A Short Course on Market Design and Matching

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Lecture 1: Introduction

Market mechanism: not always the right solution

Some examples (“What money can’t buy”, by Michael Sandel):

- Shakespeare in the Park
- Linestanding.com
- Fast tracks (airports, amusement parks, etc.)

Another example:

- Selling organs

Market mechanism: not always the right solution

- There are gains from trade.
- Negative spillovers over some segments of the market.
- Market mechanism undermines the political and social institutions.

Monetary incentives might backfire

- A fine or a fee: a daycare experiment
  Uri Gneezy, Aldo Rustichini, A Fine is a Price, *Journal of Legal Studies*, 29, 1-17, 2000
- To pay students for reading books
  Fryer R. Financial Incentives and Student Achievement: Evidence from Randomized Trials, *Quarterly Journal of Economics*, 126 (4) :1755-1798, 2011
- Paying for blood donations
  The Gift Relationship, Richard Titmuss, 1970

Monetary incentives might backfire

- Corrupting nature of money
- Intrinsic motivation is crowded out by monetary incentives.
- Monetary compensation for donating blood might crowd out the supply of blood donors

Selling organs

Kantian perspective

"Human beings are of incomparable ethical worth and admit of no equivalent. Each has value that is beyond the contingencies of supply and demand or of any other relative estimation. They are priceless. Consequently, to sell an integral human body part is to corrupt the very meaning of human dignity." (Cohen, 1999)

Market rejection as a fact

"My point is that people find some transactions repugnant. That's a reason to treat other people's intuitions about repugnant transactions with respect, even if they don't raise lower their hands at the same moment we do." (Roth, 2015)

Selling organs

Tirole's response

"Our repulsive feelings are sources of unreliable ethics inspirations." (Tirole, 2016)

Inconsistency argument

"There are many other situations when money is given to others without suggesting that the monetary exchange causes loss of dignity. We give money as a present to people celebrating their birthday. We give money as a baptismal gift. We give money to convey our condolences to the bereaved. We pin paper bills on the wedding attire of the bride or groom to symbolise our support for their partnership. The point is that there is nothing inherently suspicious about money changing hands. On the contrary, money has important symbolisms for various practices and traditions." (de Castro, 2003)

Selling organs

Becker and Elias (2007) propose a market where the health system, not the individuals, is the supplier.

- The price is not freely determined.
- They propose a scheme for calculating the price offered by the medical system.
- The calculation of this price has three components:
  - the risk of death during transplantation (price of human life: \$5m, multiplied by the risk, 0.1% = \$5K)
  - the period of convalescence
  - the decrease in the quality of life after the operation
- For a total price of \$15,000 which in addition to \$160,000 on the average cost of a kidney transplant, gives a unit cost of \$175,000.

The approach of market design

- The rejection of price mechanism is a fact and backed by the legal framework.
- This constitute a set of constraints, which cannot be removed. So, work under these constraints.
- Design a market without money for welfare improvements.

The scope

- student placement in schools,
- labor markets where workers and firms are matched,
- organizing organ donation network
- resettlement of refugees
  - “Improving refugee integration through data-driven algorithmic assignment” by Bansak, Ferwerda, Hainmueller, Dillon, Hangartner, Lawrence and Weinstein, forthcoming in *Science*.
- allocating courses to students
- implementing affirmative action
- cadet-branch matching
- allocating dormitories-houses to students

Social impact

The economics of ‘matching and market design’ has analyzed these problems and improved real-life institutions in recent years. For example, economists have helped

- NYC and Boston school districts design their school choice programs,
- medical communities reorganize their hiring procedure, and
- transplantation centers organize systematic kidney exchange mechanisms to give kidneys to as many patients as possible.

An introduction

1. the US hospital-intern market (hospital-intern matching)
2. student placement in schools (school choice)
3. allocating rooms to professors (assignment problem)
4. organizing organ donation network (kidney exchange)
5. labor markets with workers and firms (matching with contracts)

The case of US hospital-intern market

- Medical students in many countries work as residents (interns) at hospitals.
- Beginning around 1900, the American hospital-intern market was decentralized, and suffered from serious issues:
  - Students and hospitals made contracts 2 years in advance of graduation.
  - There were a lot of mismatch because students' quality and interests were unknown early in the study.
- This caused inefficiency, and doctors and hospitals tried to change their system.

The case of US hospital-intern market

- A centralized matching mechanism called NIMP (now called NRMP) was established in 1952. Students submitted rank order lists over hospitals and hospitals submitted rank order lists over students. The NIMP used these lists to decide who works where.
- The NRMP is in use now.

Two-sided matching theory

- Gale and Shapley (1962): A matching is **stable** if there is no doctor and a hospital who are not matched with each other in the prescribed match and prefer to match with each other rather than their current match.
- An unstable matching suffers from deviation by participants. On the other hand, a *stable* matching is expected to be sustainable.
  - The Deferred Acceptance Algorithm (Gale and Shapley, 1962)

School choice programs

- In many countries, children were automatically sent to a school in their neighborhoods. (In Turkey, it is still the case.)
- Recently, more and more cities in the United States and in other countries employ school choice programs: school authorities take into account preferences of children and their parents.
  - Placement mechanisms
- Typical goals of school authorities are: (1) efficient placement, (2) fairness of outcomes, (3) easy for participants to understand and use, etc.

School choice programs

- These placement mechanisms used in many cities such as Boston are flawed (Abdulkadiroğlu and Sönmez, 2003):
  1. the mechanism is manipulable, i.e., students may benefit by reporting false preferences, and
  2. the result may be neither fair nor efficient.
- New mechanisms are proposed to improve upon existing placement mechanisms (Abdulkadiroğlu and Sönmez, 2003).
- Based on this and other studies, Boston and NYC changed their student placement mechanisms: In NYC, around 30,000 students were not matched to any of their preferred schools in the old mechanism, but this number is reduced to only 3,000 after the new mechanism is adopted.

A simple theory

- a finite set of students $I = \{i_1, i_2, ..., i_n\}$;
- a finite set of schools $S = \{s_1, s_2, ..., s_m\}$;
- a strict priority profile of schools $\succ = (\succ_s)_{s \in S}$ where $\succ_s$ is the complete priority order of school s over I;
- a capacity vector $q = (q_s)_{s \in S}$ where q_s is the number of available seats at school s;
- a strict preference profile of students $P = (P_i)_{i \in I}$ such that P_i is student i's strict preferences over $S \cup \{\emptyset\}$, where $\emptyset$ stands for the option of being unassigned with $q_\emptyset = |I|$.

A simple theory

- A **matching** $\mu : I \to S \cup \{\emptyset\}$ is a function such that for each $s \in S$, $|\mu^{-1}(s)| \leq q_s$.
- A matching μ **violates the priority** of student $i \in I$ at school $s \in S$ if there exists another $j \in I$ such that $\mu(j) = s$, $s \; P_i \; \mu(i)$, and $i \succ_s j$.
- A matching μ is **stable** if for each $i \in I$ and $s \in S$, it does not violate the priority of student i at school s.

Do stable matchings exist?

Theorem (Gale and Shapley, 1962)

There always exists a stable matching.

- The proof relies on the convergence of an algorithm, the Deferred Acceptance (DA), and the fact that it always gives a *stable* matching.

The student-proposing DA Algorithm

Step 1 **(1.1)** Each student "applies" to her first choice school.
(1.2) Each school tentatively holds the most preferred applicants up to its quota and rejects all other students.

Step k ≥ 2 **(k.1)** Each student rejected in Step $k-1$ applies to her next highest choice.
(k.2) Each school considers both new applicants and the students (if any) held at Step $k-1$, tentatively holds the most preferred acceptable students up to its quota from the combined set of students, and rejects all other students.

The student-proposing DA Algorithm: An example

$\succ_{s_1}: i_1 - i_3 - i_2$
$\succ_{s_2}: i_2 - i_1 - i_3$
$\succ_{s_3}: i_2 - i_1 - i_3$

$R_{i_1}: s_2 - s_1 - s_3$
$R_{i_2}: s_1 - s_2 - s_3$
$R_{i_3}: s_1 - s_2 - s_3$