a_{17}]$, $[a_{21}, \infty_{12}, 10 \bowtie \chi_{20}]$, $[a_{22}, \infty_2, 6 \bowtie \chi_{19}]$, $[a_{22}, 14, \infty_3 \bowtie a_8]$, $[a_{22}, 5, \infty_5 \bowtie a_{16}]$, $[a_{22}, \infty_6, 13 \bowtie \chi_{19}]$, $[a_{22}, \infty_8, 4 \bowtie \chi_{20}]$, $[a_{22}, \infty_9, 12 \bowtie a_1]$, $[a_{22}, 3, \infty_{11}, \bowtie a_{19}]$, $[a_{22}, \infty_{12}, 11 \bowtie \chi_{20}]$, $[a_{23}, \infty_2, 7 \bowtie \chi_{20}]$, $[a_{23}, \infty_3, 15 \bowtie a_5]$, $[a_{23}, 6, \infty_5 \bowtie a_{17}]$, $[a_{23}, \infty_6, 14 \bowtie \chi_{19}]$, $[a_{23}, \infty_8, 5 \bowtie \chi_{18}]$, $[a_{23}, \infty_9, 13 \bowtie \chi_{20}]$, $[a_{23}, \infty_{11}, 4 \bowtie a_2]$, $[a_{23}, 12, \infty_{12} \bowtie a_{10}]$, $[a_{24}, \infty_2, 8 \bowtie \chi_{20}]$, $[a_{24}, \infty_3, 16 \bowtie a_6]$, $[a_{24}, \infty_5, 7 \bowtie a_4]$, $[a_{24}, 15, \infty_6 \bowtie a_9]$, $[a_{24}, \infty_8, 6 \bowtie \chi_{20}]$, $[a_{24}, \infty_9, 14 \bowtie \chi_{20}]$, $[a_{24}, \infty_{11}, 5 \bowtie a_2]$, $[a_{24}, \infty_{12}, 13 \bowtie a_1]$, $[\chi_1, 16, 2 \bowtie a_1]$, $[\chi_1, 1, 3 \bowtie a_8]$,

$[\chi_1, 4, 15 \bowtie a_6]$, $[\chi_1, 14, 5 \bowtie a_{10}]$, $[\chi_1, 13, 6 \bowtie a_2]$, $[\chi_1, 7, 12 \bowtie a_8]$, $[\chi_1, 8, 11 \bowtie a_8]$, $[\chi_1, 10, 9 \bowtie a_6]$, $[\chi_2, 1, 16 \bowtie a_7]$, $[\chi_2, 15, 2 \bowtie a_8]$, $[\chi_2, 14, 3 \bowtie a_2]$, $[\chi_2, 13, 4 \bowtie a_3]$, $[\chi_2, 12, 5 \bowtie a_3]$, $[\chi_2, 6, 11 \bowtie a_2]$, $[\chi_2, 10, 7 \bowtie a_6]$, $[\chi_2, 8, 9 \bowtie a_{23}]$, $[\chi_3, 3, 16 \bowtie a_{23}]$, $[\chi_3, 4, 2 \bowtie a_7]$, $[\chi_3, 1, 5 \bowtie a_4]$, $[\chi_3, 15, 6 \bowtie a_3]$, $[\chi_3, 14, 7 \bowtie a_{20}]$, $[\chi_3, 13, 8 \bowtie a_4]$, $[\chi_3, 12, 9 \bowtie a_{22}]$, $[\chi_3, 11, 10 \bowtie a_6]$, $[\chi_4, 16, 4 \bowtie a_9]$, $[\chi_4, 5, 3 \bowtie a_9]$, $[\chi_4, 2, 6 \bowtie a_4]$, $[\chi_4, 1, 7 \bowtie a_5]$, $[\chi_4, 15, 8 \bowtie a_6]$, $[\chi_4, 9, 14 \bowtie a_4]$, $[\chi_4, 13, 10 \bowtie a_8]$, $[\chi_4, 12, 11 \bowtie a_7]$, $[\chi_5, 16, 5 \bowtie a_{11}]$, $[\chi_5, 4, 6 \bowtie a_5]$, $[\chi_5, 7, 3 \bowtie a_{10}]$, $[\chi_5, 2, 8 \bowtie a_7]$, $[\chi_5, 1, 9 \bowtie a_7]$, $[\chi_5, 15, 10 \bowtie a_7]$, $[\chi_5, 14, 11 \bowtie a_{23}]$, $[\chi_5, 13, 12 \bowtie a_9]$, $[\chi_6, 6, 16 \bowtie a_{22}]$, $[\chi_6, 7, 5 \bowtie a_{12}]$, $[\chi_6, 4, 8 \bowtie a_{22}]$, $[\chi_6, 3, 9 \bowtie a_8]$, $[\chi_6, 10, 2 \bowtie a_9]$, $[\chi_6, 11, 1 \bowtie a_{24}]$, $[\chi_6, 12, 15 \bowtie a_{11}]$, $[\chi_6, 14, 13 \bowtie a_9]$, $[\chi_7, 7, 16 \bowtie a_{21}]$, $[\chi_7, 8, 6 \bowtie a_{12}]$, $[\chi_7, 5, 9 \bowtie a_{14}]$, $[\chi_7, 4, 10 \bowtie a_9]$, $[\chi_7, 3, 11 \bowtie a_{10}]$, $[\chi_7, 2, 12 \bowtie a_{10}]$, $[\chi_7, 13, 1 \bowtie a_8]$, $[\chi_7, 15, 14 \bowtie a_{10}]$, $[\chi_8, 8, 16 \bowtie a_{12}]$, $[\chi_8, 7, 9 \bowtie a_{24}]$, $[\chi_8, 6, 10 \bowtie a_{24}]$, $[\chi_8, 5, 11 \bowtie a_{24}]$, $[\chi_8, 4, 12 \bowtie a_{11}]$, $[\chi_8, 3, 13 \bowtie a_{10}]$, $[\chi_8, 2, 14 \bowtie a_{11}]$, $[\chi_8, 1, 15 \bowtie a_{12}]$, $[\chi_9, 9, 16 \bowtie a_{13}]$, $[\chi_9, 10, 8 \bowtie a_{13}]$, $[\chi_9, 11, 7 \bowtie a_{13}]$, $[\chi_9, 12, 6 \bowtie a_{13}]$, $[\chi_9, 5, 13 \bowtie a_{11}]$, $[\chi_9, 14, 4 \bowtie a_{11}]$, $[\chi_9, 3, 15 \bowtie a_{13}]$, $[\chi_9, 2, 1 \bowtie a_{14}]$, $[\chi_{10}, 10, 16 \bowtie a_{14}]$, $[\chi_{10}, 11, 9 \bowtie a_{15}]$, $[\chi_{10}, 8, 12 \bowtie a_{24}]$, $[\chi_{10}, 7, 13 \bowtie a_{12}]$, $[\chi_{10}, 6, 14 \bowtie a_{12}]$, $[\chi_{10}, 15, 5 \bowtie a_{17}]$, $[\chi_{10}, 4, 1 \bowtie a_{15}]$, $[\chi_{10}, 3, 2 \bowtie a_{14}]$, $[\chi_{11}, 11, 16 \bowtie a_{15}]$, $[\chi_{11}, 10, 12 \bowtie a_3]$, $[\chi_{11}, 13, 9 \bowtie a_{16}]$, $[\chi_{11}, 8, 14 \bowtie a_{13}]$, $[\chi_{11}, 7, 15 \bowtie a_{22}]$, $[\chi_{11}, 6, 1 \bowtie a_{16}]$, $[\chi_{11}, 5, 2 \bowtie a_{15}]$, $[\chi_{11}, 4, 3 \bowtie a_{15}]$, $[\chi_{12}, 16, 12 \bowtie a_{17}]$, $[\chi_{12}, 11, 13 \bowtie a_{18}]$, $[\chi_{12}, 10, 14 \bowtie a_{20}]$, $[\chi_{12}, 9, 15 \bowtie a_{14}]$, $[\chi_{12}, 8, 1 \bowtie a_{13}]$, $[\chi_{12}, 7, 2 \bowtie a_{16}]$, $[\chi_{12}, 6, 3 \bowtie a_{16}]$, $[\chi_{12}, 5, 4 \bowtie a_{16}]$, $[\chi_{13}, 16, 13 \bowtie a_{19}]$, $[\chi_{13}, 12, 14 \bowtie a_{21}]$, $[\chi_{13}, 15, 11 \bowtie a_{16}]$, $[\chi_{13}, 10, 1 \bowtie a_{21}]$, $[\chi_{13}, 9, 2 \bowtie a_{17}]$, $[\chi_{13}, 8, 3 \bowtie a_{17}]$, $[\chi_{13}, 7, 4 \bowtie a_{17}]$, $[\chi_{13}, 6, 5 \bowtie a_{18}]$, $[\chi_{14}, 16, 14 \bowtie a_5]$, $[\chi_{14}, 13, 15 \bowtie a_{20}]$, $[\chi_{14}, 12, 1 \bowtie a_{22}]$, $[\chi_{14}, 2, 11 \bowtie a_{17}]$, $[\chi_{14}, 10, 3 \bowtie a_{18}]$, $[\chi_{14}, 9, 4 \bowtie a_{10}]$, $[\chi_{14}, 8, 5 \bowtie a_{20}]$, $[\chi_{14}, 7, 6 \bowtie a_{19}]$, $[\chi_{15}, 16, 15 \bowtie a_{21}]$, $[\chi_{15}, 14, 1 \bowtie a_{23}]$, $[\chi_{15}, 2, 13 \bowtie a_{20}]$, $[\chi_{15}, 3, 12 \bowtie a_{19}]$, $[\chi_{15}, 4, 11 \bowtie a_{18}]$, $[\chi_{15}, 10, 5 \bowtie a_{19}]$, $[\chi_{15}, 9, 6 \bowtie a_{18}]$, $[\chi_{15}, 8, 7 \bowtie a_{19}]$, $[\chi_1, a_{14}, a_1 \bowtie a_{13}]$, $[\chi_1, a_{15}, a_2 \bowtie a_{14}]$, $[\chi_1, a_{16}, a_3 \bowtie a_{15}]$, $[\chi_1, a_{17}, a_4 \bowtie a_{16}]$, $[\chi_1, a_{18}, a_5 \bowtie a_{17}]$, $[\chi_1, a_{19}, a_6 \bowtie a_{18}]$, $[\chi_1, a_{20}, a_7 \bowtie a_{19}]$, $[\chi_1, a_{21}, a_8 \bowtie a_{20}]$, $[\chi_1, a_{22}, a_9 \bowtie a_{21}]$, $[\chi_1, a_{23}, a_{10} \bowtie a_{22}]$, $[\chi_1, a_{24}, a_{11} \bowtie a_{23}]$, $[\chi_1, a_{13}, a_{12} \bowtie a_{24}]$, $[\chi_2, a_{16}, a_1 \bowtie a_{15}]$, $[\chi_2, a_{17}, a_2 \bowtie a_{16}]$, $[\chi_2, a_{18}, a_3 \bowtie a_{17}]$, $[\chi_2, a_{19}, a_4 \bowtie a_{18}]$, $[\chi_2, a_{20}, a_5 \bowtie a_{19}]$, $[\chi_2, a_{21}, a_6 \bowtie a_{20}]$, $[\chi_2, a_{22}, a_7 \bowtie a_{21}]$, $[\chi_2, a_{23}, a_8 \bowtie a_{22}]$, $[\chi_2, a_{24}, a_9 \bowtie a_{23}]$, $[\chi_2, a_{13}, a_{10} \bowtie a_{24}]$, $[\chi_2, a_{14}, a_{11} \bowtie a_{13}]$, $[\chi_2, a_{15}, a_{12} \bowtie a_{14}]$, $[\chi_3, a_{18}, a_1 \bowtie a_{17}]$, $[\chi_3, a_{19}, a_2 \bowtie a_{18}]$, $[\chi_3, a_{20}, a_3 \bowtie a_{19}]$, $[\chi_3, a_{21}, a_4 \bowtie a_{20}]$, $[\chi_3, a_{22}, a_5 \bowtie a_{21}]$, $[\chi_3, a_{23}, a_6 \bowtie a_{22}]$, $[\chi_3, a_{24}, a_7 \bowtie a_{23}]$, $[\chi_3, a_{13}, a_8 \bowtie a_{24}]$, $[\chi_3, a_{14}, a_9 \bowtie a_{13}]$, $[\chi_3, a_{15}, a_{10} \bowtie a_{14}]$, $[\chi_3, a_{16}, a_{11} \bowtie a_{15}]$, $[\chi_3, a_{17}, a_{12} \bowtie a_{16}]$, $[\chi_4, a_{20}, a_1 \bowtie a_{19}]$, $[\chi_4, a_{21}, a_2 \bowtie a_{20}]$, $[\chi_4, a_{22}, a_3 \bowtie a_{21}]$, $[\chi_4, a_{23}, a_4 \bowtie a_{22}]$, $[\chi_4, a_{24}, a_5 \bowtie a_{23}]$, $[\chi_4, a_{13}, a_6 \bowtie a_{24}]$, $[\chi_4, a_{14}, a_7 \bowtie a_{13}]$, $[\chi_4, a_{15}, a_8 \bowtie a_{14}]$, $[\chi_4, a_{16}, a_9 \bowtie a_{15}]$, $[\chi_4, a_{17}, a_{10} \bowtie a_{16}]$, $[\chi_4, a_{18}, a_{11} \bowtie a_{17}]$, $[\chi_4, a_{19}, a_{12} \bowtie a_{18}]$, $[\chi_5, a_{22}, a_1 \bowtie a_{21}]$, $[\chi_5, a_{23}, a_2 \bowtie a_{22}]$, $[\chi_5, a_{24}, a_3 \bowtie a_{23}]$, $[\chi_5, a_{13}, a_4 \bowtie a_{24}]$, $[\chi_5, a_{14}, a_5 \bowtie a_{13}]$, $[\chi_5, a_{15}, a_6 \bowtie a_{14}]$, $[\chi_5, a_{16}, a_7 \bowtie a_{15}]$, $[\chi_5, a_{17}, a_8 \bowtie a_{16}]$, $[\chi_5, a_{18}, a_9 \bowtie a_{17}]$, $[\chi_5, a_{19}, a_{10} \bowtie a_{18}]$, $[\chi_5, a_{20}, a_{11} \bowtie a_{19}]$, $[\chi_5, a_{21}, a_{12} \bowtie a_{20}]$, $[\chi_6, a_{24}, a_1 \bowtie a_{23}]$, $[\chi_6, a_{13}, a_2 \bowtie a_{24}]$, $[\chi_6, a_{14}, a_3 \bowtie a_{13}]$, $[\chi_6, a_{15}, a_4 \bowtie a_{14}]$, $[\chi_6, a_{16}, a_5 \bowtie a_{15}]$, $[\chi_6, a_{17}, a_6 \bowtie a_{16}]$, $[\chi_6, a_{18}, a_7 \bowtie a_{17}]$, $[\chi_6, a_{19}, a_8 \bowtie a_{18}]$, $[\chi_6, a_{20}, a_9 \bowtie a_{19}]$, $[\chi_6, a_{21}, a_{10} \bowtie a_{20}]$, $[\chi_6, a_{22}, a_{11} \bowtie a_{21}]$, $[\chi_6, a_{23}, a_{12} \bowtie a_{22}]$, $[\chi_7, a_8, a_1 \bowtie a_7]$, $[\chi_7, a_9, a_2 \bowtie a_8]$, $[\chi_7, a_{10}, a_3 \bowtie a_9]$, $[\chi_7, a_{11}, a_4 \bowtie a_{10}]$, $[\chi_7, a_{12}, a_5 \bowtie a_{11}]$, $[\chi_7, a_7, a_6 \bowtie a_{12}]$, $[\chi_7, a_{20}, a_{13} \bowtie a_{19}]$, $[\chi_7, a_{14}, a_{21} \bowtie 6]$, $[\chi_7, a_{15}, a_{22} \bowtie 2]$, $[\chi_7, a_{16}, a_{23} \bowtie 2]$, $[\chi_7, a_{17}, a_{24} \bowtie 2]$, $[\chi_7, a_{18}, a_{19} \bowtie 4]$, $[\chi_8, a_{10}, a_1 \bowtie \chi_{10}]$, $[\chi_8, a_{11}, a_2 \bowtie \chi_{10}]$, $[\chi_8, a_{12}, a_3 \bowtie \chi_{10}]$, $[\chi_8, a_7, a_4 \bowtie \chi_{10}]$, $[\chi_8, a_8, a_5 \bowtie \chi_{10}]$, $[\chi_8, a_9, a_6 \bowtie \chi_{10}]$, $[\chi_8, a_{22}, a_{13} \bowtie \chi_{10}]$, $[\chi_8, a_{23}, a_{14} \bowtie \chi_{10}]$, $[\chi_8, a_{24}, a_{15} \bowtie \chi_{10}]$, $[\chi_8, a_{19}, a_{16} \bowtie \chi_{10}]$, $[\chi_8, a_{20}, a_{17} \bowtie \chi_{10}]$, $[\chi_8, a_{21}, a_{18} \bowtie \chi_{10}]$, $[\chi_9, a_1, a_9 \bowtie \chi_{10}]$, $[\chi_9, a_2, a_{10} \bowtie \chi_{10}]$, $[\chi_9, a_3, a_{11} \bowtie \chi_{10}]$, $[\chi_9, a_4, a_{12} \bowtie \chi_{10}]$, $[\chi_9, a_5, a_7 \bowtie \chi_{10}]$, $[\chi_9, a_6, a_8 \bowtie \chi_{10}]$, $[\chi_9, a_{13}, a_{21} \bowtie \chi_{10}]$, $[\chi_9, a_{14}, a_{22} \bowtie \chi_{10}]$, $[\chi_9, a_{15}, a_{23} \bowtie$

$\chi_{10}]$, $[\chi_9, a_{16}, a_{24} \bowtie \chi_{10}]$, $[\chi_9, a_{17}, a_{19} \bowtie \chi_{10}]$, $[\chi_9, a_{18}, a_{20} \bowtie \chi_{10}]$, $[\chi_{11}, a_{12}, a_1 \bowtie \chi_{13}]$, $[\chi_{11}, a_7, a_2 \bowtie \chi_{13}]$, $[\chi_{11}, a_8, a_3 \bowtie \chi_{13}]$, $[\chi_{11}, a_9, a_4 \bowtie \chi_{13}]$, $[\chi_{11}, a_{10}, a_5 \bowtie \chi_{13}]$, $[\chi_{11}, a_{11}, a_6 \bowtie \chi_{13}]$, $[\chi_{11}, a_{24}, a_{13} \bowtie \chi_{13}]$, $[\chi_{11}, a_{19}, a_{14} \bowtie \chi_{13}]$, $[\chi_{11}, a_{20}, a_{15} \bowtie \chi_{13}]$, $[\chi_{11}, a_{21}, a_{16} \bowtie \chi_{13}]$, $[\chi_{11}, a_{22}, a_{17} \bowtie \chi_{13}]$, $[\chi_{11}, a_{23}, a_{18} \bowtie \chi_{13}]$, $[\chi_{12}, a_1, a_{11} \bowtie \chi_{13}]$, $[\chi_{12}, a_2, a_{12} \bowtie \chi_{13}]$, $[\chi_{12}, a_3, a_7 \bowtie \chi_{13}]$, $[\chi_{12}, a_4, a_8 \bowtie \chi_{13}]$, $[\chi_{12}, a_5, a_9 \bowtie \chi_{13}]$, $[\chi_{12}, a_6, a_{10} \bowtie \chi_{13}]$, $[\chi_{12}, a_{13}, a_{23} \bowtie \chi_{13}]$, $[\chi_{12}, a_{14}, a_{24} \bowtie \chi_{13}]$, $[\chi_{12}, a_{15}, a_{19} \bowtie \chi_{13}]$, $[\chi_{12}, a_{16}, a_{20} \bowtie \chi_{13}]$, $[\chi_{12}, a_{17}, a_{21} \bowtie \chi_{13}]$, $[\chi_{12}, a_{18}, a_{22} \bowtie \chi_{13}]$, $[\chi_{14}, a_2, a_1 \bowtie \chi_{19}]$, $[\chi_{14}, a_6, a_3 \bowtie \chi_{19}]$, $[\chi_{14}, a_5, a_4 \bowtie \chi_{19}]$, $[\chi_{14}, a_8, a_7 \bowtie \chi_{19}]$, $[\chi_{14}, a_{12}, a_9 \bowtie \chi_{19}]$, $[\chi_{14}, a_{11}, a_{10} \bowtie \chi_{19}]$, $[\chi_{14}, a_{14}, a_{13} \bowtie \chi_{19}]$, $[\chi_{14}, a_{18}, a_{15} \bowtie \chi_{19}]$, $[\chi_{14}, a_{17}, a_{16} \bowtie \chi_{19}]$, $[\chi_{14}, a_{20}, a_{19} \bowtie \chi_{19}]$, $[\chi_{14}, a_{24}, a_{21} \bowtie \chi_{19}]$, $[\chi_{14}, a_{23}, a_{22} \bowtie \chi_{19}]$, $[\chi_{15}, a_3, a_1 \bowtie \chi_{20}]$, $[\chi_{15}, a_4, a_2 \bowtie \chi_{19}]$, $[\chi_{15}, a_6, a_5 \bowtie \chi_{19}]$, $[\chi_{15}, a_9, a_7 \bowtie \chi_{20}]$, $[\chi_{15}, a_{10}, a_8 \bowtie \chi_{19}]$, $[\chi_{15}, a_{12}, a_{11} \bowtie \chi_{19}]$, $[\chi_{15}, a_{15}, a_{13} \bowtie \chi_{20}]$, $[\chi_{15}, a_{16}, a_{14} \bowtie \chi_{19}]$, $[\chi_{15}, a_{18}, a_{17} \bowtie \chi_{19}]$, $[\chi_{15}, a_{21}, a_{19} \bowtie \chi_{20}]$, $[\chi_{15}, a_{22}, a_{20} \bowtie \chi_{19}]$, $[\chi_{15}, a_{24}, a_{23} \bowtie \chi_{19}]$, $[\chi_{16}, a_1, a_4 \bowtie \chi_{20}]$, $[\chi_{16}, a_5, a_3 \bowtie \chi_{20}]$, $[\chi_{16}, a_2, a_6 \bowtie \chi_{19}]$, $[\chi_{16}, a_7, a_{10} \bowtie \chi_{20}]$, $[\chi_{16}, a_{11}, a_9 \bowtie \chi_{20}]$, $[\chi_{16}, a_8, a_{12} \bowtie \chi_{19}]$, $[\chi_{16}, a_{13}, a_{16} \bowtie \chi_{20}]$, $[\chi_{16}, a_{17}, a_{15} \bowtie \chi_{20}]$, $[\chi_{16}, a_{14}, a_{18} \bowtie \chi_{19}]$, $[\chi_{16}, a_{19}, a_{22} \bowtie \chi_{20}]$, $[\chi_{16}, a_{23}, a_{21} \bowtie \chi_{20}]$, $[\chi_{16}, a_{20}, a_{24} \bowtie \chi_{19}]$, $[\chi_{17}, a_1, a_5 \bowtie \chi_{20}]$, $[\chi_{17}, a_4, a_6 \bowtie \chi_{20}]$, $[\chi_{17}, a_3, a_2 \bowtie \chi_{20}]$, $[\chi_{17}, a_7, a_{11} \bowtie \chi_{20}]$, $[\chi_{17}, a_{10}, a_{12} \bowtie \chi_{20}]$, $[\chi_{17}, a_9, a_8 \bowtie \chi_{20}]$, $[\chi_{17}, a_{13}, a_{17} \bowtie \chi_{20}]$, $[\chi_{17}, a_{16}, a_{18} \bowtie \chi_{20}]$, $[\chi_{17}, a_{15}, a_{14} \bowtie \chi_{20}]$, $[\chi_{17}, a_{19}, a_{23} \bowtie \chi_{20}]$, $[\chi_{17}, a_{22}, a_{24} \bowtie \chi_{20}]$, $[\chi_{17}, a_{21}, a_{20} \bowtie \chi_{20}]$, $[\chi_{18}, a_6, a_1 \bowtie 3]$, $[\chi_{18}, a_2, a_5 \bowtie 8]$, $[\chi_{18}, a_4, a_3 \bowtie 14]$, $[\chi_{18}, a_7, a_{12} \bowtie 7]$, $[\chi_{18}, a_8, a_{11} \bowtie 6]$, $[\chi_{18}, a_{10}, a_9 \bowtie 11]$, $[\chi_{18}, a_{13}, a_{18} \bowtie 12]$, $[\chi_{18}, a_{17}, a_{14} \bowtie 7]$, $[\chi_{18}, a_{16}, a_{15} \bowtie 10]$, $[\chi_{18}, a_{24}, a_{19} \bowtie 14]$, $[\chi_{18}, a_{20}, a_{23} \bowtie 8]$, $[\chi_{18}, a_{22}, a_{21} \bowtie 9]$, $[8, a_{14}, a_{20} \bowtie 6]$, $[8, a_{15}, a_{21} \bowtie 7]$, $[10, a_{16}, a_{22} \bowtie 7]$, $[10, a_{17}, a_{23} \bowtie 3]$, $[4, a_{18}, a_{24} \bowtie 3]$.

5 Appendix 5

In this appendix we give a $(K_3 + e)$-design $(V \cup W, \mathcal{B})$ of order n P_4-regularly embedding a K_2-design of order $\lfloor \frac{3+2n}{5} \rfloor$ on V, for each $n \in \{80, 81, 88, 89, 96, 97, 104, 105, 112, 113\}$.

Case $n = 80$. Let $V = \{v_0, v_1, \ldots, v_{31}\}$ and $W = \{a_0, a_1, \ldots, a_{31}\} \cup \{b_0, b_1, \ldots, b_{15}\} = \{w_0, w_1, \ldots, w_{47}\}$, where

$$w_i = \begin{cases} a_i, & i = 0, 1, \ldots, 31; \\ b_{i-32}, & i = 32, 33, \ldots, 47. \end{cases}$$

Subscripts of v and a are calculated (mod 32), subscripts of b are calculated (mod 16) and subscripts of w are calculated (mod 48). Place in $\mathcal{B}$:

- $[a_{i+j+17}, v_{i+2j+1}, v_i \bowtie b_j]$, $i \in \{0, 1, \ldots, 31\}$ and $j \in \{0, 1, \ldots, 7\}$; $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{j+7}]$, $i \in \{0, 1, \ldots, 31\}$ and $j \in \{1, 2, \ldots, 7\}$; $[b_{15}, v_i, v_{i+16} \bowtie a_{i+24}]$, $i \in \{0, 1, \ldots, 15\}$;

- $[w_{i+32}, w_{i+16}, w_i \bowtie v_{i-8}]$, $i \in \{8, 9, \ldots, 23\}$; $[w_{i+23}, w_{i+4}, w_i \bowtie v_i]$, $i \in \{8, 9, \ldots, 31\}$; $[w_{i+23}, w_{i+4}, w_i \bowtie w_{i+24}]$, $i \in \{32, 33, \ldots, 47, 0, 1, \ldots, 7\}$; $[w_{i+3}, w_{i+1}, w_i \bowtie v_i]$, $i \in \{0, 1, \ldots, 7\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+4}]$, $i \in \{8, 12, \ldots, 44\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+5}]$, $i \in \{9, 13, \ldots, 45\}$; $[w_i, w_{i+3}, w_{i+1} \bowtie w_{i+4}]$, $i \in \{10, 14, \ldots, 46\}$; $[w_{i+s}, w_{i+r}, w_i \bowtie w_{i+t}]$, $i \in \{0, 1, \ldots, 47\}$ and $(r, s, t) \in \{(5, 22, 9), (6, 13, 14), (8, 20, 15), (10, 21, 18)\}$.

Case $n = 81$. Let $V = \{v_0, v_1, \ldots, v_{32}\}$ and $W = \{a_0, a_1, \ldots, a_{32}\} \cup \{b_0, b_1, \ldots, b_{14}\}$. Subscripts of v and a are calculated (mod 33). Place in $\mathcal{B}$:

- the blocks of a $(K_3 + e)$-design on W;

- $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{j-1}]$, $i \in \{0, 1, \ldots, 32\}$ and $j \in \{1, 2, \ldots, 15\}$; $[a_{i+16}, v_{i+32}, v_i \bowtie a_i]$, $i \in \{0, 1, \ldots, 32\}$.

Case $n = 88$. Let $V = \{v_0, v_1, \ldots, v_{34}\}$ and $W = \{a_0, a_1, \ldots, a_{34}\} \cup \{b_0, b_1, \ldots, b_{17}\} = \{w_0, w_1, \ldots, w_{52}\}$, where

$$w_i = \begin{cases} a_i, & i = 0, 1, \ldots, 34; \\ b_{i-35}, & i = 35, 36, \ldots, 52. \end{cases}$$

Subscripts of v and a are calculated (mod 35), subscripts of b are calculated (mod 18) and subscripts of w are calculated (mod 53). Place in $\mathcal{B}$:

- $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{i+j}]$, $i \in \{0, 1, \ldots, 34\}$ and $j \in \{1, 2, \ldots, 17\}$;

- $[w_{i-26}, w_{i+13}, w_i \bowtie v_i]$, $i \in \{0, 1, \ldots, 52\}$; $[w_{i+3}, w_{i+1}, w_i \bowtie v_{i-17}]$, $i \in \{35, 36, \ldots, 51\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+4}]$, $i \in \{52, 3, 7, \ldots, 31\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+5}]$, $i \in \{0, 4, \ldots, 32\}$; $[w_i, w_{i+3}, w_{i+1} \bowtie w_{i+4}]$, $i \in \{1, 5, \ldots, 33\}$; $[w_{i+s}, w_{i+r}, w_i \bowtie w_{i+t}]$, $i \in \{0, 1, \ldots, 52\}$ and $(r, s, t) \in \{(5, 25, 4), (7, 22, 6), (8, 24, 17), (9, 19, 18), (11, 23, 21)\}$.

Case $n = 89$. Let $V = \{v_0, v_1, \ldots, v_{35}\}$ and $W = \{a_0, a_1, \ldots, a_{35}\} \cup \{b_0, b_1, \ldots, b_{16}\} = \{w_0, w_1, \ldots, w_{52}\}$, where

$$w_i = \begin{cases} a_i, & i = 0, 1, \ldots, 35; \\ b_{i-36}, & i = 36, 37, \ldots, 52. \end{cases}$$

Subscripts of v and a are calculated (mod 36), subscripts of b are calculated (mod 17) and subscripts of w are calculated (mod 53). Place in $\mathcal{B}$:

- $[a_{i+j+19}, v_{i+2j+1}, v_i \bowtie b_j]$, $i \in \{0, 1, \ldots, 35\}$ and $j \in \{0, 1, \ldots, 8\}$; $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{j+8}]$, $i \in \{0, 1, \ldots, 35\}$ and $j \in \{1, 2, \ldots, 7\}$; $[a_{i+8}, v_{i+16}, v_i \bowtie a_{i+9}]$, $i \in \{0, 1, \ldots, 35\}$; $[b_{16}, v_i, v_{i+18} \bowtie a_{i+18}]$, $i \in \{0, 1, \ldots, 17\}$;

- $[w_{i+3}, w_{i+1}, w_i \bowtie v_i]$, $i \in \{0, 1, \ldots, 16\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+4}]$, $i \in \{17, 21, \ldots, 49\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+5}]$, $i \in \{18, 22, \ldots, 50\}$; $[w_i, w_{i+3}, w_{i+1} \bowtie w_{i+4}]$, $i \in \{29, 23, \ldots, 51\}$; $[w_{44}, w_{30}, w_{17} \bowtie v_{17}]$; $[w_i, w_{i-26}, w_{i+13} \bowtie w_{i-1}]$, $i \in \{18, 19, \ldots, 30\}$; $[w_{i-26}, w_{i+13}, w_i \bowtie w_{i+26}]$, $i \in \{31, 32, \ldots, 43\}$; $[w_i, w_{i-26}, w_{i+13} \bowtie w_{i+26}]$, $i \in \{44, 45, \ldots, 52, 0, 1, 2, 3\}$; $[w_{i+s}, w_{i+r}, w_i \bowtie w_{i+t}]$, $i \in \{0, 1, \ldots, 52\}$ and $(r, s, t) \in \{(5, 25, 4), (7, 22, 6), (8, 24, 17), (9, 19, 18), (11, 23, 21)\}$.

Case $n = 96$. Let $V = \{v_0, v_1, \ldots, v_{38}\}$ and $W = \{a_0, a_1, \ldots, a_{38}\} \cup \{b_0, b_1, \ldots, b_{17}\}$. Subscripts of v and a are calculated (mod 39). Place in $\mathcal{B}$:

- the blocks of a $(K_3 + e)$-design on W;

- $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{j-1}]$, $i \in \{0, 1, \ldots, 38\}$ and $j \in \{1, 2, \ldots, 18\}$; $[a_{i+19}, v_{i+38}, v_i \bowtie a_i]$, $i \in \{0, 1, \ldots, 38\}$.

Case $n = 97$. Let $V = \{v_0, v_1, \ldots, v_{38}\}$ and $W = \{a_0, a_1, \ldots, a_{38}\} \cup \{b_0, b_1, \ldots, b_{18}\} = \{w_0, w_1, \ldots, w_{57}\}$, where

$$w_i = \begin{cases} a_i, & i = 0, 1, \ldots, 38; \\ b_{i-39}, & i = 39, 40, \ldots, 57. \end{cases}$$

Subscripts of v and a are calculated (mod 39), subscripts of b are calculated (mod 19) and subscripts of w are calculated (mod 58). Place in $\mathcal{B}$:

- $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{j-1}]$, $i \in \{0, 1, \ldots, 38\}$ and $j \in \{1, 2, \ldots, 19\}$;

- $[w_{i-19}, w_{i+4}, w_i \bowtie w_{i+29}]$, $i \in \{0, 1, \ldots, 28\}$; $[w_i, w_{i+4}, w_{i-19} \bowtie v_{i-19}]$, $i \in \{29, 30, \ldots, 57\}$; $[w_{i+3}, w_{i+1}, w_i \bowtie v_i]$, $i \in \{0, 1, \ldots, 9\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+4}]$, $i \in \{10, 14, \ldots, 54\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+5}]$, $i \in \{11, 15, \ldots, 55\}$; $[w_i, w_{i+3}, w_{i+1} \bowtie w_{i+4}]$, $i \in \{12, 16, \ldots, 56\}$; $[w_{i+10}, w_i, w_{i+5} \bowtie w_{i+15}]$, $i \in \{0, 2, \ldots, 56\}$; $[w_{i+s}, w_{i+r}, w_i \bowtie w_{i+t}]$, $i \in \{0, 1, \ldots, 52\}$ and $(r, s, t) \in \{(6, 28, 11), (7, 27, 13), (8, 25, 16), (9, 24, 18), (12, 26, 21)\}$.

Case $n = 104$. Let $V = \{v_0, v_1, \ldots, v_{41}\}$ and $W = \{a_0, a_1, \ldots, a_{41}\} \cup \{b_0, b_1, \ldots, b_{19}\} = \{w_0, w_1, \ldots, w_{60}\} \cup \{\infty\}$, where $\infty = b_{19}$ and

$$w_i = \begin{cases} a_i, & i = 0, 1, \ldots, 41; \\ b_{i-42}, & i = 42, 43, \ldots, 60. \end{cases}$$

Subscripts of v and a are calculated (mod 42), subscripts of b are calculated (mod 20) and subscripts of w are calculated (mod 61). Place in $\mathcal{B}$:

- $[a_{i+j+22}, v_{i+2j+1}, v_i \bowtie b_j]$, $i \in \{0, 1, \ldots, 41\}$ and $j \in \{0, 1, \ldots, 9\}$; $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{j+9}]$, $i \in \{0, 1, \ldots, 41\}$ and $j \in \{1, 2, \ldots, 9\}$; $[a_{i+10}, v_{i+20}, v_i \bowtie a_{i+11}]$, $i \in \{0, 1, \ldots, 41\}$; $[b_{19}, v_i, v_{i+21} \bowtie a_{i+21}]$, $i \in \{0, 1, \ldots, 20\}$;

- $[w_{i+3}, w_{i+1}, w_i \bowtie v_i]$, $i \in \{0, 1, \ldots, 20\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+4}]$, $i \in \{21, 25, \ldots, 57\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+5}]$, $i \in \{22, 26, \ldots, 58\}$; $[w_i, w_{i+3}, w_{i+1} \bowtie w_{i+4}]$, $i \in \{23, 27, \ldots, 59\}$; $[w_{i+30}, w_{i+4}, w_i \bowtie \infty]$, $i \in \{0, 1, \ldots, 60\}$; $[w_{i+s}, w_{i+r}, w_i \bowtie w_{i+t}]$, $i \in \{0, 1, \ldots, 60\}$ and $(r, s, t) \in \{(5, 29, 13), (6, 28, 14), (7, 27, 15), (8, 25, 16), (9, 19, 18), (11, 23, 21)\}$.

Case $n = 105$. Let $V = \{v_0, v_1, \ldots, v_{41}\}$ and $W = \{a_0, a_1, \ldots, a_{41}\} \cup \{b_0, b_1, \ldots, b_{20}\} = \{w_0, w_1, \ldots, w_{62}\}$, where

$$
w_i = \begin{cases} a_i, & i = 0, 1, \ldots, 41; \\ b_{i-42}, & i = 42, 43, \ldots, 62. \end{cases}
$$

Subscripts of v and a are calculated (mod 42), subscripts of b are calculated (mod 21) and subscripts of w are calculated (mod 63). Place in $\mathcal{B}$:

- $[a_{i+j+22}, v_{i+2j+1}, v_i \bowtie b_j]$, $i \in \{0, 1, \ldots, 41\}$ and $j \in \{0, 1, \ldots, 9\}$; $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{j+9}]$, $i \in \{0, 1, \ldots, 41\}$ and $j \in \{1, 2, \ldots, 10\}$; $[b_{20}, v_i, v_{i+21} \bowtie a_{i+21}]$, $i \in \{0, 1, \ldots, 20\}$;

- $[w_{i+22}, w_{i+21}, w_i \bowtie v_{i-11}]$, $i \in \{0, 1, \ldots, 41\}$; $[w_{i+22}, w_i, w_{i+21} \bowtie v_{i+21}]$, $i \in \{42, 43, \ldots, 62\}$; $[w_{i+s}, w_{i+r}, w_i \bowtie w_{i+t}]$, $i \in \{0, 1, \ldots, 62\}$ and $(r, s, t) \in \{(2, 31, 10), (3, 30, 12), (4, 28, 14), (5, 20, 16), (6, 19, 23), (7, 18, 25), (8, 17, 26)\}$.

Case $n = 112$. Let $V = \{v_0, v_1, \ldots, v_{44}\}$ and $W = \{a_0, a_1, \ldots, a_{44}\} \cup \{b_0, b_1, \ldots, b_{21}\} = \{w_0, w_1, \ldots, w_{63}\} \cup \{\infty_1, \infty_2, \infty_3\}$, where $\infty_1 = a_{44}$, $\infty_2 = b_0$, $\infty_3 = b_1$ and

$$
w_i = \begin{cases} a_i, & i = 0, 1, \ldots, 43; \\ b_{i-42}, & i = 42, 43, \ldots, 63. \end{cases}
$$

Subscripts of v and a are calculated (mod 45), subscripts of b are calculated (mod 22) and subscripts of w are calculated (mod 64). Place in $\mathcal{B}$:

- $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{j-1}]$, $i \in \{0, 1, \ldots, 44\}$ and $j \in \{1, 2, \ldots, 22\}$;

- $[\infty_3, \infty_2, \infty_1 \bowtie v_{44}]$; $[w_{i+31}, w_{i+4}, w_i \bowtie v_i]$, $i \in \{12, 13, \ldots, 43\}$; $[w_{i+31}, w_{i+4}, w_i \bowtie w_{i+32}]$, $i \in \{44, 45, \ldots, 63, 0, 1, \ldots, 11\}$; $[w_{i+3}, w_{i+1}, w_i \bowtie v_i]$, $i \in \{0, 1, \ldots, 11\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+4}]$, $i \in \{12, 16, \ldots, 60\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+5}]$, $i \in \{13, 17, \ldots, 61\}$; $[w_i, w_{i+3}, w_{i+1} \bowtie w_{i+4}]$, $i \in \{14, 18, \ldots, 62\}$; $[w_{i+30}, w_{i+5}, w_i \bowtie \infty_1]$, $i \in \{0, 1, \ldots, 63\}$; $[w_{i+29}, w_{i+6}, w_i \bowtie \infty_2]$, $i \in \{0, 1, \ldots, 63\}$; $[w_{i+20}, w_{i+7}, w_i \bowtie \infty_3]$, $i \in \{0, 1, \ldots, 63\}$; $[w_{i+s}, w_{i+r}, w_i \bowtie w_{i+t}]$, $i \in \{0, 1, \ldots, 52\}$ and $(r, s, t) \in \{(8, 22, 15), (9, 21, 18), (11, 28, 19), (10, 26, 24)\}$.

Case $n = 113$. Let $V = \{v_0, v_1, \ldots, v_{44}\}$ and $W = \{a_0, a_1, \ldots, a_{44}\} \cup \{b_0, b_1, \ldots, b_{22}\} = \{w_0, w_1, \ldots, w_{66}\} \cup \{\infty\}$, where $\infty = a_0$ and

$$
w_i = \begin{cases} a_{i+1}, & i = 0, 1, \ldots, 43; \\ b_{i-44}, & i = 44, 45, \ldots, 66. \end{cases}
$$

Subscripts of v and a are calculated (mod 45), subscripts of b are calculated (mod 23) and subscripts of w are calculated (mod 67). Place in $\mathcal{B}$:

- $[a_{i+j}, v_{i+2j}, v_i \bowtie b_{i+j}]$, $i \in \{0, 1, \ldots, 44\}$, $j \in \{1, 2, \ldots, 22\}$ and $(i, j) \neq (0, 22)$; $[a_{22}, v_{44}, v_0 \bowtie a_0]$;

- $[w_{i+33}, w_{i+4}, w_i \bowtie v_{i+1}]$, $i \in \{0, 1, \ldots, 66\}$; $[w_{i+3}, w_{i+1}, w_i \bowtie v_{i-21}]$, $i \in \{44, 45, \ldots, 66\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+4}]$, $i \in \{0, 4, \ldots, 40\}$; $[w_i, w_{i+1}, w_{i+3} \bowtie w_{i+5}]$, $i \in \{1, 5, \ldots, 41\}$; $[w_i, w_{i+3}, w_{i+1} \bowtie w_{i+4}]$, $i \in \{2, 6, \ldots, 42\}$; $[w_{i+32}, w_{i+5}, w_i \bowtie \infty]$, $i \in \{0, 1, \ldots, 66\}$; $[w_{i+s}, w_{i+r}, w_i \bowtie w_{i+t}]$, $i \in \{0, 1, \ldots, 52\}$ and $(r, s, t) \in \{(6, 26, 17), (7, 21, 18), (8, 23, 19), (9, 25, 28), (10, 22, 30), (11, 24, 31)\}$.