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Trap-imbalanced fermion mixtures

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We analyze the ground-state phases of two-component ($\sigma \equiv \{\uparrow,\downarrow\}$) population- and mass-balanced ($N_\uparrow = N_\downarrow$ and $m_\uparrow = m_\downarrow$) but trap-imbalanced ($\omega_\uparrow \neq \omega_\downarrow$) fermion mixtures as a function of interaction strength from the weak attraction Bardeen-Cooper-Schrieffer (BCS) to the strong attraction Bose-Einstein condensation (BEC) limit. In the BCS limit, we find that the unpolarized superfluid (UPS) fermions exist away from the central core of the trapping potentials, and are surrounded by partially σ-polarized normal (PσPN) fermions. As the interactions increase towards unitarity, we find that the central PσPN core first transitions to a UPS, and then expands towards the edges until the entire mixture becomes a UPS in the BEC limit.

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Ultracold atomic physics experiments with two-component fermion mixtures have enabled the study of novel superfluid and insulating phases which have not been possible in other systems. For instance, the tuning of attractive fermion-fermion interactions has permitted the ground state of the system to evolve from a weak fermion attraction Bardeen-Cooper-Schrieffer (BCS) limit of loosely bound and largely overlapping Cooper pairs to a strong fermion attraction limit of tightly bound and small bosonic molecules that undergo Bose-Einstein condensation (BEC) [1–6]. For mass- and population-balanced mixtures, in agreement with the early theoretical predictions [7], these experiments have shown that the BCS-BEC evolution is just a crossover.

Recently, the ground-state phase diagram of mass-balanced but population-imbalanced fermion mixtures have been theoretically analyzed showing that the BCS-BEC evolution is not a crossover but quantum phase transitions occur between normal and superfluid phases [8,9]. In addition, phase separation between superfluid (paired) and normal (excess) fermions has been shown. Motivated by these predictions, there have been several experiments with mass-balanced but population-imbalanced fermion mixtures [10,11], leading to an intensive theoretical activity [12–19]. Since exotic superfluid phases [i.e., Fulde-Ferrell and Larkin-Ovchinnikov (FFLO) [20,21]] can be potentially realized, imbalanced fermion mixtures are currently of interest to many communities ranging from atomic and molecular to condensed- and nuclear-matter physics. For instance, the ground-state phase diagrams of mass- and population-imbalanced fermion mixtures have been recently analyzed showing quantum and topological phase transitions [22–27].

In this manuscript, we analyze the ground-state phases of two-component ($\sigma \equiv \{\uparrow,\downarrow\}$) population- and mass-balanced ($N_\uparrow = N_\downarrow$ and $m_\uparrow = m_\downarrow$) but trap-imbalanced ($\omega_\uparrow \neq \omega_\downarrow$) fermion mixtures as a function of interaction strength, where ω_σ is the trapping frequency of σ component. The ground state involves very rich shell-structures consisting of unpolarized superfluid (UPS) and unpolarized normal (UPN) as well as partially σ-polarized normal (PσPN) and fully σ-polarized normal (FσPN) fermions. Our results are schematically shown in Fig. 1, and are as follows. In the BCS limit shown in Fig. 1(b), we find that the UPS fermions exist only away from the central core of the trapping potentials, and are surrounded by PσPN fermions. As the interactions increase towards unitarity, we find that the central PσPN core first transitions to a UPS as shown in Fig. 1(c), and then expands towards the edges until the entire mixture becomes a UPS in the BEC limit as shown in Fig. 1(d).

Pairing Hamiltonian. To obtain these results, we start with the Hamiltonian density (in units of $\hbar = k_B = 1$),

$$H(\mathbf{r}) = \sum_\sigma \psi_\sigma^\dagger(\mathbf{r}) K_\sigma(\mathbf{r}) \psi_\sigma(\mathbf{r}) - g \Psi_{\uparrow,\downarrow}^\dagger(\mathbf{r}) \Psi_{\uparrow,\downarrow}(\mathbf{r}), \qquad (1)$$

where $\psi_\sigma^\dagger(\mathbf{r})$ creates a pseudospin-σ fermion at position $\mathbf{r}$, and $\Psi_{\uparrow,\downarrow}^\dagger(\mathbf{r}) = \psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow^\dagger(\mathbf{r})$ is the pair creation operator. In Eq. (1), $g>0$ is the strengh of the attractive fermion-fermion interactions, and we defined $K_\sigma(\mathbf{r}) = -\nabla^2/(2m_\sigma) - \mu_\sigma(\mathbf{r})$, where $\mu_\sigma(\mathbf{r}) = \mu_\sigma - V_\sigma(\mathbf{r})$ is the local chemical potential. The global chemical potential μ_σ fixes the density $n_\sigma = N_\sigma/V$ of each type of fermion independently, where N_σ is the number of σ fermions and V is the volume. The term $V_\sigma(\mathbf{r}) = m_\sigma(\omega_{\sigma,x}^2 x^2 + \omega_{\sigma,y}^2 y^2 + \omega_{\sigma,z}^2 z^2)/2$ corresponds to the trapping potential, which is assumed to be harmonic in space.

In the momentum space, within the local-density (LD) approximation, the local mean-field (MF) Hamiltonian can be written as $H_\mathbf{Q}(\mathbf{r}) = \Sigma_{\mathbf{k},\sigma} \xi_{\mathbf{k},\sigma}(\mathbf{r}) a_{\mathbf{k},\sigma}^\dagger a_{\mathbf{k},\sigma} - \Delta_\mathbf{Q}(\mathbf{r}) \Sigma_\mathbf{k} (a_{\mathbf{k}+\mathbf{Q}/2,\uparrow}^\dagger a_{-\mathbf{k}+\mathbf{Q}/2,\downarrow}^\dagger + \text{H.c.}) + \Delta_\mathbf{Q}^2(\mathbf{r})/g$, where $\mathbf{Q}$ is the center-of-mass momentum of individual Cooper pairs, and $\xi_{\mathbf{k},\sigma}(\mathbf{r}) = \epsilon_{\mathbf{k},\sigma} - \mu_\sigma(\mathbf{r})$ with $\epsilon_{\mathbf{k},\sigma} = |\mathbf{k}|^2/(2m_\sigma)$. Here, $\Delta_\mathbf{Q}(\mathbf{r})$ is the local MF order parameter, which is assumed to be real without loss of generality, and defined by $\Delta_\mathbf{Q}(\mathbf{r}) = g \Sigma_\mathbf{k} \langle a_{-\mathbf{k}+\mathbf{Q}/2,\downarrow} a_{\mathbf{k}+\mathbf{Q}/2,\uparrow} \rangle$, where $\langle \cdots \rangle$ implies a thermal average.

Self-consistency equations. The local MF Hamiltonian can now be solved by using standard techniques [8,18,22]. The order parameter $\Delta_\mathbf{Q}(\mathbf{r})$ is determined by

$$\frac{MV}{4\pi a_F} = \sum_\mathbf{k} \left\{ \frac{1}{2\epsilon_\mathbf{k}} - \frac{1 - f[E_{\mathbf{k},\mathbf{Q},\uparrow}(\mathbf{r})] - f[E_{\mathbf{k},\mathbf{Q},\downarrow}(\mathbf{r})]}{2E_{\mathbf{k},\mathbf{Q}}(\mathbf{r})} \right\}, \quad (2)$$

where $\epsilon_\mathbf{k} = (\epsilon_{\mathbf{k},\uparrow} + \epsilon_{\mathbf{k},\downarrow})/2$ is the average kinetic energy, $f(x) = 1/[\exp(x/T)+1]$ is the Fermi function, and $E_{\mathbf{k},\mathbf{Q}}(\mathbf{r}) = [\xi_{\mathbf{k},\mathbf{Q}}^2(\mathbf{r}) + \Delta_\mathbf{Q}^2(\mathbf{r})]^{1/2}$ with $\xi_{\mathbf{k},\mathbf{Q}}(\mathbf{r}) = [\xi_{\mathbf{k}+\mathbf{Q}/2,\uparrow}(\mathbf{r})$

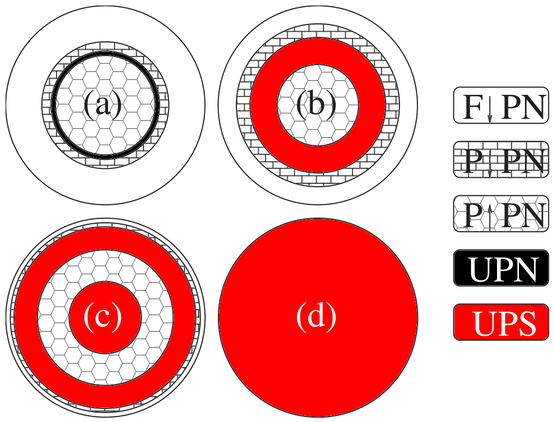

FIG. 1. (Color online) Diagrams showing shell-structures of two-component ($\sigma\equiv\{\uparrow,\downarrow\}$) population- and mass-balanced ($N_\uparrow=N_\downarrow$ and $m_\uparrow=m_\downarrow$) but trap-imbalanced ($\omega_\uparrow>\omega_\downarrow$) fermion mixtures for (a) noninteracting, (b) weak, (c) intermediate, and (d) strong attraction regimes. Here, ω_σ is the trapping frequency of σ fermions. The colored regions correspond to unpolarized superfluid (UPS, red), unpolarized normal (UPN, black), partially ↑-polarized normal (P↑PN, rectangular pattern), partially ↓-polarized normal (P↓PN, hexagonal pattern), and fully ↓-polarized normal (F↓PN, white) phases.

$+\xi_{-\mathbf{k}+\mathbf{Q}/2,\downarrow}(\mathbf{r})]/2$. Here, $E_{\mathbf{k},\mathbf{Q},\sigma}(\mathbf{r})=E_{\mathbf{k},\mathbf{Q}}(\mathbf{r})+s_\sigma[\xi_{\mathbf{k}+\mathbf{Q}/2,\uparrow}(\mathbf{r})-\xi_{-\mathbf{k}+\mathbf{Q}/2,\downarrow}(\mathbf{r})]/2$ is the quasiparticle energy when $s_\uparrow=1$ or the negative of the quasihole energy when $s_\downarrow=-1$. Notice that we elliminate g in favor of the fermion–fermion scattering length a_F via the usual regularization $1/g=-MV/(4\pi a_F)+\Sigma_{\mathbf{k}}1/(2\epsilon_{\mathbf{k}})$, where $M=2m_\uparrow m_\downarrow/(m_\uparrow+m_\downarrow)$ is twice the reduced mass of ↑ and ↓ fermions. Equation (2) has to be solved self-consistently with the number equations $N_\sigma=\int d\mathbf{r}n_\sigma(\mathbf{r})$, where

$$n_\sigma(r)=\frac{1}{V}\sum_{\mathbf{k}}\{u^2_{\mathbf{k},\mathbf{Q}}(\mathbf{r})f[E_{\mathbf{k},\mathbf{Q},\sigma}(r)]+v^2_{\mathbf{k},\mathbf{Q}}(\mathbf{r})f[-E_{\mathbf{k},\mathbf{Q},-\sigma}(\mathbf{r})]\},\tag{3}$$

is the local density of σ fermions. Here, $u^2_{\mathbf{k},\mathbf{Q}}(\mathbf{r})=[1+\xi_{\mathbf{k},\mathbf{Q}}(\mathbf{r})/E_{\mathbf{k},\mathbf{Q}}(\mathbf{r})]/2$ and $v^2_{\mathbf{k},\mathbf{Q}}(\mathbf{r})=[1-\xi_{\mathbf{k},\mathbf{Q}}(\mathbf{r})/E_{\mathbf{k},\mathbf{Q}}(\mathbf{r})]/2$. Since the pseudospin symmetry is broken in population-, mass-, and/or trap-imbalanced Fermi gases, one needs to solve all three equations self-consistently for all $\mathbf{Q}$, and determine the value of $\mathbf{Q}$ that minimizes the free energy [9,20,21].

Having established the theoretical formalism, next we analyze the ground-state phases of trap-imbalanced fermion mixtures as a function of a_F. For this purpose, we first discuss the numerical results and then provide analytical insight into the problem.

Numerical calculations. In this manuscript, we assume that the trapping potentials are isotropic in space such that $V_\sigma(\mathbf{r})=m_\sigma\omega_\sigma^2r^2/2$ where $r=|\mathbf{r}|$. In addition, we do not explicitly consider the FFLO-like ($Q\neq0$) superfluid phase [20,21], and limit the numerical calculations to the $Q=0$ phase. However, this phase may be present in the weakly attracting imbalanced fermion mixtures, but only in a narrow parameter

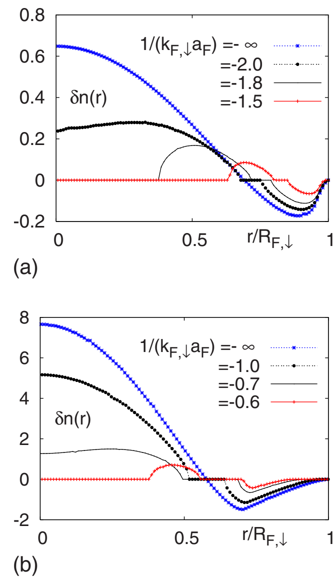

FIG. 2. (Color online) Local polarization $\delta n(r)=n_\uparrow(r)-n_\downarrow(r)$ [in units of $k^3_{F,\downarrow}/(2\pi)^3$] versus radius r (in units of $R_{F,\downarrow}$) is shown for population- and mass-balanced ($N_\uparrow=N_\downarrow$ and $m_\uparrow=m_\downarrow$) but trap-imbalanced fermion mixtures, where the trapping frequencies are (a) $\omega_\uparrow=1.1\omega_\downarrow$ and (b) $\omega_\uparrow=2\omega_\downarrow$. While the unpolarized regions are UPS, the polarized regions include both FFLO superfluid and PσPN and FσPN phases.

space, as discussed below. We also set $N_\uparrow=N_\downarrow$ and $m_\uparrow=m_\downarrow$, and consider two cases (a) $\omega_\uparrow=1.1\omega_\downarrow$ and (b) $\omega_\uparrow=2\omega_\downarrow$. The numerical calculation involves self-consistent solutions of Eqs. (2) and (3) for $\Delta_0(r)$, $n_\sigma(r)$, and μ_σ. For instance, in Fig. 2, we show the local polarization $\delta n(r)=n_\uparrow(r)-n_\downarrow(r)$ as a function of a_F, characterizing the noninteracting, and weak, intermediate and strong attraction regimes.

We numerically find that the ground state involves very rich shell structures consisting of UPS and UPN as well as PσPN and FσPN phases, depending on the particular value of a_F as shown in Fig. 1. To understand these shell structures, next we analyze the noninteracting, and weakly and strongly attracting limits, which are analytically tractable.

Noninteracting fermion mixtures. To understand the interacting trap-imbalanced fermion mixtures, it is useful to analyze first the noninteracting case when $g=0$ or $a_F\to0^-$. In this limit, the mixture is in normal phase such that the superfluid order parameter vanishes at all space $\Delta_Q(r)=0$, and that the global chemical potentials are identical to the global Fermi energies $\mu_\sigma=\epsilon_{F,\sigma}$ at zero temperature ($T=0$). Thus, Eq. (3) reduces to $n_\sigma(r)=(1/V)\Sigma_{\mathbf{k}}f[\xi_{\mathbf{k},\sigma}(r)]$, and at $T=0$ is given by $n_\sigma(r)=(1/V)\Sigma_{k<k_{F,\sigma}(r)}1$, where $k_{F,\sigma}(r)$ is the local Fermi momentum defined by $\epsilon_{F,\sigma}=k^2_{F,\sigma}/(2m_\sigma)=k^2_{F,\sigma}(r)/(2m_\sigma)+V_\sigma(r)$. This leads to $n_\sigma(r)=m^3_\sigma\omega^3_\sigma(R^2_{F,\sigma}-r^2)^{3/2}/(6\pi^2)$, where $R_{F,\sigma}$ is the Thomas-Fermi radius of σ fermions defined by $\epsilon_{F,\sigma}=m_\sigma\omega^2_\sigma R^2_{F,\sigma}/2$, such that $k_{F,\sigma}=m_\sigma\omega_\sigma R_{F,\sigma}$ is the global Fermi momentum. Then, the number of σ fermions is found by integrating $n_\sigma(r)$ over $\mathbf{r}$ where $r\leqslant R_{F,\sigma}$, leading to $N_\sigma=k^3_{F,\sigma}R^3_{F,\sigma}/48$. Setting $N_\uparrow=N_\downarrow$ gives $R_{F,\uparrow}/R_{F,\downarrow}=[m_\downarrow\omega_\downarrow/(m_\uparrow\omega_\uparrow)]^{1/2}$, and therefore, a trap-imbalanced fermion mixture can be realized when the condition $m_\uparrow\omega_\uparrow\neq m_\downarrow\omega_\downarrow$ is satisfied.

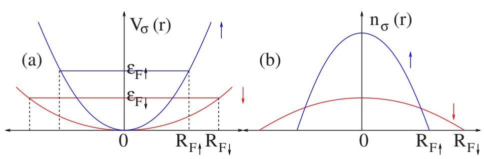

FIG. 3. (Color online) Schematic (a) trap (V_σ) and (b) density (n_σ) profiles are shown for non-interacting $[1/(k_{F,\downarrow}a_F)=-\infty]$ population- and mass-balanced ($N_\uparrow=N_\downarrow$ and $m_\uparrow=m_\downarrow$) but trap-imbalanced ($\omega_\uparrow>\omega_\downarrow$) fermion mixtures.

When $m_\uparrow\omega_\uparrow>m_\downarrow\omega_\downarrow$, since $R_{F,\uparrow}>R_{F,\downarrow}$, there are more $\uparrow$ fermions near the center of the trap while $\downarrow$ fermions are in excess near the edges as shown in Fig. 3. In addition, the local densities of $\uparrow$ and $\downarrow$ fermions are equal only at radius $r_c=R_{F,\downarrow}[m_\downarrow\omega_\downarrow(\omega_\uparrow-\omega_\downarrow)/(m_\uparrow\omega_\uparrow^2-m_\downarrow\omega_\downarrow^2)]^{1/2}$ satisfying $\delta n(r_c)=0$. Therefore, in this case, the ground state corresponds to a P$\uparrow$PN for $0\leqslant r<r_c$, to a UPN for $r=r_c$, to a P$\downarrow$PN for $r_c<r\leqslant R_{F,\uparrow}$, and to a P$\downarrow$PN for $R_{F,\uparrow}<r\leqslant R_{F,\downarrow}$. For instance, when $\omega_\uparrow=2\omega_\downarrow$, we find $r_c\approx 0.58R_{F,\downarrow}$ and $R_{F,\uparrow}\approx 0.71R_{F,\downarrow}$, and these three phases can be seen in Fig. 2(b) when $1/(k_{F,\downarrow}a_F)=-\infty$. The shell structure of this case is schematically shown in Fig. 1(a). Having finite and attractive fermion-fermion interactions changes this simple noninteracting picture dramatically, which is discussed next.

Weakly attracting fermion mixtures. When $g>0$ or $a_F\lesssim 0$, the normal mixture may become unstable against formation of Cooper pairs at some regions of the trap such that $\Delta_Q(r)\neq 0$. According to the BCS theory of superconductivity, this is always the case for chemical potentially balanced mixtures, $\delta\mu(r)=[\mu_\uparrow(r)-\mu_\downarrow(r)]/2=0$ for all r, no matter how weak the g is. This suggests that, for an arbitrarilly small g, trap-imbalanced fermion mixtures first become unstable against superfluidity at radius r_c, where $\delta n(r_c)=0$ and $\delta\mu(r_c)=0$.

In the weakly attracting BCS limit when $g\ll\{\epsilon_{F,\uparrow},\epsilon_{F,\downarrow}\}$, the local order parameter is obtained from Eq. (2), and is given by $\Delta_0(r)=(8/e^2)\mu(r)\exp\{\pi/[2k_F(r)a_F]\}$, which is valid when $\mu(r)\gg\Delta_0(r)$. Here, $\mu(r)=[\mu_\uparrow(r)+\mu_\downarrow(r)]/2$ is the effective local Fermi energy and $k_F(r)=[2M\mu(r)]^{1/2}$ is the effective local Fermi momentum. Notice that, $\Delta_0(r)$ has a maximum at the center of the trap and it vanishes towards the edges. Furthermore, for $\mu(r)\geqslant 0$, Eq. (3) suggests that while the local superfluid phase is unpolarized when $|\delta\mu(r)|\leqslant\Delta_0(r)$, it is polarized when $|\delta\mu(r)|>\Delta_0(r)$. However, it is also known that the polarized superfluid phase is unstable in this limit since the normal phase with $\Delta_0(r)=0$ is favored with a lower free energy [8,9,22].

Therefore, for mass-balanced ($m_\uparrow=m_\downarrow$) mixtures, a UPS phase with $Q=0$ minimizes the free energy only when $0\leqslant|\delta\mu(r)|\lesssim 0.71\Delta_0(r)$, however, further increase in $|\delta\mu(r)|$ causes a first order transition to a PσPN phase when $|\delta\mu(r)|\gtrsim 0.71\Delta_0(r)$ [28,29]. In this weakly attracting BCS limit, this suggests that the UPS fermions are surrounded by the PσPN fermions, and that they exist only away from the central core of the trapping potentials. For instance, these phases can be seen in Fig. 2(b) when $1/(k_{F,\downarrow}a_F)=-1$ or -0.7, and are schematically shown in Fig. 1(b). Notice that similar shell structures have been also reported for purely mass-imbalanced mixtures [24,26,27].

As the interactions increase, $\Delta_0(r)$ increases at a faster rate near $r=0$ due to the faster increase in local fermion densities, which causes an additional first order transition from PσPN to UPS at $r=0$. For instance, when $\omega_\uparrow=2\omega_\downarrow$, this transition occurs at $1/(k_{F,\downarrow}a_F)\sim -0.65$ as can be seen in Fig. 2(b), and the shell structure of this case is schematically shown in Fig. 1(c). Notice that this shell structure does not occur with purely mass-imbalanced mixtures [24,26,27]. Further increasing the interactions towards unitarity, we find that the central UPS region expands towards the edges.

In passing to the strongly attracting BEC limit, we make two comments. First, it is known that an FFLO-like superfluid phase with $\Delta_Q(r)=\Delta(r)\exp[iQ(r)r]$ and $Q(r)\sim 2.4M|\delta\mu(r)|/k_F(r)$ may also exist in a small parameter space when $0.71\Delta_0(r)\lesssim|\delta\mu(r)|\lesssim 0.75\Delta_0(r)$ [20,21]. This phase resides between the UPS and PσPN phases, and is separated from the UPS phase by a first order and from the PσPN phase by a second order transition. Notice that FFLO shells are not shown in Figs. 1(b) and 1(c). Second, the inclusion of fluctuations beyond the MF would reduce $\Delta_0(r)$, and therefore the transitions discussed above are likely to occur at higher values of $1/(k_{F,\downarrow}a_F)$ than our MF predictions. While the weakly attracting MF description is strictly valid for $1/(k_{F,\downarrow}a_F)\ll 0$, it still serves as a qualitative estimator for the phase boundaries until $1/(k_{F,\downarrow}a_F)\lesssim -0.5$. However, this description can not be used for $1/(k_{F,\downarrow}a_F)\gtrsim 0$, which is discussed next.

Strongly attracting fermion mixtures. In the strong fermion attraction (BEC) limit when $g\gg\{\epsilon_{F,\uparrow},\epsilon_{F,\downarrow}\}$ or $a_F\gtrsim 0$, imbalanced fermion mixtures can be described by a mixture of weakly repulsing Bose molecules and Fermi atoms, where the Bose molecules correspond to paired $\uparrow$ and $\downarrow$ fermions, and the Fermi atoms correspond to unpaired fermions [13,22]. However, in population-balanced ($N_\uparrow=N_\downarrow$) mixtures, all $\uparrow$ and $\downarrow$ fermions are paired to form Bose molecules, and therefore the equation of motion at $T=0$ is

$$-\mu_B(r)\Psi_B(r)+U_{BB}|\Psi_B(r)|^2\Psi_B(r)=\frac{\nabla^2\Psi_B(r)}{2m_B},\qquad(4)$$

which is of the Gross-Pitaevskii form, where $\Psi_B(r)=[M^2a_F/(8\pi)]^{1/2}\Delta_0(r)$ is the local BEC order parameter, $\mu_B(r)=2\mu(r)-\epsilon_b=\mu_B-V_B(r)$ is the local chemical potential, $U_{BB}=4\pi a_{BB}/m_B$ is the repulsive interaction, and $m_B=m_\uparrow+m_\downarrow$ is the mass of the molecular bosons. Here, $\mu_B=\mu_\uparrow+\mu_\downarrow-\epsilon_b$ is the chemical potential, $V_B(r)=V_\uparrow(r)+V_\downarrow(r)$ is the trapping potential, $\epsilon_b=-1/(Ma_F^2)$ is the binding energy, and $a_{BB}=(m_B/M)a_F$ (within the Born approximation) is the boson-boson scattering length of the molecules. Notice that identification of $V_B(r)=m_B\omega_B^2r^2/2$ as the molecular trapping potential leads to $\omega_B=[(m_\uparrow\omega_\uparrow^2+m_\downarrow\omega_\downarrow^2)/(m_\uparrow+m_\downarrow)]^{1/2}$, which is the effective trapping frequency felt by the molecular bosons. Therefore, we find in the BEC limit that the ground state of trap-imbalanced fermion mixtures is the BEC of molecular bosons for the entire trap.

These results are strictly valid for $1/(k_{F,\downarrow}a_F)\gg 0$, but they still serve as a qualitative estimator for the phase boundaries until $1/(k_{F,\downarrow}a_F)\gtrsim 1$. For instance, when $\omega_\uparrow=2\omega_\downarrow$, we find that the central UPS region expands, and the inner UPS and normal regions shrink towards the edges with respect to Fig. 1(c), as $1/(k_{F,\downarrow}a_F)$ is increased. Notice that the expansion of central UPS towards the edges is similar to the one observed with population-imbalanced fermion mixtures [10,11]. However, in our case, the normal regions vanish beyond a critical $1/(k_{F,\downarrow}a_F)$, and the entire mixture becomes a UPS [30]. When $\omega_\uparrow=2\omega_\downarrow$, this occurs at $1/(k_{F,\downarrow}a_F)\sim-0.1$, and the shell-structure of this case is schematically shown in Fig. 1(d). Having analyzed the ground-state phases, next we discuss briefly the validity of our results and also their experimental realization in atomic systems.

Experimental realization. In this manuscript, we mainly rely on the LD, MF, and isotropic trap approximations. In LD approximation, the mixture is treated as locally homogenous, and this approximation is valid as long as the number of fermions is large [12,18,19], which is typically satisfied in atomic systems. In MF approximation, the superfluid order parameter is treated at the saddle-point level, and the fluctuations are not included [22]. This description is qualitatively valid throughout the BCS-BEC evolution only at low temperatures [7], which can be reached in atomic systems. Lastly, in isotropic trap approximation, the traps are assumed to be isotropic, while the atomic traps are typically elongated in one direction. However, the anisotropy of traps is not expected to affect the shell structure of superfluid and normal phases other than causing shells to have elliptical rather than circular cross sections.

In atomic systems, trap-imbalanced fermion mixtures can be realized in several ways. For instance, in the case of magnetically trapped systems, trapping two different hyperfine states ($\uparrow$ and $\downarrow$) of a particular atom (i.e., ^{6}Li or ^{40}K) which have different magnetic moments (i.e., $\mathcal{M}_\uparrow>\mathcal{M}_\downarrow$) corresponds to a situation where $m_\uparrow=m_\downarrow$ and $\omega_\uparrow>\omega_\downarrow$. Likewise, in optically trapped systems, asymmetrically detuning the laser frequency with respect to two hyperfine states may produce a state-dependent optical trap. Furthermore, trap-imbalanced fermion mixtures can be naturally realized with two-species fermion mixtures [24,26,27] (i.e., ^{6}Li and ^{40}K) in both magnetically and optically trapped systems due to their different mass and also to hyperfine properties.

Conclusions. We analyzed the ground-state phases of two-component population- and mass-balanced but trap-imbalanced fermion mixtures as a function of fermion-fermion interactions. In the BCS limit, we found that the UPS fermions are surrounded by PσPN fermions, and exist only away from the central core of the trapping potentials. As the interactions increase towards unitarity, we found that the central PσPN core first transitions to a UPS, and then expands towards the edges until the entire mixture becomes a UPS in the BEC limit.

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[30] In the strongly attracting limit of a population- and trap-imbalanced mixture, unpaired excess fermions (paired bosons) are pushed away from the center of the trapping potentials when fermions with loose (tight) traps are sufficiently in excess.