PHYSICAL REVIEW A **77**, 041607(R) (2008)

Trapped *p*-wave superfluids: A local-density approach

M. Iskin and C. J. Williams

Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, Maryland 20899-8423, USA

(Received 25 January 2008; published 16 April 2008)

The local-density approximation is used to study the ground state superfluid properties of harmonically trapped *p*-wave Fermi gases as a function of fermion-fermion attraction strength. While the density distribution is bimodal on the weakly attracting BCS side, it becomes unimodal with increasing attraction and saturates towards the Bose-Einstein condensate (BEC) side. This nonmonotonic evolution is related to the topological gapless-to-gapped phase transition, and may be observed via radio-frequency spectroscopy since quasiparticle transfer current requires a finite threshold only on the BEC side.

DOI: 10.1103/PhysRevA.77.041607 PACS number(s): 03.75.Hh, 03.75.Ss, 05.30.Fk

Recent experiments measuring momentum distribution, collective modes, order parameter, quantized vortices, etc. have provided strong evidence for observation of a superfluid phase in two-component cold atomic mixtures, interacting with short-range attractive *s*-wave interactions [1–6]. These experiments have also shown evidence that the ground state of these *s*-wave mixtures evolves smoothly from a paired Bardeen-Cooper-Schrieffer (BCS) superfluid to a molecular Bose-Einstein condensate (BEC) as the attractive interaction varies from weak to strong values, marking the first demonstration of theoretically predicted BCS-BEC crossover [7–9].

On the other hand, there has also been substantial experimental progress studying *p*-wave interactions to observe triplet superfluidity [10–14]. However, controlling the *p*-wave interactions is proving much more difficult due to the narrow nature of *p*-wave Feshbach resonances as well as to the more dramatic two- and also three-body losses [10–14]. These experiments still motivated considerable theoretical interest predicting quantum and topological phase transitions [15–20]. More recently, *p*-wave molecules have been produced and their two-body properties have been studied [21], opening the possibility of studying many-body properties of *p*-wave superfluids in the near future.

In this Rapid Communication, unlike the previous works on homogenous systems [15–20], we study the ground state superfluid properties of harmonically trapped *p*-wave Fermi gases as a function of fermion-fermion attraction strength. Our main results are as follows. While we find that the density distribution is bimodal on the weakly attracting BCS side where the local chemical potentials are positive everywhere inside the trap, it becomes unimodal with increasing attraction and saturates towards the BEC side where the local chemical potentials become negative. This nonmonotonic evolution is related to the topological gapless-to-gapped phase transition occurring in *p*-wave superfluids, and is in sharp contrast with the *s*-wave case where the superfluid phase is always gapped leading to a smooth crossover. Lastly, we propose that the phase transition found in the *p*-wave case may be observed via radio-frequency spectroscopy since quasiparticle transfer current requires a finite threshold only on the BEC side, which is in sharp contrast with the crossover physics found in the *s*-wave case where a finite threshold is required throughout BCS-BEC evolution.

Local-density (LD) approach. To obtain these results, we consider a harmonic trapping potential, separate the relative motion from the center-of-mass one, and use LD approximation to describe the latter. In this approximation, the system is treated as locally homogenous at every position inside the trap, and it is valid as long as the number of fermions is large which is typically satisfied in cold atomic systems [22]. In order to describe the pairing correlations occurring in the relative coordinates, we use the BCS mean field (MF) formalism and neglect fluctuations. The MF description is qualitatively valid throughout BCS-BEC evolution only at the low temperatures considered here [7–9], and it has been extensively applied to cold atomic systems describing qualitatively the experimental observations.

Therefore we start with the following local MF Hamiltonian (in units of $\hbar = k_B = 1$)

$$H_\ell(r) = \sum_{\mathbf{k},\sigma} \xi_\ell(r,\mathbf{k}) a^\dagger_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma} + \frac{|\Delta_\ell(r)|^2}{g} - \sum_{\mathbf{k}} [\Delta_\ell(r,\mathbf{k}) a^\dagger_{\mathbf{k},\uparrow} a^\dagger_{-\mathbf{k},\downarrow} + \text{H.c.}], \qquad (1)$$

where $\ell = 0$ ($\ell = 1$) corresponds to *s*-wave (*p*-wave) systems, $a^\dagger_{\mathbf{k},\sigma}$ creates a pseudospin σ fermion with momentum $\mathbf{k}$, and $\xi_\ell(r,\mathbf{k}) = \epsilon(\mathbf{k}) - \mu_\ell(r)$ is the dispersion with $\epsilon(\mathbf{k}) = k^2/(2M)$ and $\mu_\ell(r) = \mu_\ell - V(r)$. While the global chemical potential μ_ℓ fixes the total number of fermions, the local chemical potential $\mu_\ell(r)$ includes the trapping potential $V(r) = M\omega_0^2 r^2/2$. In Eq. (1), $\Delta_\ell(r,\mathbf{k}) = \Delta_\ell(r)\Gamma_\ell(\mathbf{k})$ is the local MF order parameter where $\Delta_\ell(r) = g\Sigma_{\mathbf{k}}\Gamma_\ell(\mathbf{k})\langle a_{-\mathbf{k},\uparrow} a_{\mathbf{k},\downarrow}\rangle$ describes the spatial dependence, such that $g > 0$ is the strengh of the attractive fermion-fermion interactions and $\langle \cdots \rangle$ implies a thermal average. $\Gamma_\ell(\mathbf{k})$ determines the symmetry of the order parameter given by $\Gamma_\ell(\mathbf{k}) = W_\ell(k)\Sigma_m \lambda_{\ell,m} Y_{\ell,m}(\hat{\mathbf{k}})$ where $W_\ell(k) = k^\ell k_0/(k^2 + k_0^2)^{(\ell+1)/2}$ describes the momentum dependence [16,19]. Here, $k_0 \sim R_0^{-1}$ sets the momentum scale where R_0 is the interaction range in real space.

In this Rapid Communication, we assume $\Delta_\ell(r)$ to be real, and consider only the $\lambda_{1,0} = 1$ and $\lambda_{1,\pm 1} = 0$ symmetry. This choice is motivated by recent experiments [11,14], where it was found that the magnetic dipole-dipole interactions between valence electrons split *p*-wave Feshbach resonances that belong to different m states. Thus $m=0$ and ± 1 resonances may be tuned and studied independently if the splitting is large enough in comparison to the experimental resolution. However, we emphasize that our discussion applies qualitatively to other p-wave symmetries as well.

The local MF Hamiltonian can be solved by using standard techniques [16,19], leading to a set of nonlinear equations for $\Delta_\ell(r)$ and $\mu_\ell(r)$. These equations are

$$\frac{MV}{4\pi a_\ell k_0^{2\ell}} = \sum_{\mathbf{k}} \left[\frac{W_\ell^2(k)}{2\epsilon(\mathbf{k})} - \frac{4\pi\Gamma_\ell^2(\mathbf{k})}{2E_\ell(r,\mathbf{k})} \tanh\frac{E_\ell(r,\mathbf{k})}{2T} \right], \quad (2)$$

$$n_\ell(r) = \frac{1}{V}\sum_{\mathbf{k},\sigma} \left[\frac{1}{2} - \frac{\xi_\ell(r,\mathbf{k})}{2E_\ell(r,\mathbf{k})} \tanh\frac{E_\ell(r,\mathbf{k})}{2T} \right], \quad (3)$$

where a_ℓ is the experimentally relevant scattering parameter which regularizes g, $E_\ell(r,\mathbf{k})=\sqrt{\xi_\ell^2(r,\mathbf{k})+\Delta_\ell^2(r,\mathbf{k})}$ is the local quasiparticle energy, T is the temperature, and V is the volume. In Eq. (3), $n_\ell(r)$ is the local density of fermions, and the total number of fermions N is fixed by $N=\int d\mathbf{r}\, n_\ell(r)$. Notice that a_ℓ has units of length (volume) in the s-wave (p-wave) case, and that our self-consistent solutions also describe the single pseudospin p-wave systems (except for the σ summations here and throughout), which are presented next.

BCS-BEC evolution in homogenous systems. To understand the ground state properties of harmonically trapped p-wave superfluids within the LD approach, it is very useful to analyze first the homogenous s- and p-wave systems where $V(r)=0$. Thus next we discuss the s-wave case where $\Delta_s(\mathbf{k})=\Delta_s W_0(k) Y_{0,0}(\hat{\mathbf{k}})$ with $Y_{0,0}(\hat{\mathbf{k}})=1/\sqrt{4\pi}$, and compare these results with the p-wave case where $\Delta_p(\mathbf{k})=\Delta_p W_1(k) Y_{1,0}(\hat{\mathbf{k}})$ with $Y_{1,0}(\hat{\mathbf{k}})=\sqrt{3/(4\pi)}\cos(\theta_{\mathbf{k}})$. In the numerical calculations, while we mainly consider $k_0=100k_F$ to describe realistically the short-ranged atomic interactions, some of the $k_0=10k_F$ results are also shown for comparison.

In Fig. 1(a), we show Δ_s and μ_s at zero temperature ($T=0$) for the s-wave case where the BCS-BEC evolution range in $1/(k_F a_s)$ is of order 1. Notice that Δ_s grows continuously without saturation with increasing attraction, while μ_s decreases continuously from the Fermi energy $\epsilon_F=k_F^2/(2M)$ on the BCS side to the half of the binding energy $\epsilon_{b,s}/2=-1/(2Ma_s^2)$ on the BEC side [9]. Here, k_F is the Fermi momentum which fixes the total density $n=\Sigma_\sigma k_F^3/(6\pi^2)$ of fermions. Thus we conclude that the evolution of Δ_s and μ_s as a function of $1/(k_F a_s)$ is analytic throughout, and BCS-BEC evolution is a smooth crossover [7–9].

In Fig. 1(b), we show Δ_p and μ_p at $T=0$ for the p-wave case where the BCS-BEC evolution range in $1/(k_0 k_F^2 a_p)$ is of order 1. Notice that Δ_p is exponentially small but still finite in the BCS limit when $\mu_p \approx \epsilon_F$ and it grows rapidly with increasing attraction but almost saturates for large $1/(k_0 k_F^2 a_p)$, while μ_p decreases continuously from ϵ_F on the BCS side to $\epsilon_{b,p}/2=-1/(Mk_0 a_p)$ on the BEC side. However, both Δ_p and μ_p are nonanalytic exactly when $\mu_p=0$ at $1/(k_F^3 a_p)\approx 0.45$, which occurs on the BEC side of unitarity ($|a_p|\to\infty$). We note that the nonanalyticity of μ_p is barely seen in Fig. 1(b), and it is more explicit in derivatives of μ_p.

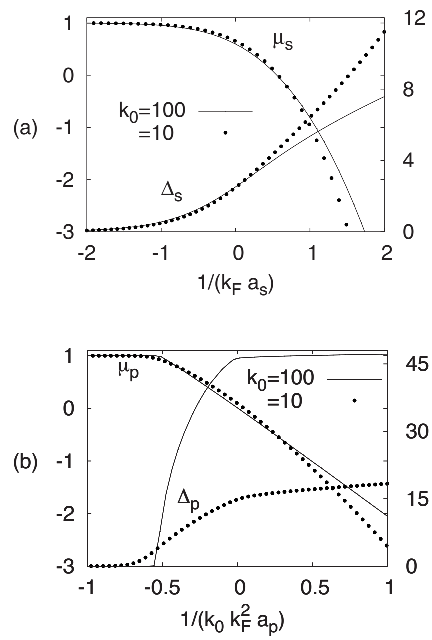

FIG. 1. We show (in units of ϵ_F) the chemical potential μ_ℓ (left y axis) and the amplitude of the order parameter Δ_ℓ (right y axis) for (a) s-wave systems as a function of $1/(k_F a_s)$ and (b) p-wave systems as a function of $1/(k_0 k_F^2 a_p)$. Here, solid (dotted) lines correspond to $k_0=100k_F$ ($k_0=10k_F$).

Thus in the p-wave case, BCS-BEC evolution is not a crossover, but a quantum phase transition occurs [15,17,19].

This phase transition can be understood as follows. The quasiparticle excitation spectrum $E_\ell(\mathbf{k})$ is gapless when the conditions $\Delta_\ell(\mathbf{k})=0$ and $\xi_\ell(\mathbf{k})=0$ are both satisfied for some $\mathbf{k}$-space regions. While the second condition is satisfied for both s- and p-wave symmetries on the BCS side where $\mu_\ell>0$, the first condition is only satisfied by the p-wave order parameter. Therefore unlike the s-wave case, $E_p(\mathbf{k})$ is gapless on the BCS side ($\mu_p>0$) but it is gapped on the BEC side ($\mu_p<0$), leading to the phase transition discussed above [7,15,19,23]. Having discussed the ground state of homogenous systems, next we analyze the trapped case.

BCS-BEC evolution in trapped systems. For this purpose, similar to the analysis of homogenous systems, first we discuss the s-wave case where $\Delta_s(r,\mathbf{k})=\Delta_s(r) W_0(k) Y_{0,0}(\hat{\mathbf{k}})$, and compare these results with the p-wave case where $\Delta_p(r,\mathbf{k})=\Delta_p(r) W_1(k) Y_{1,0}(\hat{\mathbf{k}})$. In the numerical calculations, we again choose $k_0=100k_F$ where $k_F=M\omega_0 r_F$ is the global Fermi momentum. Here, r_F is the Thomas-Fermi radius determined by $V(r_F)=\epsilon_F=k_F^2/(2M)$ and fixes the total number of fermions to $N=\Sigma_\sigma k_F^3 r_F^3/48$.

Within the LD approximation, the density distribution of trapped noninteracting ($g=0$ or $a_\ell\to 0^-$) gas is $n_\ell(r)=\Sigma_\sigma k_F^3(r)/(6\pi^2)$, where $k_F(r)$ is the local Fermi momentum determined by $\mu_\ell=k_F^2(r)/(2M)+V(r)$ with $\mu_\ell=\epsilon_F$. Therefore both $k_F(r)$ and $n_\ell(r)$ are highest at the center of the trap as can be also seen in Figs. 2(a) and 2(b) when $1/(k_F a_s)=-\infty$ and $1/(k_0 k_F^2 a_p)=-\infty$, respectively.

In the presence of weak attraction, while μ_ℓ deviates from ϵ_F, the density distribution is still well-described by the noninteracting expression. For fixed N, $n_\ell(r)$ is expected to

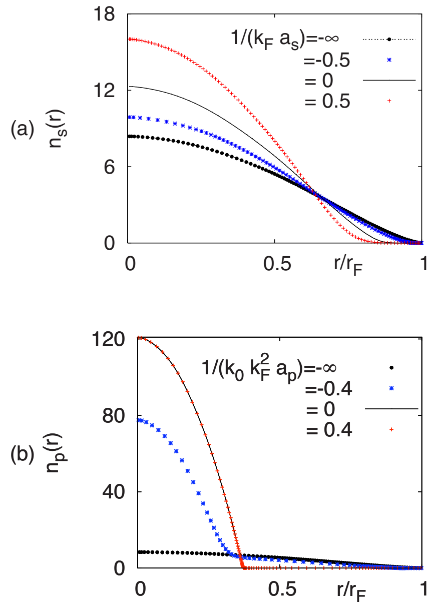

FIG. 2. (Color online) We show [in units of $k_F^3/(2\pi)^3$] the density distribution $n_\ell(r)$ for (a) s- and (b) p-wave systems as a function of the trap radius r (in units of r_F). Here, we set $k_0=100k_F$.

squeeze and increase towards the center of the trap since μ_ℓ decreases with increasing attraction. The squeezing effect of the weak attractive interactions can be seen in Fig. 2(a) for the s-wave and in Fig. 2(b) for the p-wave systems when $1/(k_F a_s)=-0.5$ and $1/(k_0 k_F^2 a_p)=-0.4$, respectively.

However, weakly attracting p-wave superfluids have bimodal density distribution as shown in Fig. 2(b), which is in sharp contrast with the unimodal s-wave distribution of Fig. 2(a). This difference can be understood from the homogenous results shown in Figs. 1(a) and 1(b) as follows. First, since $k_F(r)$ decreases away from the center of the trap, the local s- and p-wave scattering parameters $1/[k_F(r)a_s]$ and $1/[k_0 k_F^2(r)a_p]$, respectively, increase as a function of r if a_s and a_p are fixed. Second, notice in Fig. 1(b) that Δ_p increases rapidly from exponentially small to larger values as a function of $1/(k_0 k_F^2 a_p)$, unlike Δ_s which increases smoothly as shown in Fig. 2(a). These two observations combined show that the almost noninteracting $n_p(r)$ distribution towards the tail is due to finite but exponentially small $\Delta_p(r)$.

With increasing attraction towards unitarity, while the unimodal $n_s(r)$ distribution smoothly squeezes further as shown in Fig. 2(a) for $1/(k_F a_s)=0$ and 0.5, the bimodal $n_p(r)$ distribution becomes unimodal and saturates as shown in Fig. 2(b) for $1/(k_0 k_F^2 a_p)=0$ and 0.4. This difference can be understood at best on the BEC side where strongly attracting fermion pairs form weakly repulsive local molecules which can be well-described by the Bogoliubov theory [9,19]. On this side, the size of the s-wave molecules decreases to arbitrarily small values as $\xi_{B,s}\sim a_s>0$ when $k_0 a_s \gg 1$, leading to arbitrarily weak molecule-molecule repulsion $U_{BB,s}=2\pi a_{BB,s}/M$ where $a_{BB,s}=2a_s$ within the Born approximation [9]. However, the size of the p-wave molecules saturates to small but finite values as $\xi_{B,p}\sim 1/k_0$ when $k_0^3 a_p \gg 1$, leading also to a weak but finite molecular repulsion $U_{BB,p}=2\pi a_{BB,p}/M$ where $a_{BB,p}=9/k_0$ [19]. Since our LD approximation recovers the Thomas-Fermi approximation for the resultant molecules in the BEC limit, $n_p(r)$ saturates rapidly for $1/(k_0 k_F^2 a_p)>0$ due to the presence of weak but finite molecular repulsion.

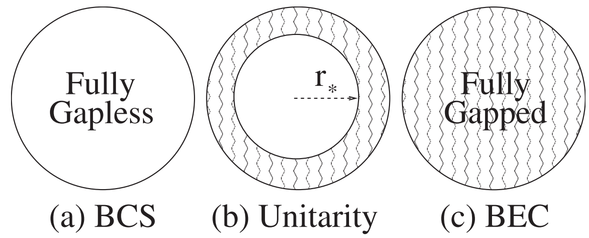

FIG. 3. Schematic diagrams showing (a) a fully gapless superfluid on the BCS side when $\mu_p(0\lesssim r\lesssim r_F)>0$, (b) a partially gapped superfluid around unitarity when $\mu_p(0\lesssim r\lesssim r_*)>0$ but $\mu_p(r>r_*)<0$, and (c) a fully gapped superfluid on the BEC side when $\mu_p(r\gtrsim 0)<0$.

This nonmonotonic evolution of $n_p(r)$ is also related to the topological phase transition discussed above for the homogenous systems. In the trapped case, the local quasiparticle excitation spectrum $E_\ell(r,\mathbf{k})$ at position r is gapless when the conditions $\Delta_\ell(r,\mathbf{k})=0$ and $\xi_\ell(r,\mathbf{k})=0$ are both satisfied for some $\mathbf{k}$-space regions. While these conditions are both satisfied everywhere inside the trap leading to a fully gapless superfluid on the BCS side, they are only satisfied around the center of the trap close to unitarity leading to a partially gapped superfluid. Further increasing the attraction towards the BEC limit, the second condition is not satisfied, and the entire superfluid becomes fully gapped. These phases are schematically shown in Figs. 3(a)–3(c), respectively, and next we discuss their experimental detection.

Radio-frequency (rf) spectroscopy. The gapless-to-gapped phase transition discussed above may be observed in cold atomic systems via, for instance, rf spectroscopy, where atoms are transferred from one hyperfine state to another generating a quasiparticle current [22,24–26]. This is analogous to electrons tunneling from a superconducting to normal metal, and it has been used in atomic systems to observe pairing correlations in unpolarized [25] as well as polarized [26] mixtures.

The local quasiparticle transfer current, within the LD approximation, is given by [22,24,27]

$$I_\ell(r,\omega) = t_F^2 \sum_{\mathbf{k}} A_\ell[\mathbf{k},\xi_\ell(r,\mathbf{k})-\omega] F[\xi_\ell(r,\mathbf{k})-\omega], \qquad (4)$$

where t_F is the transfer amplitude, $A_\ell(\mathbf{k},x)$ is the spectral function corresponding to the superfluid state, and $F(x)=1/[\exp(x/T)+1]$ is the Fermi function. Here, $\omega=\omega_L-\omega_H$ is the effective detuning where ω_L and ω_H are rf laser frequency and hyperfine splitting, respectively. We evaluate Eq. (4) with the standard BCS spectral functions $A_\ell(\mathbf{k},\epsilon)=2\pi\{u_\ell^2(r,\mathbf{k})\delta[\epsilon-E_\ell(r,\mathbf{k})]+v_\ell^2(r,\mathbf{k})\delta[\epsilon+E_\ell(r,\mathbf{k})]\}$, where $u_\ell^2(r,\mathbf{k})=0.5[1+\xi_\ell(r,\mathbf{k})/E_\ell(r,\mathbf{k})]$ and $v_\ell^2(r,\mathbf{k})=0.5[1-\xi_\ell(r,\mathbf{k})/E_\ell(r,\mathbf{k})]$ are coherence factors, and $\delta(x)$ is the delta function.

At $T=0$, the s-wave current $I_s(r,\omega)$ can be evaluated analytically leading to $I_s(r,\omega)$

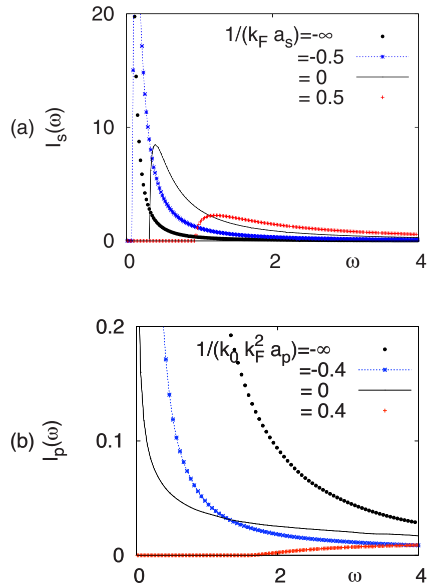

FIG. 4. (Color online) We show (in units of $\rho_F t_F^2/2$) the quasiparticle transfer current $I_\ell(\omega)$ for homogenous (a) s- and (b) p-wave systems as a function of the effective detuning ω (in units of ϵ_F). Here, we set $k_0=100k_F$.

$=\pi\rho_F t_F^2[\Delta_s(r)/(4\pi\omega)]^2\sqrt{C(r,\omega)}\theta(\omega)\theta[C(r,\omega)]$, where $\rho_F=MVk_F/(2\pi^2)$ is the density of states, $C(r,\omega)=[\omega^2-\Delta^2(r)/(4\pi)]/(2\omega)+\mu_s(r)$, and $\theta(x)$ is the theta function. Therefore, $I_s(r,\omega)$ flows when the threshold $\omega_{th,s}(r)\geq-\mu_s(r)+\sqrt{\mu_s^2(r)+\Delta_s^2(r)/(4\pi)}\geq 0$ is reached, which reduces to $\omega_{th,s}(r)\geq\Delta_s^2(r)/[8\pi\mu_s(r)]$ in the BCS and $\omega_{th,s}(r)\geq 2|\mu_s(r)|$ in the BEC limit. Therefore $\omega_{th,s}(r)\neq 0$ everywhere inside the trap throughout BCS-BEC evolution [22,24–26]. These finite detuning thresholds can be also seen in Fig. 4(a), where we show $I_s(\omega)$ for homogenous systems.

The p-wave current $I_p(r,\omega)$ is difficult to evaluate analytically. However, unlike the s-wave case, we expect that $\omega_{th,p}(r)\neq 0$ everywhere inside the trap only on the BEC side beyond unitarity where $E_p(r,\mathbf{k})$ is gapped, and also that $\omega_{th,p}(r)=0$ everywhere inside the trap on the BCS side where $E_p(r,\mathbf{k})$ is gapless. The absence (presence) of finite thresholds in gapless (gapped) superfluids can be seen in our homogenous results shown in Fig. 4(b). Notice that, while $\omega_{th,p}=0$ for $1/(k_0k_F^2a_p)=-\infty$, -0.4, and 0, a finite threshold is required for $1/(k_0k_F^2a_p)=0.4$. Based on these results, we hope that spatially resolved rf spectroscopy measurements (similar to [26]) may be used to identify all three phases proposed in Fig. 3.

Conclusions. To summarize, we showed that while the density distribution of p-wave systems is bimodal on the weakly attracting BCS side, it saturates and becomes unimodal with increasing attraction towards the BEC side. We discussed that this nonmonotonic evolution is related to the topological gapless-to-gapped phase transition occurring in p-wave superfluids, and is in sharp contrast with the s-wave case where the superfluid phase is always gapped leading to a smooth crossover. Lastly, we proposed that this phase transition may be observed via rf spectroscopy since quasiparticle transfer current requires a finite threshold only on the BEC side, which is in sharp contrast with the s-wave case where a finite threshold is required throughout BCS-BEC evolution.

[1] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. **92**, 040403 (2004).
[2] M. Bartenstein *et al.*, Phys. Rev. Lett. **92**, 203201 (2004).
[3] J. Kinast, S. L. Hemmer, M. E. Gehm, A. Turlapov, and J. E. Thomas, Phys. Rev. Lett. **92**, 150402 (2004).
[4] T. Bourdel *et al.*, Phys. Rev. Lett. **93**, 050401 (2004).
[5] G. B. Partridge, K. E. Strecker, R. I. Kamar, M. W. Jack, and R. G. Hulet, Phys. Rev. Lett. **95**, 020404 (2005).
[6] M. W. Zwierlein *et al.*, Nature (London) **435**, 1047 (2005).
[7] A. J. Leggett, J. Phys. (Paris), Colloq. **C7**, 19 (1980).
[8] P. Noziéres and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).
[9] J. R. Engelbrecht, M. Randeria, and C. A. R. Sade Melo, Phys. Rev. B **55**, 15153 (1997).
[10] C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. **90**, 053201 (2003).
[11] C. Ticknor, C. A. Regal, D. S. Jin, and J. L. Bohn, Phys. Rev. A **69**, 042712 (2004).
[12] J. Zhang *et al.*, Phys. Rev. A **70**, 030702(R) (2004).
[13] C. H. Schunck *et al.*, Phys. Rev. A **71**, 045601 (2005).
[14] K. Günter, T. Stoferle, H. Moritz, M. Kohl, and T. Esslinger, Phys. Rev. Lett. **95**, 230401 (2005).
[15] S. S. Botelho and C. A. R. Sá de Melo, J. Low Temp. Phys. **140**, 409 (2005).
[16] T.-L. Ho and R. B. Diener, Phys. Rev. Lett. **94**, 090402 (2005).
[17] V. Gurarie, L. Radzihovsky, and A. V. Andreev, Phys. Rev. Lett. **94**, 230403 (2005).
[18] C.-H. Cheng and S.-K. Yip, Phys. Rev. Lett. **95**, 070404 (2005).
[19] M. Iskin and C. A. R. Sá de Melo, Phys. Rev. Lett. **96**, 040402 (2006); Phys. Rev. A **74**, 013608 (2006).
[20] K. B. Gubbels and H. T. C. Stoof, Phys. Rev. Lett. **99**, 190406 (2007).
[21] J. P. Gaebler, J. T. Stewart, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. **98**, 200403 (2007).
[22] Y. Ohashi and A. Griffin, Phys. Rev. A **72**, 013601 (2005).
[23] G. E. Volovik, *Exotic Properties of Superfluid* ^{3}He (World Scientific, Singapore, 1992).
[24] P. Törma and P. Zoller, Phys. Rev. Lett. **85**, 487 (2000).
[25] C. Chin *et al.*, Science **305**, 1128 (2004).
[26] Y. Shin, C. H. Schunck, A. Schirotzek, and W. Ketterle, Phys. Rev. Lett. **99**, 090403 (2007).
[27] G. Mahan, *Many-Particle Physics* (Kluwer-Plenum, New York, 2000), Chap. 8.