



Production, Manufacturing and Logistics

Evaluation of cycle-count policies for supply chains with inventory inaccuracy and implications on RFID investments



A. Gürhan Kök*, Kevin H. Shang

Fuqua School of Business, Duke University, Durham, NC 27708, United States

ARTICLE INFO

Article history:

Received 18 January 2012

Accepted 25 January 2014

Available online 6 February 2014

Keywords:

Inventory inaccuracy

Cycle counts

Multi-echelon systems

RFID

ABSTRACT

Inventory record inaccuracy leads to ineffective replenishment decisions and deteriorates supply chain performance. Conducting cycle counts (i.e., periodic inventory auditing) is a common approach to correcting inventory records. It is not clear, however, how inaccuracy at different locations affects supply chain performance and how an effective cycle-count program for a multi-stage supply chain should be designed. This paper aims to answer these questions by considering a serial supply chain that has inventory record inaccuracy and operates under local base-stock policies. A random error, representing a stock loss, such as shrinkage or spoilage, reduces the physical inventory at each location in each period. The errors are cumulative and are not observed until a location performs a cycle count. We provide a simple recursion to evaluate the system cost and propose a heuristic to obtain effective base-stock levels. For a two-stage system with identical error distributions and counting costs, we prove that it is more effective to conduct more frequent cycle counts at the downstream stage. In a numerical study for more general systems, we find that location (proximity to the customer), error rates, and counting costs are primary factors that determine which stages should get a higher priority when allocating cycle counts. However, it is in general not effective to allocate all cycle counts to the priority stages only. One should balance cycle counts between priority stages and non-priority stages by considering secondary factors such as lead times, holding costs, and the supply chain length. In particular, more cycle counts should be allocated to a stage when the ratio of its lead time to the total system lead time is small and the ratio of its holding cost to the total system holding cost is large. In addition, more cycle counts should be allocated to downstream stages when the number of stages in the supply chain is large. The analysis and insights generated from our study can be used to design guidelines or scorecard systems that help managers design better cycle-count policies. Finally, we discuss implications of our study on RFID investments in a supply chain.

© 2014 Elsevier B.V. All rights reserved.

1. Introduction

Inventory record inaccuracy refers to the discrepancy between physical inventory held in stock and the record of inventory stored in the information system of a firm. Shrinkage, spoilage, misplaced inventories, and transaction errors (e.g., scanning errors and incorrectly counting products) contribute to inaccurate inventory information. Inaccurate inventory information leads to ineffective replenishment decisions, which, in turn, result in poor service levels and higher inventory costs. This is a major issue affecting supply chain performance in manufacturing, distribution and retail

settings. For example, DeHoratius and Raman (2008) found records to be inaccurate 65% of the items stored at a publicly traded retailer. According to ECR Europe (2003), the value of lost inventory due to shrinkage in 2000 was €13.4 billion for retailers and €4.6 billion for manufacturers in Europe.

A common method to mitigate the impact of inventory inaccuracy is to conduct cycle counts. Companies usually implement cycle-count policies according to an ABC classification scheme, i.e., classifying products into A, B, C classes based on product attributes, such as volume, error rate and value, and assigning a count cycle to each class (Jordan, 1994). Motivated by empirical studies and business practice, researchers recently have developed various analytical models, aiming to study this problem more rigorously (see Section 2 below). To our knowledge, all of the existing analytical results are on single-location models. Nevertheless, major apparel retailers and consumer-packaged goods companies

* Corresponding author. Address: Gurhan Kök, Fuqua School of Business, Duke University, 100 Fuqua Drive, Durham, NC 27708, United States. Tel.: +1 919 660 8039.

E-mail addresses: gurhan.kok@duke.edu (A.G. Kök), khshang@duke.edu (K.H. Shang).

have significant inaccuracy problems across their supply chains. Managers have limited knowledge of the extent of inaccuracy at different locations and their impact on overall supply chain performance (Delen, Hardgrave, & Sharda, 2009; Hardgrave, Aloysius, & Goyal, 2009, 2013). In this study, we investigate the impact of inventory inaccuracy on the supply chain performance. In particular, we aim to answer the following questions: (1) What is the impact of inventory inaccuracy at a location on the entire supply chain performance? (2) Given the fact that cycle counts are costly, which locations should have more frequent counts? (3) How should different product attributes and system characteristics be taken into account when designing cycle-count policies in a supply chain? The answers to these questions are not readily available in the academic and business literatures. While one may argue that record inaccuracy at downstream locations has a greater impact because of proximity to customers, inaccuracy at an upstream location affects the supply for all downstream locations. Similarly, one may argue that maintaining accurate records at a location with a longer lead time is more important because such locations are slower to respond to demand changes. On the other hand, such locations generally have more pipeline and safety stock, which implies that they would be affected less by record inaccuracy.

In this paper, we consider a periodic-review, N -stage serial supply chain with inventory record inaccuracy. Random customer demand arrives at stage 1. Stage 1 replenishes from stage 2, stage 2 from stage 3, and so on, and stage N from an outside supplier with ample supply. There are constant transportation lead times between stages. Record inaccuracy is caused by a random inventory loss that reduces physical inventory. (See Section 2 for a discussion on the other causes of inventory inaccuracy.) To describe inventory inaccuracy, we use the term “nominal” to signify inventory records stored in the computer system, and “actual” to signify the physical inventory levels. Each stage implements a cycle-count policy to correct the inventory records. A fixed stage-specific inspection cost is incurred for each cycle count. Errors at each stage are cumulative and they are not observed by the information system unless the stage conducts a cycle count. Thus, the discrepancy between the nominal inventory and the actual inventory equals the accumulated error since the last cycle count. The material flow is controlled by local base-stock policies. That is, if the local nominal inventory order position at a stage is less than a target base-stock level, the stage places an order to raise the nominal inventory position to the target level. Such a replenishment scheme is commonly seen in practice and most firms have computer systems automate this process. The objective is to minimize the actual average total supply-chain cost per period.

To evaluate the system cost, we derive the actual local inventory variables for any given cycle-count policy. However, obtaining these local inventory variables requires characterizing the local demand for each stage. One key contribution of this paper is that we provide a simple procedure to evaluate the system cost. This procedure recursively evaluates the cost for each echelon (a stage and all of its downstream stages) starting from echelon 1 to echelon N . Moreover, this recursion leads to a heuristic to find local base-stock levels. A numerical study suggests that the heuristic is effective. We then use these results to answer the research questions.

For two-stage systems, regarding the impact of errors, we show that it is more costly to have the same error occur at the downstream stage than at the upstream stage. Furthermore, regarding the cycle-count policies, we show that it is always a better strategy to assign more cycle counts to the downstream stage than to the upstream stage when both stages have identical errors and counting costs. For more general systems, we categorize system parameters into two groups in terms of their effect on cycle-count policy decisions. Primary factors determine which stages should get a higher priority when allocating cycle counts. They include location

(position in the supply chain), error rate, and counting cost. All else being equal, downstream locations should be assigned more frequent counts. However, a significantly higher error rate or lower counting cost at an upstream stage may reverse the result. We observe that the marginal benefit of cycle counts is decreasing in its frequency. Thus, one should not allocate all cycle counts to a single location. We therefore suggest using secondary factors to determine whether the policy should strongly favor the high-priority stages or allocate counts in a more balanced way. The secondary factors include lead time and holding cost structure and the supply chain length. We find that more cycle counts should be allocated to a stage if the ratio of the stage’s lead time to the total system lead time is small and the ratio of the stage’s holding cost to the total holding cost is large. Furthermore, more cycle counts should be allocated to downstream stages if the supply chain is long (i.e., the number of stages in the supply chain is large).

Our research questions have implications for investment decisions on Radio Frequency Identification (RFID) systems. Specifically, if a location installs RFID readers, this will, in principle, eliminate inventory inaccuracy by continuously monitoring the inventory. This has the same effect as conducting a cycle count in each period in our model. In fact, many RFID applications in retail supply chains involve hand-held RFID readers, whose role is to facilitate cycle counting. Because RFID implementation can be very costly (Kearney, 2004), many industry practitioners are concerned about where RFID readers should be installed in a supply chain (Chappell et al., 2002). Labor savings and investment costs of RFID systems can be quantified in each stage in isolation from others (Chopra & Sodhi, 2007), but it is not clear how one can quantify the impact of reducing inaccuracy at a particular stage on supply chain performance. The guidelines developed in this paper have implications on how to prioritize RFID investments in a supply chain.

The rest of the paper is organized as follows. Section 2 discusses major causes of inventory inaccuracy in supply chains and reviews the literature related to each of the causes. Section 3 presents a single-stage model and discusses the model set-up. Section 4 introduces the serial system and provides a scheme to evaluate the average total cost per period. Section 5 presents the lower bound and the heuristic algorithm. Section 6 presents our analytical and numerical results and provide answers to the research questions. Section 7 concludes. All proofs are provided in Appendix A.

2. Causes of inventory inaccuracy and related literature

Empirical research and industry reports indicate that shrinkage, transaction errors, and misplaced items are the main reasons that cause inventory inaccuracy (ECR Europe, 2003; Raman & Ton, 2003; Sheppard & Brown, 1993). In the following, we describe each type of inaccuracy and its impact on the supply chain.

Shrinkage, also known as stock loss, may be due to theft by shoppers or employees, and spoiled and damaged inventory. The impact of shrinkage on the actual inventory is one-sided: it always reduces the actual inventory. Thus, the inventory record is always greater than or equal to the actual inventory when shrinkage is the only reason that causes inventory inaccuracy. Our model considers this type of inaccuracy. In a supply chain, inventory shrinkages between locations are often independent because losing a unit of inventory at one location is not related to that at another location.

Shrinkage is found to be the dominant cause of inventory inaccuracy in many empirical studies. Kang and Gershwin (2005) report two firms for which the majority of the inventory inaccuracy is due to stock loss. Similarly, ECR Europe (2001, 2003) conduct a comprehensive study over more than 200 companies and suggest that stock loss errors may be more prominent than others. In these studies, shrinkage is estimated to be about 1.7% of the sales for

retailers and 0.56% for manufacturers. The National Retail Security Survey of 118 retailers (Hollinger, 2003) reports similar results. These studies emphasize the stockout effects of shrinkage. Chen (2013) proposes a data-driven optimal inspection policy for a single-item model. Fleisch and Tellkamp (2005) conduct a simulation study on the impact of inventory shrinkage for a three-echelon supply chain. Arguably, shrinkage has the most direct impact on supply chain performance because shrinkage at an upstream location will directly affect the stock availability of its downstream location, which may, in turn, lead to significant stockouts or backlogs for the system. In order to directly address this issue affecting many supply chains, we consider only shrinkage-type inaccuracy in the present paper.

Transaction errors refer to errors caused by misidentification of the items or miscounting of the items in inbound and outbound processes. This transaction will make the actual inventory larger (smaller) than the nominal inventory. Thus, the impact of transaction error on the actual inventory is two-sided: it may increase or decrease the actual inventory. Iglehart and Morey (1972) is the first paper to consider transaction errors in a single-stage model. The inventory system implements a base-stock policy and the objective is to find the cycle-count frequency that minimizes inspection and inventory holding costs subject to a pre-specified inaccuracy probability. In an empirical study of one retailer's records, DeHoratius and Raman (2008) report that the difference between physical inventory and inventory record can be positive or negative. They also find that the extent of errors increases with sales volume, the number of stages in the supply chain, and product variety. There are recent papers that investigate optimal and effective inventory and inspection policies for single-stage model, e.g., Kk and Shang (2007), DeHoratius, Mersereau, and Schrage (2008), Sahin, Buzacott, and Dallery (2008), Deshpande, Schwarz, and Raju (2009), Sahin and Dallery (2009), and Mersereau (2012). To our knowledge, there has been no work on investigating the impact of transaction errors on a multi-stage supply chain.

Misplaced or inaccessible inventory refers to inventory that is physically at the facility, but its exact location is unknown. The impact of misplaced inventory is also one-sided: it reduces actual inventory levels. However, unlike shrinkage, misplaced inventory is not permanently lost; it may be recovered and added to the actual inventory after a cycle count. Rekik, Sahin, Jemai, and Dallery (2008) investigate the impact of different types of misplacement errors for a retailer–supplier system under a news-vendor framework. amdereli and Swaminathan (2009) study inventory decisions and coordination contracts for a system similar to Rekik et al.

Incorporating all error types in a model without any simplifying assumptions is a difficult task even for single-stage systems. Atali, Lee, and zer (2005) and Bensoussan, akanyıldırım, and Sethi (2007) consider multiple types of errors in a single-stage model. The former assumes that errors occur independently from the current inventory levels in order to derive inventory policies. The latter develops a heuristic feedback policy due to the dimensionality of the problem. We refer the reader to Chen and Mersereau (2013) for a review on inventory inaccuracy issues and models.

Our research also pertains to the literature with RFID. One of the benefits of RFID systems is greater visibility of material flow and order progress. Gaukler, zer, and Hausman (2008) and Kim, Klabjan, and Simchi-Levi (2007) study expediting strategies given the accurate order progress information from upstream stages. Bottani, Montanari, and Volpi (2010) investigates the impact of RFID on mitigating the bullwhip effect in the Italian FMCG supply chain. Pei and Klabjan (2010) study the optimal inventory policy for a serial system in which RFID systems provide order progress information for replenishments and shipments. They compare the expected costs of the model with different levels of RFID

deployment and find that broader deployment reduces costs. This group of papers focuses on gaining visibility into order movements that are usually modeled with a stochastic lead time. Our focus is different in that we compare the benefit of reducing inaccuracy at different stages in a supply chain. For a review of the literature on RFID-related models and future applications, see Zipkin (2006) and Lee and zer (2007).

The present paper builds on the literature on multi-echelon inventory systems, particularly the classical serial supply chain model studied by Clark and Scarf (1960), Chen and Zheng (1994), and many others. We incorporate inventory record inaccuracy and cycle-count policies in the serial system by introducing unobserved inventory errors at each stage. This leads to an additional dimension in the inventory dynamics and expands the set of decision variables.

Finally, this paper is related to inventory/production systems with random yields. For single-stage yield models under periodic review, Henig and Gerchak (1990) show that base-stock type policies are optimal. Lee and Yano (1988) study a multi-stage model for a single production run to meet a single-period demand. Liu, So, and Zhang (2010) consider the impact of supply reliability on inventory and pricing decisions. On a related topic, Bollapragada, Rao, and Zhang (2004) consider assembly systems with random capacity at each location that are controlled with base-stock policies. In random yield and random capacity models, the uncertainty at every stage is observable in each period after it is realized, which removes any uncertainty about inventory levels in the supply chain. In our model, however, the errors at a stage are not observed and accumulate until a cycle count is performed.

3. Single-stage system

In this section we consider a single-stage system. We use this model to illustrate the assumptions and the concept of nominal and actual inventory variables. This discussion sets the stage for the serial system.

Consider a periodic-review inventory system that orders according to a base-stock policy. Time is divided into periods of length one and the periods are numbered $0, 1, 2, \dots$. There is a constant lead time of L periods. Customer demand follows a Poisson process with rate λ . Let $[t, t+r)$ and $[t, t+r]$ denote the time interval over periods $t, t+1, \dots, t+r-1$ and periods $t, t+1, \dots, t+r$, respectively. Let $D_{intt}, t+r)$ and $D[t, t+r]$ denote the total demand that occurs in $[t, t+r)$ and $[t, t+r]$, respectively. We denote $D[r)$ and $D[r]$ the total demand in r and $r+1$ periods for the long-run average cost calculations. We assume that a random loss of inventory occurs in each period that reduces the level of actual inventory. We call the random inventory loss an error and we assume that the errors follow a Poisson process with rate μ . Let $\xi[t, t+r)$ and $\xi[t, t+r]$ denote the accumulated error in $[t, t+r)$ and $[t, t+r]$, respectively, and $\xi[r)$ and $\xi[r]$ denote total error accumulated in r and $r+1$ periods for the long-run average cost calculations. The system implements a cycle-count policy to correct the inventory record. That is, the stage conducts a cycle count every T periods, where T represents the cycle-count interval. The system incurs a fixed cost of K every time a cycle count is conducted. Errors in the system accumulate and are not observed until a cycle count is conducted. As a result, the discrepancy between actual and nominal inventory positions is nondecreasing between two cycle counts. One may view this system as consisting of two processes: (1) The actual inventory dynamics and the cost evaluation process based on the actual inventory and (2) the ordering process in which orders are placed based on the nominal inventory order position (or inventory record). Below is the sequence of events in each period.

- (1) The stage receives a shipment at the beginning of a period,
- (2) random error and random customer demand occurs during the period,
- (3) if the current period is a cycle-count period, then the stage conducts a cycle count,
- (4) the stage places an order according to the nominal inventory order position,
- (5) costs are evaluated at the end of the period.

We assume that the random demand and error processes during the period are interleaving Poisson processes.¹ The actual inventory level at the end of a period is the beginning actual inventory level minus the sum of the two Poisson random variables, i.e., one period of demand and one period of error, regardless of our assumptions about the order of the two processes.

We now describe how these events affect the nominal and actual inventory variables. We define the following inventory variables to facilitate the subsequent discussion.

$AIOP^-(t)$ = Actual inventory order position at the beginning of period t ,

$AIOP(t)$ = Actual inventory order position after placing an order at the end of period t ,

$AIL^-(t)$ = Actual inventory level at the beginning of period t after receiving a shipment,

$AIL(t)$ = Actual inventory level after error and demand occur,

$B(t)$ = Actual backlogged consumer demand after error and demand occur.

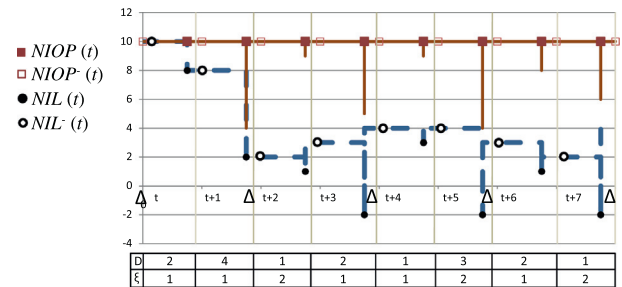
Let $NIOP^-(t)$, $NIOP(t)$, $NIL^-(t)$, and $NIL(t)$ denote the corresponding nominal inventory variables. In event (1), both $AIL^-(t)$ and $NIL^-(t)$ increase because of the arriving shipment. In event (2), random error reduces the actual inventory variables and random demand reduces the nominal and actual inventory variables. $AIL(t)$ and $AIOP(t)$ are determined by the total demand from the two random processes. In event (3), the nominal inventory variables are aligned with the actual ones if the stage conducts a cycle count. In event (4), the stage places an order according to the nominal inventory order position. That is, at the end of a period, if the nominal inventory order position is less than the local base-stock level s , the stage places an order that raises the nominal inventory order position to s ; otherwise, the stage does not order. Thus, event (4) determines both $NIOP(t)$ and $AIOP(t)$ (or equivalently, $NIOP^-(t + 1)$ and $AIOP^-(t + 1)$). In event (5), costs are evaluated based on $AIL(t)$. For this purpose, define $[x]^+ = \max\{x, 0\}$ and $[x]^- = \max\{0, -x\}$. A linear holding cost h is incurred for each unit of actual on-hand inventory $[AIL(t)]^+$ per period and a backorder cost b for each unit of actual backlogged consumer demand $B(t)$ per period. The system is backlogged if $AIL(t)$ is negative. However, only a portion of that backlog is caused by the customer demand process, while the rest is due to the error process. The distribution of the actual backlogged demand $B(t)$ depends on the total backlog level $[AIL(t)]^-$ as described later in this section. The objective is to minimize the average total cost per period.

We make the following remarks about the model assumptions. First, the assumption of independent and identically distributed errors implies that the error distribution is independent of the inventory level. While certain types of inventory losses, such as invisible demand may satisfy this assumption, other error types such as shrinkage or spoilage can occur only when inventory level is positive. We make this assumption for tractability of the model. Modeling the dependence on inventory level would require

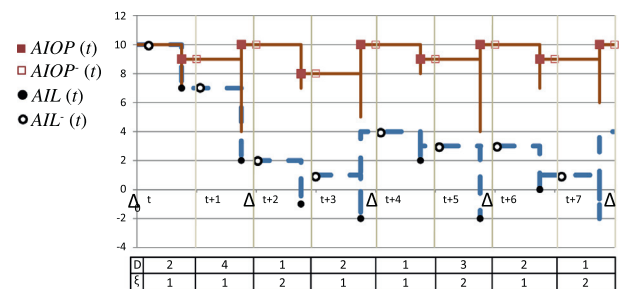
keeping track of periodic demand and error during lead time as a vector. This would cause the size of the state space to grow intractably large, which is similar to the difficulty in the lost-sales system. Second, for tractability, we assume that the discrepancy between the actual inventory and nominal inventory is reconciled only when a cycle count is performed. There is no correction of the records due to the occurrence of a backorder. That is, if the computer shows the item in stock but it is actually not available, the demand is backlogged and the backorder information is not used to correct the records. Because we ignore the occurrence of such corrections in our characterization of the system state, the present model can be seen as an approximation to the systems that utilize the backlog information to correct records when the backlog probability is small. Many companies either do not have proper systems in place to update their computer records or do not allow line workers to change system records to ensure data integrity. Third, demand fulfillment and cost calculation are based only on actual on-hand inventory $(AIL)^+$ and actual backlogged demand B . In daily operations, a company may do its accounting based on the inventory records. However, at the end of a year, the annual cost according to the information system may be different from the actual annual cost, which is realized based on the actual inventory dynamics. This difference is due to the inaccuracy of the records. Therefore, a reconciliation by the accounting department is always necessary.

An example

We use an example to illustrate the inventory dynamics under the nominal and actual inventory schemes. Consider a single-stage system with $T = 2$, $L = 2$, and $s = 10$. Fig. 1(a) illustrates the dynamics of the nominal inventory order position and inventory level. Fig. 1(b) illustrates the corresponding actual inventory variables. The demand and error levels in each period are given in the figure. The system has conducted a cycle count at the end of period $t - 1$, so there is no error in the system at the beginning of period t . The future cycle-count periods are $\{t + 1, t + 3, t + 5, \dots\}$ and the cycle-count periods are marked with the “ Δ ” symbol.



(a)



(b)

Fig. 1. A single-stage system with $L = 2$, $T = 2$, and $s = 10$. (a) The dynamics of the nominal inventory order position and inventory level. (b) The dynamics of the corresponding actual inventory variables.

¹ The model can be generalized to allow for general demand and error distributions if periodic demand were realized first, depleted the inventory and then the error occurred.

Let us first focus on the nominal inventory variables shown in Fig. 1(a). We assume that $NIOP^-(t) = NIL^-(t) = 10$ at the beginning of period t . Because the error in period t is not observed, $NIOP$ and NIL change in the same way as do those in the standard base-stock policy with demand of two units, that is, $NIOP^-(t+1) = 10$ and $NIL^-(t+1) = 8$. In period $t+1$, four units of demand is recorded and an accumulated error of $\xi[t, t+2] = 2$ is detected during the cycle count. Hence, $NIL(t+1) = 2$. Finally, the stage places an order of six units to bring $NIOP(t+1) = 10$, which is equal to $NIOP^-(t+2)$. The rest of the dynamics can be developed with the same logic. Clearly, the actual inventory dynamics are different because the nominal system does not realize that the error affects the physical inventory level in every period.

Fig. 1(b) presents the corresponding actual inventory variables. At the beginning of period t , $AIOP^-(t) = AIL^-(t) = 10$. As a result of two units of demand and one unit of error, we have $AIL(t) = 7$. The stage orders two units according to the nominal inventory order position, and we get $AIOP(t) = AIOP^-(t+1) = 9$. Now, consider period $t+5$. At the beginning of the period, the stage receives shipment of one unit sent at the end of period $t+2$. Hence, $AIL^-(t+5) = AIL(t+4) + 1 = 3$. An error of two units and a demand of three units are realized during the period and we have $AIL(t+5) = -2$. Depending on the sequence of the error and demand realization during the period, some or all of these two units of backlogs may be due to customer demand. We discuss later how we account for backlogs due to demand and error separately. At the beginning of period $t+6$, the stage receives a shipment of five units and $AIL^-(t+6) = -2 + 5 = 3$.

We now show how to evaluate the average system cost per period. The inventory dynamics result in a regenerative process. Specifically, if the system conducts a cycle count at the end of period $t-1$, the nominal inventory order position is set equal to the actual position at the beginning of period t , i.e., $NIOP^-(t) = AIOP^-(t)$. Because the errors are nonnegative, the nominal inventory terms are always greater than or equal to the actual terms. Hence, after a cycle count, the corrected nominal inventory position cannot be more than the target base-stock level s and the system can order up to the target level after aligning the nominal terms with the actual ones. In order to analyze this regenerative process that has a cycle length of T periods, we set the beginning of periods $t, t+T, t+2T, \dots$ as the regenerative epochs. The long-run average cost per period is equal to the expected cost between two consecutive regenerative epochs divided by the cycle length. The $AIOP^-$ in the cycle-count interval $[t, t+T)$ determines AIL^- and AIL in $[t+L, t+L+T)$ as follows.

Proposition 1. For $r = 0, \dots, T-1$,

$$\begin{aligned} NIOP^-(t+r) &= s, \\ AIOP^-(t+r) &= s - \xi[t, t+r] \quad (1) \\ AIL^-(t+L+r) &= AIOP^-(t+r) - D[t+r, t+L+r] - \xi[t+r, t+L+r], \\ AIL(t+L+r) &= AIOP^-(t+r) - D[t+r, t+L+r] - \xi[t+r, t+L+r], \quad (2) \end{aligned}$$

and $B(t+L+r)$ is binomially distributed with parameters $[AIL(t+L+r)]^-$ and $\lambda/(\lambda + \mu)$.

The second equation is due to the replenishment policy that orders to cover the demand in the no-cycle-count periods and the demand and the accumulated error in the cycle-count period. The third and fourth equations are similar to those in the standard base-stock models and reflect the effect of the lead time. The last statement gives the distribution of backlogged customer demand given the total backlog level. From the moment AIL falls to zero until the end of the period, two Poisson processes reduce AIL , but only customer arrivals cause a backlog penalty. The process after AIL falls to zero is independent of the demand and error realizations before that point because of the memoryless property of

the Poisson process. Hence, the fraction of total backlog that is due to customer demand follows a Binomial distribution with number of trials equal to $[AIL(t+L+r)]^-$ and success probability equal to the ratio of mean demand to mean error plus mean demand. Thus, the expected value of $B(t+L+r)$ given $AIL(t+L+r)$ is simply $(\lambda/(\lambda + \mu))AIL(t+L+r)^-$. This accounts for the system incurring the backorder penalty for only customer demands when the actual inventory level is less than or equal to zero, but not for the error process. In the example in Fig. 1, the actual inventory level in period $t+5$ is three units after replenishment and minus two units at the end of the period (after two units of error and three units of demand). The actual backlogged demand level, however, depends on the realized sequence of the error and demand occurrences. Proposition 1 tells us that $B(t+5)$ is distributed according a binomial distribution with parameters $(2, \lambda/(\lambda + \mu))$.

For expositional simplicity, define $\hat{b} = b\lambda/(\lambda + \mu)$. The average inventory cost per period is given by

$$\begin{aligned} G(s, T) &= \frac{1}{T} \left(\sum_{r=0}^{T-1} \mathbf{E} [hAIL(t+L+r)^+ + bB(t+L+r)] \right) \\ &= \frac{1}{T} \left(\sum_{r=0}^{T-1} \mathbf{E} [hAIL(t+L+r) + (\hat{b} + h)(AIL(t+L+r)^-)] \right). \end{aligned}$$

The distribution of $AIL(t+L+r)$ is determined by $AIOP^-(t+r)$ given by (1). Note that $\xi[t + \lfloor r/T \rfloor T, t+r]$ in that expression yields r periods of error for $r = 0, 1, 2, \dots, T-1$. Define

$$g(y, r) = \mathbf{E} [h(y - \xi[r] - D[L] - \xi[L]) + (\hat{b} + h)(y - \xi[r] - D[L] - \xi[L])^-].$$

The average inventory cost per period, $G(s, T)$, and the average total cost per period including the counting cost, $C(s, T)$, can be expressed as follows.

Proposition 2. $G(s, T) = (1/T) \sum_{r=0}^{T-1} g(s, r)$ and $C(s, T) = G(s, T) + K/T$.

Note that each $g(y, r)$ function is the same as a single-period inventory cost function with different demand variables. Because each of them is convex, average cost $G(s, T)$ is also convex in the base-stock level s for fixed T . Let $s(T) = \arg \min_s C(s, T)$ be the optimal base-stock level. Define $G(T) = G(s(T), T)$ as the minimum inventory cost and $C(T) = C(s(T), T)$ as the optimal total cost for a given cycle-count policy.

Proposition 3. For $T \geq 2$, (1) $s(T-1) \leq s(T)$, (2) $G(T-1) \leq G(T)$.

Proposition 3 states that the optimal base-stock level and the optimal inventory cost increase with the cycle-count interval length T . When the cycle-count interval is longer, more periods of error accumulate in the system. Thus, the system has to maintain a higher stock level to cover the inventory loss and the additional uncertainty. The higher stock level leads to a higher optimal system cost. The impact of the cycle-count interval on $C(T)$ depends on how K/T compares with inventory related costs.

4. Series systems

We now consider an N -stage series system. Customer demand occurs at stage 1. Stage 1 is replenished by stage 2, stage 2 by stage 3, and so on, and stage N by an outside source with ample supply. The lead time between stage j and $j+1$ is L_j periods. Let $\bar{L}_j = \sum_{i=j}^N L_i$, the cumulative lead time from stage j to stage N . Each stage orders according to a local base-stock policy. At the beginning of a period, if stage j 's local inventory order position is lower than the base-stock level s_j , the stage places an order to raise its nominal inventory order position to s_j ; otherwise, the stage does not order.

Let $S_j \stackrel{\text{def}}{=} \sum_{i=1}^j s_i$. Denote $D_j[t, t+r)$ and $D_j[t, t+r]$ the local demand occurring in periods $[t, t+r)$ and $[t, t+r]$ for stage j , respectively. Note that under a local base-stock policy, stage $j-1$'s order is stage j 's local demand. A nonnegative random inventory loss, or error, occurs at each stage every period, reducing the physical inventory level. We assume that the error at stage j is Poisson with rate μ_j and is independent and identically distributed across periods. Also, the error process at a stage is independent of the other stages. Let $\xi_j[t, t+r)$ and $\xi_j[t, t+r]$ denote the accumulated error in periods $[t, t+r)$ and $[t, t+r]$ at stage j . We use $\xi_j[r)$ and $\xi_j[r]$ for the accumulated error of r and $r+1$ periods in the long-run average cost calculations, respectively. Each stage conducts a cycle-count policy to correct its inventory record. Let T_j be the cycle-count interval for stage j and K_j be the counting cost at stage j . We let variables in bold font denote the vector of that variable for $j=1, \dots, N$. That is, $\mathbf{s} = (s_1, \dots, s_N)$, $\mathbf{T} = (T_1, \dots, T_N)$. Similarly for $\boldsymbol{\mu}$, \mathbf{L} , \mathbf{h} , and \mathbf{K} .

For stage 1, the sequence of events is the same as that for the single-stage system. For stages $j \geq 2$, the sequence of events is similar to stage 1. Specifically,

- (1) at the beginning of a period, stage j receives a shipment sent L_j periods ago,
- (2) random error occurs during the period,
- (3) downstream order (or local demand) arrives at the end of a period,
- (4) stage j sends a shipment to stage $j-1$,
- (5) stage j conducts a cycle count, if the current period is a cycle-count period,
- (6) stage j places an order according to the nominal inventory order position at the end of a period,
- (7) costs are evaluated based on the actual inventory level.

Below we discuss how to evaluate the average total cost per period. In Section 4.1, we show how to evaluate the cost using local inventory variables. In Section 4.2, we provide an alternative approach, which evaluates the cost using the corresponding echelon variables. This echelon approach simplifies the computation significantly and sets the stage for the subsequent analysis.

4.1. Evaluation: local approach

To explain how these events affect inventory variables, we define the following local inventory variables.

- $AIO P_j^-(t)$ = Actual local inventory order position at stage j at the beginning of period t ,
- $AIO P_j^+(t)$ = Actual local inventory order position after ordering at stage j at the end of period t ,
- $AIL_j^-(t)$ = Actual local inventory level at stage j at the beginning of period t after receiving a shipment placed L_j periods ago,
- $AIL_j^+(t)$ = Actual local inventory level at stage j at the end of period t .

The effect of the sequence of events on the actual inventory variables is as follows. In event (1), stage j receives a shipment, which raises the actual inventory level and determines $AIL_j^-(t)$. In event (2), the error reduces $AIO P_j^-(t)$ and $AIL_j^-(t)$. In event (3), a downstream order (local demand) arrives, which reduces $AIO P_j^-(t)$. In event (4), stage j ships as much as possible to fulfill stage $j-1$'s order, and $AIL_j^+(t)$ is determined. In event (5), the nominal and actual local inventory variables are aligned after a cycle count is conducted. In event (6), $AIO P_j^+(t)$ is determined after an order is placed. This $AIO P_j^+$ level is carried over to the next period, i.e.,

$AIO P_j^+(t) = AIO P_j^-(t+1)$. In event (7), costs are evaluated based on $AIL_j^+(t)$. Let h_j denote the echelon holding cost, $h'_j = \sum_{i=j}^N h_i$ the local holding cost for stage j , and b the backorder cost at stage 1. The inventory holding cost for stage j at the end of period t is $h'_j [AIL_j^+(t)]^+$, $j=1, \dots, N$, and the expected cost of backlogged customer demand at stage 1 is $bE[B(t)] = \hat{b} [AIL_1^-(t)]^-$, where $\hat{b} = b\lambda/(\lambda + \mu_1)$.

We assume that all stages conduct a cycle count at the end of period $t-1$. Let $T = lcm\{T_1, \dots, T_N\}$, where $lcm\{\}$ is an operator that generates the least common multiplier. The common cycle-count periods for all stages are $t-1, t+T-1, t+2T-1, \dots$. After a common cycle-count period, all stages have zero error. Thus, we set the beginning of periods $t, t+T, t+2T, \dots$ as regenerative epochs. Without loss of generality, we will focus on a regenerative cycle consisting of periods $t+r, r=0, 1, \dots, T-1$. For expositional simplicity, define

$$r_j = r + N - j, \quad j = 1, 2, \dots, N.$$

As we shall see, $AIO P_j^-(t + \bar{L}_{j+1} + r_j)$ and the local demand D_j jointly determine the actual local inventory level $AIL_j^+(t + \bar{L}_j + r_j)$ for stage $j, j=N, N-1, \dots, 1$.

To illustrate this, we first characterize $AIO P_j^+$. Similar to the expression for the order position for the single-stage system in (1), we need to adjust the order position down for the error accumulated in the system since the last inspection. Let $\lfloor \cdot \rfloor$ be the round-down operator. Under the considered inventory and cycle-count policy, we have

$$AIO P_j^-(t + \bar{L}_{j+1} + r_j) = s_j - \xi_j \left[t + \left\lfloor \frac{\bar{L}_{j+1} + r_j}{T_j} \right\rfloor T_j, t + \bar{L}_{j+1} + r_j \right). \quad (3)$$

We next characterize the local demand D_j for stage j . Note that the local demand for stage j is the order placed by stage $j-1$. Thus, we can sequentially characterize these local demands from stage 1, stage 2, etc., up to stage N . Let t be a regenerative epoch and ℓ be any nonnegative integer. Clearly, $D_1[t, t+\ell) = D[t, t+\ell)$. For stage $j=2, 3, \dots, N$,

$$D_j[t, t+\ell) = D_{j-1}[t, t+\ell) + \xi_{j-1} \left[\left\lfloor \frac{t}{T_{j-1}} \right\rfloor T_{j-1}, \left\lfloor \frac{t+\ell}{T_{j-1}} \right\rfloor T_{j-1} \right),$$

$$D_j[t, t+\ell] = D_{j-1}[t, t+\ell] + \xi_{j-1} \left[\left\lfloor \frac{t}{T_{j-1}} \right\rfloor T_{j-1}, \left\lfloor \frac{t+\ell+1}{T_{j-1}} \right\rfloor T_{j-1} \right].$$

As seen from these equations, stage $j-1$ simply passes on all the orders it receives from downstream to upstream stage j and, in the cycle-count periods, the local errors that it observes (if any). With these local demands and $AIO P_j^+$, we can characterize $AIL_j^+(t + \bar{L}_j + r_j)$ as we did for the single-stage system in (2). This process starts from stage N . Note that

$$AIL_N^-(t + L_N + r_N) = AIO P_N^-(t + r_N) - D_N[t + r_N, t + L_N + r_N) - \xi_N[t + r_N, t + L_N + r_N), \quad (4)$$

$$AIL_N^+(t + L_N + r_N) = AIO P_N^-(t + r_N) - D_N[t + r_N, t + L_N + r_N] - \xi_N[t + r_N, t + L_N + r_N]. \quad (5)$$

It is more difficult to characterize $AIL_j^+(t + \bar{L}_j + r_j), j < N$, because stage j may not always receive what it orders. To reflect this fact, we define the following inventory terms:

- $AIP_j^-(t)$ = actual local inventory in-transit position at the beginning of period t after a shipment is received;
- $AIP_j^+(t)$ = actual local inventory in-transit position at the

Now, consider stage $j (< N)$ at the end of period $t + \bar{L}_{j+1} + r_{j+1}$. If stage $j+1$ has fulfilled stage j 's order in this period,

$$AIP'_j(t + \bar{L}_{j+1} + r_{j+1}) = AIOp'_j(t + \bar{L}_{j+1} + r_{j+1}).$$

On the other hand, if stage $j + 1$ does not have sufficient stock to fulfill stage j 's order, then stage $j + 1$ has a backlog. In this case, $All'_{j+1}(t + \bar{L}_{j+1} + r_{j+1}) < 0$ and

$$AIP'_j(t + \bar{L}_{j+1} + r_{j+1}) = AIOp'_j(t + \bar{L}_{j+1} + r_{j+1}) + All'_{j+1}(t + \bar{L}_{j+1} + r_{j+1}). \tag{6}$$

Note that the backlog may be due to the error and local demand at stage $j + 1$. The above equation suggests that this backlog reduces the actual inventory in-transit position at stage j .²

Combining these two cases together, we have

$$AIP'_j(t + \bar{L}_{j+1} + r_{j+1}) = AIOp'_j(t + \bar{L}_{j+1} + r_{j+1}) + \min\{0, All'_{j+1}(t + \bar{L}_{j+1} + r_{j+1})\}, \tag{7}$$

or equivalently,

$$AIP'_j(t + \bar{L}_{j+1} + r_j) = AIOp'_j(t + \bar{L}_{j+1} + r_j) + \min\{0, All'_{j+1}(t + \bar{L}_{j+1} + r_{j+1})\}. \tag{8}$$

Given the distribution of $AIP'_j(t + \bar{L}_{j+1} + r_j)$, we can characterize the distribution of future inventory levels similarly to stage N . We have

$$All'_j(t + \bar{L}_j + r_j) = AIP'_j(t + \bar{L}_{j+1} + r_j) - D_j[t + \bar{L}_{j+1} + r_j, t + \bar{L}_j + r_j] - \xi_j[t + \bar{L}_{j+1} + r_j, t + \bar{L}_j + r_j], \tag{9}$$

$$All'_j(t + \bar{L}_j + r_j) = AIP'_j(t + \bar{L}_{j+1} + r_j) - D_j[t + \bar{L}_{j+1} + r_j, t + \bar{L}_j + r_j] - \xi_j[t + \bar{L}_{j+1} + r_j, t + \bar{L}_j + r_j]. \tag{10}$$

In summary, we can characterize $All'_j(t + \bar{L}_j + r_j)$ through (3), (8), (9) and (10) recursively, starting from stage N to stage 1. We are able to characterize $All'_j(t)$ for any period t by repeating this process for all regenerative cycles. The average inventory cost per period is

$$G(\mathbf{s}, \mathbf{T}) = \frac{1}{T} \sum_{r=0}^{T-1} \mathbf{E} \left[\left(\sum_{j=1}^N (h'_j (All'_j(t+r))^+ + h_j IT_{j-1}(t+r)) + \hat{b} (All'_1(t+r))^- \right) \right], \tag{11}$$

where $IT_{j-1}(t)$ is the total inventory in transit to stage $j - 1$ at the end of period t , and $IT_0(t) \equiv 0$. Because all customer demand will be fulfilled and all errors will be recovered, $\mathbf{E}[IT_{j-1}(t)] = (\mathbf{E}[D] + \sum_{i=1}^{j-1} \mathbf{E}[\xi_i])L_{j-1}$.

We end this section with a connection to the literature on the bullwhip phenomenon. Lee, Padmanabhan, and Whang (1997) demonstrate four reasons that can lead to the bullwhip effect in supply chains. The above characterization of the local demand for each stage indicates that the mean and variance of D_j are larger than those of D_{j-1} . Furthermore, the increase in variance is larger if cycle-count interval of stage $(j - 1)$ is longer. Thus, inventory loss across locations in a supply chain is another factor that may contribute to the bullwhip effect.

4.2. Evaluation: echelon approach

The above procedure to obtain the total cost per period and the computation is quite complicated because we need to compute the distribution of the local demand D_j for all stages and periods. We next provide a simple echelon scheme to evaluate the total system cost. This scheme allows us to bypass the computation of the distribution of D_j . Below we define the echelon inventory variables and present the relationships between the local and echelon inventory variables. These relationships are standard and can be found in Zipkin (2000).

² This is because of the assumption that errors can occur when stage $j + 1$'s actual inventory level is non-positive. When the backlog at stage $j + 1$ is filled after receiving a shipment in subsequent periods, AIP'_j will be restored back to $AIOp'_j$.

$AIOp'_j(t)$ = Actual echelon inventory-order position at stage j at the beginning of period $t = \sum_{i=1}^j AIOp'_i(t)$,

$All'_j(t)$ = Actual echelon inventory level at stage j at the beginning of period $t = All'_j(t) + \sum_{i=1}^{j-1} AIOp'_i(t)$,

$All_j(t)$ = Actual echelon inventory level at stage j at the end of period t ,

$AIP'_j(t)$ = Actual echelon inventory in-transit position at stage j at the beginning of period $t = AIP'_j(t) + \sum_{i=1}^{j-1} AIOp'_i(t)$.

The expression for $All_j(t)$ can be similarly established by removing “-” in the variables in the equation for $All'_j(t)$.

With these definitions, the total cost per period $G(\mathbf{s}, \mathbf{T})$ in (11) can be expressed as follows.

$$G(\mathbf{s}, \mathbf{T}) = \frac{1}{T} \sum_{r=0}^{T-1} \mathbf{E} \left[\left(\sum_{j=1}^N h_j All_j(t+r) + (\hat{b} + h'_1) (All_1(t+r))^- \right) \right]. \tag{12}$$

Note that this is equal to

$$G(\mathbf{s}, \mathbf{T}) = \frac{1}{T} \sum_{r=0}^{T-1} \mathbf{E} \left[\left(\sum_{j=1}^N h_j All_j(t + \bar{L}_j + r_j) + (\hat{b} + h'_1) (All_1(t + \bar{L}_1 + r_1))^- \right) \right]. \tag{13}$$

The only difference between the above cost functions above is that the time index for stage j is shifted by $\bar{L}_j + r_j - r$ periods in the second equation. Nevertheless, every period is evaluated exactly once and the long-run average total cost per period is the same with either formulation.

Our next task is to provide a simple recursion to obtain $All_j(t + \bar{L}_j + r_j)$. From the relationship between the local variables given by (8)–(10) and the definitions of echelon variables, we can derive the following.

Proposition 4. For $j = N, N - 1, \dots, 1$,

$$AIP'_j(t + \bar{L}_{j+1} + r_j) = \min\{AIOp'_j(t + \bar{L}_{j+1} + r_j), All_{j+1}(t + \bar{L}_{j+1} + r_{j+1})\}, \tag{14}$$

$$All'_j(t + \bar{L}_j + r_j) = AIP'_j(t + \bar{L}_{j+1} + r_j) - D[t + \bar{L}_{j+1} + r_j, t + \bar{L}_j + r_j] - \sum_{i=1}^j \xi_i[t + \bar{L}_{j+1} + r_j, t + \bar{L}_j + r_j], \tag{15}$$

$$All_j(t + \bar{L}_j + r_j) = AIP'_j(t + \bar{L}_{j+1} + r_j) - D[t + \bar{L}_{j+1} + r_j, t + \bar{L}_j + r_j] - \sum_{i=1}^j \xi_i[t + \bar{L}_{j+1} + r_j, t + \bar{L}_j + r_j], \tag{16}$$

where $\bar{L}_{N+1} = 0$ and $All'_{N+1}(t + r) = \infty$.

Given $AIOp'_j(t + \bar{L}_{j+1} + r_j)$, we can use Proposition 4 to obtain $AIP'_j(t + \bar{L}_{j+1} + r_j)$, which further determines All'_j and All_j in period $t + \bar{L}_j + r_j$. From (3) and the definition of the echelon order position, we get

$$AIOp'_j(t + \bar{L}_{j+1} + r_j) = \sum_{i=1}^j AIOp'_i(t + \bar{L}_{j+1} + r_j) = S_j - \sum_{i=1}^j \xi_i[t + \lfloor (\bar{L}_{j+1} + r_j) / T_i \rfloor T_i, t + \bar{L}_{j+1} + r_j]. \tag{17}$$

Thus, with Proposition 4 and (17), we can obtain the total cost in Eq. (13).

Below we provide a bottom-up recursion to evaluate the total cost per period. The idea is to recursively evaluate the cost for echelon j , for $j = 1, \dots, N$ by assuming that echelon j has ample supply from its upstream stage. Define the following notation to represent the number of errors accumulated since the last cycle count at stage j .

$$\mathbb{M}_{T_j}(\ell) = t + \ell - \left(t + \left\lfloor \frac{\ell}{T_j} \right\rfloor T_j \right).$$

This notation yields the number of periods since the last cycle count at stage j , given a regenerative epoch t and a time length of ℓ periods.

Proposition 5. For $r = 0, 1, 2, \dots, T - 1$, define

$$g_1(y, r) = \mathbf{E}[h_1(y - \xi_1[\mathbb{M}_{T_1}(\bar{L}_2 + r_1)] - D[L_1] - \xi_1[L_1]) + (\hat{b} + h'_1)(y - \xi_1[\mathbb{M}_{T_1}(\bar{L}_2 + r_1)] - D[L_1] - \xi_1[L_1])^-]. \quad (18)$$

For $j = 2, \dots, N$ and $r = 0, 1, 2, \dots, T - 1$, define

$$g_j(y, r) = \mathbf{E} \left[h_j \left(y - \sum_{i=1}^j \xi_i[\mathbb{M}_{T_i}(\bar{L}_{j+1} + r_j)] - D[L_j] - \sum_{i=1}^j \xi_i[L_j] \right) g_{j-1} \times \left(\min \left\{ S_{j-1}, y - \sum_{i=1}^j \xi_i[\mathbb{M}_{T_i}(\bar{L}_{j+1} + r_j)] + \sum_{i=1}^{j-1} \xi_i[\mathbb{M}_{T_i}(\bar{L}_j + r_{j-1})] - D[L_j] - \sum_{i=1}^j \xi_i[L_j] \right\}, r \right) \right]. \quad (19)$$

Then, $G(\mathbf{s}, \mathbf{T}) = (1/T) \sum_{r=0}^{T-1} g_N(S_N, r)$ and $C(\mathbf{s}, \mathbf{T}) = \sum_{j=1}^N \frac{K_j}{T_j} + G(\mathbf{s}, \mathbf{T})$.

The recursion in the proposition enables us to easily evaluate the system cost for a given set of base-stock levels. In the next section, we use this recursion to generate a heuristic solution for the optimal base-stock levels and develop a lower bound on the optimal cost.

5. Lower bound and heuristic

The above bottom-up recursion is similar to that for the classic multi-echelon inventory problem (e.g., Chen, 1999; Chen & Zheng, 1994). However, unlike the classic problem, the optimal solution cannot be obtained by minimizing $(1/T) \sum_{r=0}^{T-1} g_j(S_j, r)$ recursively. The reason is as follows. In the first stage, each $g_1(y, r)$ function is convex, but we choose a single S_1 that minimizes $\sum_{r=0}^{T-1} g_1(S_1, r)$. In the second stage, $g_2(y, r)$ calls $g_1(\min\{S_1, y\}, r)$, and g_2 is no longer convex, because S_1 is not the minimizer of individual $g_1(y, r)$ functions. As a result, we cannot guarantee the convexity or unimodality of the functions $g_j(y, r)$ for $j \geq 2$. In Section 5.1, we derive a lower bound on the total cost per period for any local base-stock policy. In Section 5.2, we present a heuristic algorithm based on the recursion above. As we shall show in a numerical study, the heuristic solution is near-optimal and therefore can be used to draw insights about supply chains with inventory record inaccuracy.

5.1. Lower bound

We establish a lower bound to the total cost under any local base-stock policy, i.e., a lower bound to the minimal cost that can be achieved. We compute the lower bound by replacing S_j in (19) with the minimizer of each $g_j(y, r)$ function, which yields convex cost functions at each recursion step. Define a series of \underline{g}_j functions recursively: Let $\underline{g}_1(y, r) = g_1(y, r)$ and

$$S_1(r) = \arg \min_y \underline{g}_1(y, r). \quad (20)$$

Define

$$\underline{g}_{1,1}(y, r) = \begin{cases} \underline{g}_1(y, r), & \text{if } y \leq S_1(r) \\ \underline{g}_1(S_1(r), r), & \text{otherwise.} \end{cases}$$

Note that $\underline{g}_{1,1}(y, r)$ function is decreasing and convex if $y \leq S_1(r)$ and is constant if $y > S_1(r)$. Also, $\underline{g}_{1,1}(y, r) = \underline{g}_1(\min\{S_1(r), y\}, r)$.

With these definitions, the inventory cost part of the total cost in (13) can be expressed as

$$\begin{aligned} G(\mathbf{s}, \mathbf{T}) &= \frac{1}{T} \sum_{r=0}^{T-1} \mathbf{E} \left[\sum_{j=2}^N h_j \text{All}_j(t + \bar{L}_j + r_j) + \underline{g}_1((\text{AIP}_1^-(t + \bar{L}_2 + r_1) + \xi_1[\mathbb{M}_{T_1}(\bar{L}_2 + r_1))), r) \right] \\ &\geq \frac{1}{T} \sum_{r=0}^{T-1} \mathbf{E} \left[\sum_{j=2}^N h_j \text{All}_j(t + \bar{L}_j + r_j) + \underline{g}_{1,1}((\text{AIP}_1^-(t + \bar{L}_2 + r_1) + \xi_1[\mathbb{M}_{T_1}(\bar{L}_2 + r_1))), r) \right] \\ &\geq \frac{1}{T} \sum_{r=0}^{T-1} \mathbf{E} \left[\sum_{j=2}^N h_j \text{All}_j(t + \bar{L}_j + r_j) + \underline{g}_{1,1}(\text{All}_2(t + \bar{L}_2 + r_2) + \xi_1[\mathbb{M}_{T_1}(\bar{L}_2 + r_1)], r) \right] \\ &= \frac{1}{T} \sum_{r=0}^{T-1} \mathbf{E} \left[\sum_{j=2}^N h_j \text{All}_j(t + \bar{L}_j + r_j) + \underline{g}_1(\min\{S_1(r), (\text{All}_2(t + \bar{L}_2 + r_2) + \xi_1[\mathbb{M}_{T_1}(\bar{L}_2 + r_1)])\}, r) \right]. \quad (21) \end{aligned}$$

The first equality follows from (16) and (18). The last inequality holds because $\text{All}_2(t + \bar{L}_2 + r_2) \geq \text{AIP}_1^-(t + \bar{L}_2 + r_1)$ and $\underline{g}_{1,1}$ is a decreasing function.

For $j = 2, \dots, N - 1$, suppose that $S_{j-1}(r)$ is known. We substitute $S_{j-1}(r)$ for S_{j-1} in (19), and define the resulting function as $\underline{g}_j(y, r)$. In this case, because we are replacing $S_{j-1}(r)$, the minimizer of each individual function, in $\underline{g}_{j-1}(\min\{S_{j-1}, y\}, r)$, the resulting $\underline{g}_j(y, r)$ functions are convex in y . Let

$$S_j(r) = \arg \min_y \underline{g}_j(y, r).$$

Define

$$\underline{g}_{j,j}(y, r) = \begin{cases} \underline{g}_j(y, r), & \text{if } y \leq S_j(r) \\ \underline{g}_j(S_j(r), r), & \text{otherwise.} \end{cases}$$

Also, $\underline{g}_j(y, r) = \underline{g}_j(\min\{S_j(r), y\}, r)$.

With these definitions, (21) can be continued as

$$\begin{aligned} (21) &= \frac{1}{T} \sum_{r=0}^{T-1} \mathbf{E} \left[\sum_{j=3}^N h_j \text{All}_j(t + \bar{L}_j + r_j) + \underline{g}_2((\text{AIP}_2^-(t + \bar{L}_3 + r_2) + \xi_2[\mathbb{M}_{T_2}(\bar{L}_3 + r_2))), r) \right] \\ &\geq \dots \\ &= \frac{1}{T} \sum_{r=0}^{T-1} \mathbf{E} [h_N (\text{All}_N(t + L_N + r_N)) \\ &\quad + \underline{g}_{N-1} \left(\min \left\{ S_{N-1}(r), \left(\text{All}_N(t + \bar{L}_N + r_N) + \sum_{i=1}^{N-1} \xi_i[\mathbb{M}_{T_i}(\bar{L}_N + r_{N-1})] \right) \right\}, r \right)] \\ &= \frac{1}{T} \sum_{r=0}^{T-1} g_N(S_N, r). \end{aligned}$$

Clearly, $(1/T) \sum_{r=0}^{T-1} \underline{g}_N(S_N, r)$ is convex. Define

$$S_N^* = \arg \min_{S_N} \sum_{r=0}^{T-1} \underline{g}_N(S_N, r).$$

Define $\underline{C}(\mathbf{T}) \stackrel{\text{def}}{=} \sum_{j=1}^N \frac{K_j}{T_j} + \frac{1}{T} \sum_{r=0}^{T-1} \underline{g}_N(S_N^*, r)$. This provides a lower bound to the optimal total supply chain cost given the class of static local

base-stock policies, that is, $\underline{C}(\mathbf{T}) \leq C(\mathbf{s}, \mathbf{T})$ for any \mathbf{s} . This lower-bound cost function reduces to the one in Chen and Zheng (1994) when there are no errors in the system. Proposition 6 summarizes these results.

Proposition 6. For fixed \mathbf{T} ,

- (1) $g_j(y, r)$ is convex in y , for $j = 1, \dots, N$.
- (2) $\underline{C}(\mathbf{T})$ is a lower bound on $C(\mathbf{s}, \mathbf{T})$ for any \mathbf{s} .

In summary, we can use the recursion defined in (18) and (19) to obtain the lower bound. For stage 1, set $\underline{g}_1(y, r) = g_1(y, r)$ and let $S_1(r) = \arg \min_y g_1(y, r)$. For $j = 2, \dots, N - 1$, suppose that $S_{j-1}(r)$ is known. We substitute $S_{j-1}(r)$ for S_{j-1} in (19), and define the resulting function as $\underline{g}_j(y, r)$ and let $S_j(r) = \arg \min_y \underline{g}_j(y, r)$. We compute $\underline{g}_N(y, r)$ function with $S_{N-1}(r)$ and set $S_N^* = \arg \min_y \frac{1}{T} \sum_{r=0}^{T-1} \underline{g}_N(y, r)$. The lower bound is given by $\underline{C}(\mathbf{T})$.

5.2. Heuristic

The lower bound cost is strictly less than the optimal total cost because we are not able to find a single S_j to represent $S_j(r)$ in each iteration during the construction of the lower bound. We present an algorithm that recursively solves (18) and (19) to obtain a heuristic solution \tilde{S}_j . Specifically, let $\tilde{g}_1(y, r) = g_1(y, r)$, $\tilde{g}_1(y) = \sum_{r=0}^{T-1} \tilde{g}_1(y, r)$, and $\tilde{S}_1 = \arg \min_y \tilde{g}_1(y)$.

For $j = 2, \dots, N$, suppose that \tilde{S}_{j-1} is known. We substitute \tilde{S}_{j-1} for S_{j-1} in (19), and define the resulting function as $\tilde{g}_j(y, r)$. We define $\tilde{g}_j(y) = \sum_{r=0}^{T-1} \tilde{g}_j(y, r)$ and find its first minimizer, \tilde{S}_j . Because $\tilde{g}_j(y)$ is not necessarily convex, we find \tilde{S}_j via a line search. The heuristic solution is given by

$$\tilde{s}_1 = \tilde{S}_1, \quad \tilde{s}_j = \tilde{S}_j - \tilde{S}_{j-1}, \quad j = 2, \dots, N.$$

Define $C(\mathbf{T}) = C(\tilde{\mathbf{s}}, \mathbf{T})$, as the heuristic cost. Notice that the proposed heuristic generates the optimal base-stock level for stage 1 because $\tilde{g}_1(y)$ is convex, and $\tilde{g}_1(\tilde{s}_1) = \sum_r \tilde{g}_1(\tilde{s}_1, r)$ gives the minimum possible cost for stage 1. The heuristic also generates the exact optimal solution when $T_j = 1$ for $j < N$, because the resulting heuristic cost is equal to the lower bound cost.

We now provide an analytical result on the impact of cycle-count policy for a two-stage system.

Theorem 1. For a two-stage system with identical error distribution and identical counting costs at both stages (i.e., $\mu_1 = \mu_2$ and $K_1 = K_2$), we have $C(1, T) < C(T, 1)$ for $T > 1$.

Theorem 1 states that it is more effective to conduct more frequent cycle counts at a downstream stage for a two-stage system with identical error distributions. Intuitively, when stage 1 conducts a cycle count, it will order the demand plus the error from stage 2. From the perspective of stage 2, this error will reduce stage 2's inventory position in the same fashion as if it had conducted a cycle count and the same amount of error was identified. Thus, conducting a cycle count at stage 1 essentially mitigates the impact of error at both stages. This result holds for any (L_1, L_2) and (h_1, h_2) . In addition, this result holds for any $K_1 \leq K_2$, but may not hold otherwise, depending on the scale of counting costs relative to the scale of inventory costs. Notice that Theorem 1 holds for the optimal base-stock levels, because the heuristic yields the optimal base-stock level for stage 1, and the proof shows the result for any echelon inventory level y in the stage 2 cost function. This result demonstrates that, all else being equal, more attention should be

paid to the downstream location. Nevertheless, designing an effective cycle-count policy requires an understanding of how the relative impact changes with supply chain properties such as the lead times and the supply chain length. We conduct a numerical study to answer these questions in Section 6.

6. Numerical study

This section provides a numerical study to investigate how system parameters affect cycle-count policies. These numerical observations will lead to guidelines for designing effective cycle-count policies.

We use our heuristic base-stock policy to answer these questions through a sensitivity analysis study. We focus our discussion on both two-stage and four-stage systems. The parameters we use in the sensitivity analysis are as follows: For the two-stage system, the parameters³ are

$$\lambda = 20, \quad (h_1, h_2) \in \{(1, 3), (2, 2), (3, 1)\}, \quad (L_1, L_2) \in \{(1, 5), (3, 3), (5, 1)\}, \\ (\mu_1, \mu_2) \in \{(1, 0), (0, 1), (1, 1), (2, 2)\}, \quad K_j \in \{2, 6, 10, 14, 18, 22, 26, 30\} \text{ for all } j.$$

These parameters are chosen to represent various supply chain configurations. In addition, we consider three service levels $\alpha = \{0.8, 0.9, 0.95\}$, and set $\hat{b} = (h_1 + h_2)\alpha / (1 - \alpha)$. Finally, we consider the following 16 cycle-count policies:

$$(T_1, T_2) \in \left\{ (1, 1), (1, 3), (3, 1), (1, 6), (6, 1), (1, 12), (12, 1), (2, 2), \right. \\ \left. (2, 4), (3, 3), (2, 12), (4, 4), (3, 12), (6, 6), (6, 12), (12, 12) \right\}.$$

The largest cycle-count interval, 12, corresponds to annual inventory counts if the inventory review period is a month.

For the four-stage system, the parameters are

$$\lambda = 20, \quad \mathbf{h} \in \{(1, 1, 3, 3), (2, 2, 2, 2), (3, 3, 1, 1)\}, \\ \mathbf{L} \in \{(1, 1, 5, 5), (3, 3, 3, 3), (5, 5, 1, 1)\}, \\ \boldsymbol{\mu} \in \{(1, 1, 1, 1), (1, 1, 2, 1)\}, \quad \mathbf{K} = (10, 10, 10, 10).$$

In addition, we set $\hat{b} = 72$, representing 90% service level. For this test, we consider 36 cycle-count policies listed in Table 1 ranging from (1, 1, 1, 1) to (12, 12, 12, 12).

We first verify the effectiveness of our heuristic solution under the above chosen parameters. We define the percentage difference between the heuristic cost and the lower bound cost as

$$\frac{C(\tilde{\mathbf{s}}, \mathbf{T}) - \underline{C}(\mathbf{T})}{\underline{C}(\mathbf{T})} \times 100\%.$$

In this study, we find that the average difference is 0.22% and 0.65% for the two-stage systems with error rates (1, 1) and with error rates (2, 2), respectively. The average difference is 0.29% for four-stage systems. We conclude that our heuristic base-stock levels are near optimal. We therefore use it to answer the above research questions.

6.1. Sensitivity analysis and observations

The purpose of the sensitivity analysis study is to examine the impact of varying one specific parameter on the total cost. Therefore, we need to select a base case as a reference point. We select the following two-stage system as our base case: $\boldsymbol{\mu} = (1, 1)$, $\mathbf{L} = (3, 3)$, $\mathbf{h} = (2, 2)$, $\mathbf{K} = (10, 10)$, and $\hat{b} = 36$. Similarly, we choose the following base case for the four-stage system: $\boldsymbol{\mu} = (1, 1, 1, 1)$, $\mathbf{L} = (3, 3, 3, 3)$, $\mathbf{h} = (2, 2, 2, 2)$, $\mathbf{K} = (10, 10, 10, 10)$, and $\hat{b} = 72$. We

³ We tested another set of experiment with normal demand and error distributions. All the subsequent insights remain valid. For the normal distribution case, we assume that the error and demand distributions are i.i.d between periods, and an error occurs at the beginning of a period, followed by the demand.

Table 1
Optimal cycle-count policy for a two-stage system with base case parameters.

	K_2								
	2	6	10	14	18	22	26	30	
K_1	2	2,3	2,6	2,6	2,6	2,6	2,6	2,12	2,12
	6	3,4	3,4	4,6	4,6	4,6	4,6	3,12	3,12
	10	4,3	4,6	4,6	4,6	4,6	4,6	4,6	4,12
	14	4,3	4,6	4,6	4,6	4,6	4,6	4,6	4,12
	18	6,4	6,4	4,6	4,6	4,6	4,6	4,6	4,12
	22	6,4	6,4	6,4	4,6	4,6	4,6	4,6	6,12
	26	6,4	6,4	6,4	6,4	6,6	6,6	6,12	6,12
	30	6,4	6,4	6,4	6,4	6,6	6,6	6,12	6,12

measure the impact of cycle-count policies by computing the percentage reduction in the optimal cost relative to the worst case, $C(12, \dots, 12)$, which has minimal correction of errors. Define

$$\Delta(\mathbf{T}) = \frac{C(12, \dots, 12) - C(\mathbf{T})}{C(12, \dots, 12)} \times 100\%.$$

For simplicity, we use $\mathbf{T}' \succ \mathbf{T}''$ to denote that a cycle-count policy \mathbf{T}' achieves more percentage cost reduction than policy \mathbf{T}'' . Clearly, different parameter sets may lead to larger or smaller magnitude of the effects. However, we are interested more in the insights obtained by observing the direction of the change in the total cost in response to the system parameters.

6.2. Observations

We summarize main observations from the numerical study below. Notice that a curve in Figs. 2–7 is obtained by starting with a base case and changing only the variable specified in the horizontal axis. For example, in Fig. 2, the “diamond” shape curve shows the cost reduction for the four-stage base case under different cycle-count policies; the “square” shape curve shows the same when we increase the error rate at stage 3 in the same four-stage case.

1. Fig. 2 shows the impact of conducting cycle counts on different locations in four-stage systems. We find that the result in Theorem 1 holds for longer supply chains as well: With identical error rates and counting costs, a downstream cycle count is more effective than an upstream cycle count. A cycle count at an upstream stage could be more beneficial to the system only when the error rate at that location is significantly higher than the downstream locations.

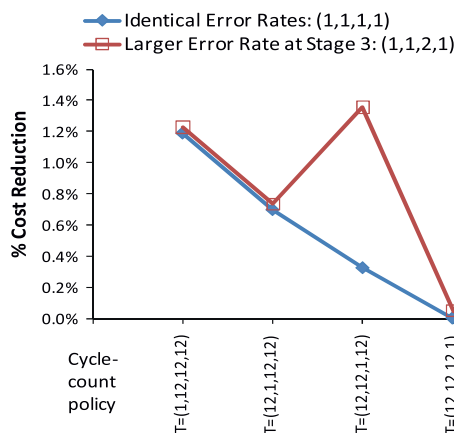


Fig. 2. Impact of error rates.

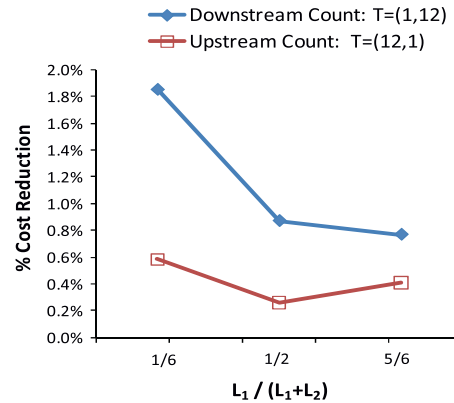


Fig. 3. Impact of lead times ($N = 2$).

- Fig. 3 shows the benefit of conducting cycle counts under various lead time ratios for the two-stage base case. We particularly consider two cycle-count policies: $(T_1, T_2) = (1, 12)$ and $(12, 1)$. The former (the latter) represents conducting more frequent counts at stage 1 (stage 2). From the top curve, we observe that the benefit of cycle counts at the downstream location decreases as the share of the downstream lead time increases. Since a shorter lead time implies less stock, this result suggests that a leaner downstream stage is more vulnerable to the error.
- Fig. 4(a) and (b) illustrate the two conclusions observed above for the four-stage base system.
- The benefit of cycle counts at a location increases if the ratio of the holding cost to the total holding cost increases. Intuitively, when the holding cost is larger, the stage will carry less inventory, making the impact of errors more significant.
- Fig. 5 illustrates the impact of cycle counts on the number of stages for both $N = 2$ and $N = 4$ cases. To make a fair comparison, we change the lead time parameters of the two-stage system by setting $\mathbf{L} = (6, 6)$ and $\mathbf{h} = (4, 4)$. This ensures that total lead time and total holding cost of two-stage system are the same as the four-stage system. We find that the impact of conducting cycle count at the most upstream stage relative to the impact of conducting cycle counts at stage 1 for $N = 4$ (i.e., 0.1–1.2%) is smaller than that for $N = 2$ (i.e., 2.1–3.5%). This observation suggests that cycle counts should be focused on both stage 1 and stage 2 (on stage 1) when $N = 4$ ($N = 2$). Thus, a rough guideline based on this conclusion for systems with similar error rates and counting costs is that cycle counts should be focused more on the downstream half of the supply chain.
- Fig. 6 shows the marginal benefit of assigning more than one cycle count for the base example. Starting from $\mathbf{T} = (12, 12)$, we change T_1 (T_2) only to increase the downstream (upstream) counts. As shown, both upstream and downstream counts exhibit diminishing rates of return. More importantly, the first additional upstream cycle count may be more beneficial to the system than the third additional downstream count. This result immediately suggests that it is not a good strategy to allocate all cycle counts to a single location. Instead, cycle counts should be allocated across locations in a dynamic way.
- Based on observation 6, an effective cycle-count policy must assign the first few cycle counts downstream but then keep a reasonably balanced allocation of the total cycle counts available. Fig. 7(a) shows that $(2, 2) \succ (1, 12)$, where the former has 12 counts and the latter has 13 counts per year. (Note that this comparison holds even when $K_1 = K_2 = 0$, indicating that the difference is mostly due to the inventory costs.) Hence, a balanced policy is usually better than a policy that heavily

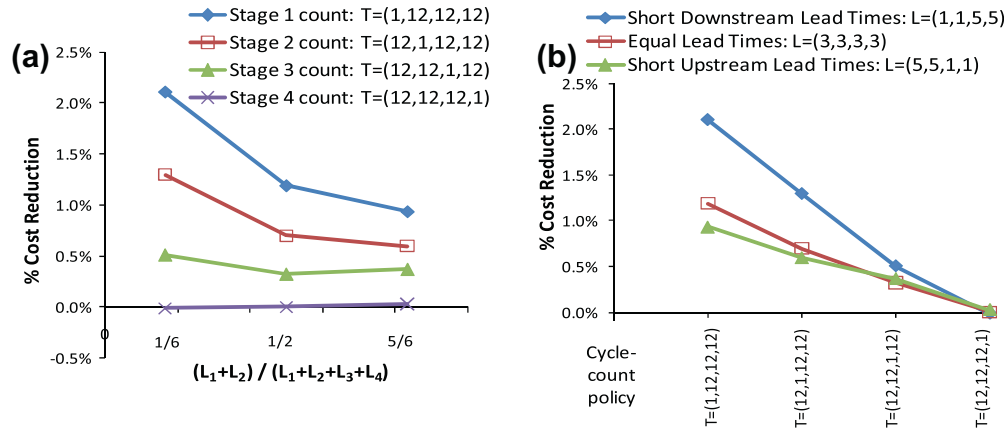


Fig. 4. Impact of lead times ($N = 4$).

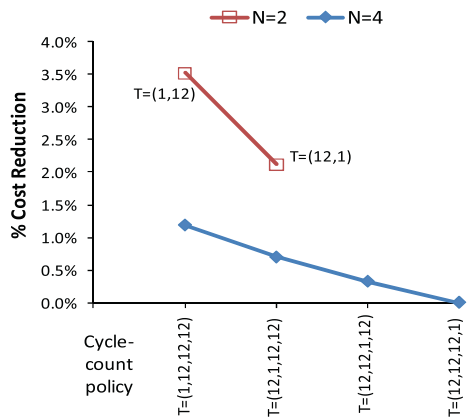


Fig. 5. Impact of cycle count location.

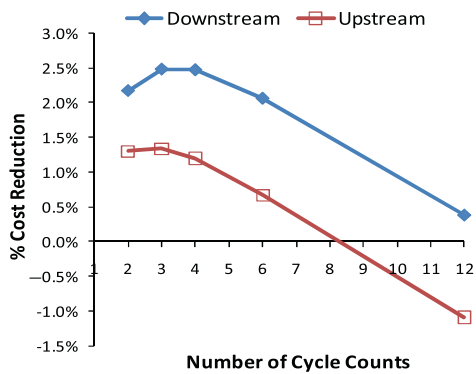


Fig. 6. Impact of number of cycle counts.

favors downstream locations. Nevertheless, we know that as the downstream lead time share decreases downstream counts have a larger impact. Fig. 7(b) compares policies (4, 12) and (6, 6), both with four counts per year. While $(6, 6) \succ (4, 12)$ when $L_1/(L_1 + L_2) \geq 1/2$, we have $(4, 12) \succ (6, 6)$ as that ratio decreases. Hence, the balance should be tilted slightly towards downstream locations in supply chains with short downstream lead times. These guidelines carry over to longer supply chains.

8. The above observations are drawn from the base cases, in which each stage has identical counting cost. Clearly, stage counting

costs directly affect cycle-count decisions. While observation 1 suggests that downstream counts are more beneficial to the system when counting costs are identical, the result will be reversed if downstream counting costs are significantly higher than upstream counting costs. Table 1 illustrates the optimal cycle-count policies for various counting cost combinations in a two-stage system. The optimal cycle-count policy is obtained by searching over all 36 feasible policies with $T_j \in \{1, 2, 3, 4, 6, 12\}$. For example, as shown in the table, the best cycle-count policy for the base case example with $K_1 = K_2 = 10$ is $T = (4, 6)$. As K_1 increases, downstream cycle-count frequency decreases. To compensate for that, the upstream cycle-count frequency may increase. Similarly, as K_2 increases, upstream cycle-count frequency decreases and downstream cycle-count frequency generally increases. Note that it is optimal to count more frequently at the upstream stage rather than the downstream stage only when K_2 is significantly lower than K_1 . In most cases, even when $K_1/K_2 > 2$, it is optimal to count more frequently downstream, indicating that location in the supply chain should play a very significant role in counting decisions. In addition, the optimal cycle-count policy is (4, 6) for the base case and a wide range of counting costs around that, indicating that optimal cycle-count policy is not very sensitive to the counting cost structure.

Based on these observations, we categorize the factors that affect cycle-count decisions into two groups. Primary factors determine which stages should get priority (i.e., higher cycle-count frequency). These factors are (i) stage (position in the supply chain) (ii) error rate, and (iii) counting cost. Secondary factors determine whether the policy should strongly favor the high-priority stages or allocate counts in a somewhat balanced way. These are (i) lead time structure, (ii) holding cost structure, and (iii) the length of the supply chain.

These two groups of factors can be used to provide the following guidelines for designing effective cycle-count policies. When allocating a fixed budget or a fixed number of cycle counts in a supply chain, a manager should first consider primary factors by allocating first few cycle counts to downstream locations and locations with significantly higher error rate or low counting cost. Then, the manager should consider the secondary factors to allocate the remaining cycle counts. For example, if the ratio of the lead time of a downstream stage to the total system lead time is large, the manager should allocate the remaining cycle counts to keep a more balanced distribution rather than focusing on the downstream stage. On the other hand, if the supply chain is long,

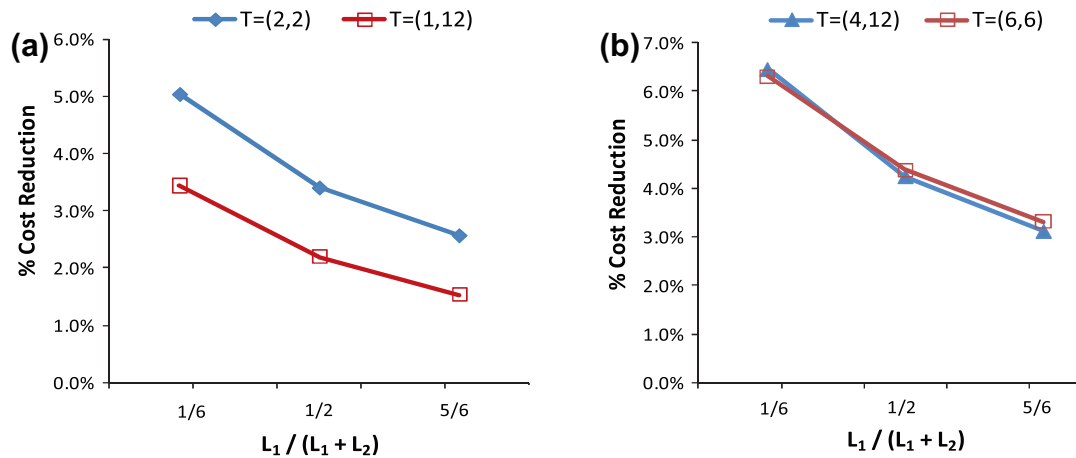


Fig. 7. Effectiveness of balanced cycle count policies.

the manager should allocate the remaining counts to favor the few downstream stages.

Finally, our model and analysis also provide implications on RFID investments from the perspective of reducing inventory inaccuracy. Consider the following settings with RFID. (i) In some settings with tagged products, RFID provides perpetual information in locations where readers are installed. This scenario can be evaluated with our framework. For example, for a production line at a factory with a six-stage process, a reader in stages 3 and 5 is equivalent to a cycle-count policy (12, 12, 1, 12, 1, 12). (ii) In some other settings, the cost of readers may be small relative to the cost of RFID tags. In that case, the problem is to decide at which stage the tags should be put on the products. For example, if stage 3 and its downstream stages have tagged products, this is equivalent to policy (1, 1, 1, 12, 12, 12). (iii) In the apparel industry, it is common to use hand-held RFID readers to do cycle-counts. In this case, the store staff performs cycle counts using the handheld readers near shelves or hangers. These cycle counts are faster (less costly) and possibly more accurate than manual counts. Our results on cycle-count policies are directly relevant for this setting. RFID investment determines which stages have relatively low counting costs and therefore count more often. In any of these scenarios, a supply chain manager needs to do a cost-benefit analysis for the RFID investment. It has been suggested in the literature that from the perspective of labor cost savings, upstream stages (back-end operations) should be RFID-enabled first (Chopra & Sodhi, 2007). Our results suggest that, from the perspective of record inaccuracy, all else being equal, downstream stages should get priority.

7. Conclusion

Inventory record inaccuracy is a prevalent issue that affects supply chain performance. While common mitigation approaches such as conducting cycle counts and installing inventory tracking systems have been adopted by many firms, there are no clear guidelines in the literature on how to design cycle-count policies from the perspective of the entire supply chain. This paper aims to shed light on this issue.

For a two-stage system, we prove that with identical errors and counting costs at both stages, it is always more effective to conduct more frequent cycle counts at the downstream stage. In a numerical study, we find that the above conclusion holds for more general systems unless an upstream stage has a significantly higher error rate or lower counting cost. In particular, more cycle counts should be allocated to a downstream stage if the proportion of

its lead time to the system lead time is small, the proportion of its holding cost to the total holding cost is large, or the number of stages in the supply chain is large. However, because the marginal benefit of cycle counts is decreasing in its frequency, it is not a good strategy to assign all the cycle counts to the downstream stages. Based on our analytical and numerical results, we conclude that, in addition to the existing attributes such as volume, error rates, value and counting costs suggested by practitioners, companies should consider the following attributes when designing system-wide cycle-count programs: location in the supply chain, lead times, holding costs, and the number of stages in the supply chain.

There are several interesting future research directions from both an academic and practical point of view. First, for tractability, we assume that the errors are independent of the on-hand inventory levels. While in practice errors should depend on the on-hand inventory level, we believe that our key insight remains valid. This is because the downstream stage in general holds more inventory at optimality in a supply chain, which implies that the error is larger at downstream stages. Thus, more cycle counts should be allocated to downstream stages. Nevertheless, from a practical perspective, it remains an open question how to optimize the system with inventory-dependent errors. Second, we assume that record inaccuracy is removed only if a cycle-count is performed. Some companies may have systems in place to correct inventory records even in the no-cycle-count periods if a backorder occurs and the record shows the product in-stock. It would be interesting to generalize the current model to accommodate such practice. Third, we consider only stationary base-stock policies. However, other replenishment policies that depend on accumulated errors should be more effective. It would be interesting to investigate the value yielded from a more sophisticated error-dependent policy. Finally, the serial system is a special case of supply chain systems. It is important to study the cycle count policies for more general systems, such as assembly and distribution systems. The modeling approach (i.e., inventory shrinkage and cycle counts), evaluation schemes and the optimization procedure may be useful for analyzing these general systems.

Acknowledgements

The authors would like to thank Zhijie Tao and Hang Bai for their help with computation, Bill Hardgrave for useful discussions and the Editor, and the anonymous reviewers for their helpful suggestions.

Appendix A. Proofs

Proposition 3. Let Δ denote the difference of a function, i.e., $\Delta f(y) = f(y + 1) - f(y)$. We only show $T = 2$ case as the general T case can be proven similarly. Note that

$$C(y, 2) = \frac{1}{2}(g(y, 0) + g(y, 1)),$$

$$C(y, 1) = g(y, 0) = \frac{1}{2}(g(y, 0) + g(y, 0)).$$

To show part (1), we only need to show $\Delta g(y, 0) + \Delta g(y, 0) \geq \Delta g(y, 0) + \Delta g(y, 1)$, or equivalently, $\Delta g(y, 0) \geq \Delta g(y, 1)$ for all y .

$$\Delta g(y, 0) = h_1 - (\hat{b} + h'_1)P(D[L] + \xi[L] > y),$$

$$\Delta g(y, 1) = h_1 - (\hat{b} + h'_1)P(D[L] + \xi[L] + \xi[1] > y).$$

Because $D[L] + \xi[L] + \xi[1] \geq_{st} D[L] + \xi[L]$, we have $P(D[L] + \xi[L] > y) \leq P(D[L] + \xi[L] + \xi[1] > y)$, for all y . Thus, the inequality $\Delta g(y, 0) \geq \Delta g(y, 1)$ holds.

For part (2),

$$g(y, 1) = E[h(y - D[L] - \xi[L] - \xi[1])^+ + \hat{b}(y - D[L] - \xi[L] - \xi[1])^-] \\ \geq E[h(y - D[L] - \xi[L] - E[\xi[1]])^+ + \hat{b}(y - D[L] - \xi[L] - E[\xi[1]])^-] \equiv g'(y)$$

for all y due to Jensen's inequality. Let s' be the minimizer of $g'(y)$. It is easy to see that $g'(s') = g(s(1), 0)$. Thus, we have

$$C(2) = (1/2)(g(s(2), 1) + g(s(2), 0)) \geq (1/2)(g'(s(2)) + g(s(2), 0)) \\ \geq (1/2)(g'(s') + g(s(2), 0)) \geq (1/2)(g(s(1), 0) + g(s(1), 0)) = C(1).$$

Proposition 4. We first show (14). By definition, $AIP_j(t) = AIP'_j(t) + AIOp_{j-1}(t)$. Adding $AIOp_{j-1}(t + \bar{L}_{j+1} + r_{j+1})$ on both sides of (6) yields

$$AIP_j(t + \bar{L}_{j+1} + r_{j+1}) = AIOp'_j(t + \bar{L}_{j+1} + r_{j+1}) + \min\{0, AIL'_{j+1}(t + \bar{L}_{j+1} + r_{j+1})\} \\ + AIOp_{j-1}(t + \bar{L}_{j+1} + r_{j+1}) \\ = AIOp_j(t + \bar{L}_{j+1} + r_{j+1}) + \min\{0, AIL'_{j+1}(t + \bar{L}_{j+1} + r_{j+1})\} \\ = \min\{AIOp_j(t + \bar{L}_{j+1} + r_{j+1}), AIL'_{j+1}(t + \bar{L}_{j+1} + r_{j+1})\}.$$

Eq. (14) follows immediately.

We next show (16). Eq. (15) can be proved similarly.

For notational simplicity, let $t_{j+1} = t + \bar{L}_{j+1} + r_j$ and $t_j = t + \bar{L}_j + r_j$. By definition, $AIL_j(t) = AIL'_j(t) + \sum_{i=1}^{j-1} AIOp'_i(t)$. Thus,

$$AIL_j(t_j) = AIL'_j(t_j) + \sum_{i=1}^{j-1} AIOp'_i(t_j) \\ = AIP'_j(t_{j+1}) - D_j[t_{j+1}, t_j] - \xi_j[t_{j+1}, t_j] \\ + \sum_{i=1}^{j-1} (AIOp'_i(t_{j+1}) - D_i[t_{j+1}, t_j] + D_{i+1}[t_{j+1}, t_j] - \xi_i[t_{j+1}, t_j]) \\ = AIP'_j(t_{j+1}) + \sum_{i=1}^{j-1} AIOp'_i(t_{j+1}) - D[t_{j+1}, t_j] - \sum_{i=1}^j \xi_i[t_{j+1}, t_j] \\ = AIP_j(t_{j+1}) - D[t_{j+1}, t_j] - \sum_{i=1}^j \xi_i[t_{j+1}, t_j].$$

Proposition 5. We only prove the $N = 2$ case. The general N case can be proved similarly. When $N = 2, \bar{L}_3 = 0, \bar{L}_2 = L_2$ and $\bar{L}_1 = L_1 + L_2$. The total cost per period in (13) becomes

$$\frac{1}{T} \sum_{t=0}^{T-1} E[h_2 ALL_2(t + \bar{L}_2 + r_2) + h_1 ALL_1(t + \bar{L}_1 + r_1) + (\hat{b} + h'_1)(ALL_1(t + \bar{L}_1 + r_1))^-] \\ = \frac{1}{T} \sum_{t=0}^{T-1} E[h_2 (AIOp'_2(t + r_2) - D[t + r_2, t + L_2 + r_2]) - \sum_{i=1}^2 \xi_i[t + r_2, t + L_2 + r_2]) \\ + h_1 (AIP_1(t + L_2 + r_1) - D[t + L_2 + r_1, t + \bar{L}_1 + r_1] - \xi_1[t + L_2 + r_1, t + \bar{L}_1 + r_1]) \\ + (\hat{b} + h'_1)(AIP_1(t + L_2 + r_1) - D[t + L_2 + r_1, t + \bar{L}_1 + r_1] - \xi_1[t + L_2 + r_1, t + \bar{L}_1 + r_1])^-],$$

where

$$AIP_1^-(t + L_2 + r_1) = \min\{AIOp_1^-(t + L_2 + r_1), ALL_2(t + L_2 + r_2)\}.$$

Eq. (22) is due to Proposition 4.

Note that from (17) and Proposition 4,

$$AIOp'_2(t + r_2) = S_2 - \sum_{i=1}^2 \xi_i \left[t + \left\lfloor \frac{r_2}{T_i} \right\rfloor T_i, t + r_2 \right) = S_2 - \sum_{i=1}^2 \xi_i [M_{T_i}(r_2)],$$

$$AIOp_1^-(t + L_2 + r_1) = s_1 - \xi_1 [M_{T_1}(L_2 + r_1)],$$

$$ALL_2(t + L_2 + r_2) = S_2 - \sum_{i=1}^2 \xi_i [M_{T_i}(r_2)] - D[L_2] - \sum_{i=1}^2 \xi_i [L_2].$$

We can substitute these values into (22) and obtain the total cost as a function system parameters. It can be verified that the recursion yields the same total cost.

Theorem 1. For simplicity, we only prove the result for $T = 2$. The general T case can be proven in a similar manner. Define $(a \wedge b) = \min\{a, b\}$, $D_1^1 = D[L_1] + \xi_1[L_1]$, and $D_2^{1,2} = D[L_2] + \xi_1[L_2] + \xi_2[L_2]$. We use superscripts "A" and "B" to represent the cost functions and the optimal base-stock levels associated with $C(1, 2)$ and $C(2, 1)$, respectively. Define

$$s_1^A = \arg \min\{g_1^A(y, 0) + g_1^A(y, 1)\},$$

$$s_1^B = \arg \min\{g_1^B(y, 0) + g_1^B(y, 1)\}.$$

Showing $C(2, 1) > C(1, 2)$ is equivalent to showing

$$g_2^B(y, 0) + g_2^B(y, 1) \geq g_2^A(y, 0) + g_2^A(y, 1), \quad \forall y. \tag{23}$$

For A, $\xi_1[M_{T_1}(t)] = 0$ for any $t, \bar{L}_2 = L_2, \bar{L}_3 = 0, r_2 = r, r_1 = r + 1$. We have

$$g_1^A(y, 0) = E[h_1(y - D_1^1) + (b + h'_1)(y - D_1^1)^-], \\ g_1^A(y, 1) = E[h_1(y - D_1^1) + (b + h'_1)(y - D_1^1)^-], \\ g_2^A(y, 0) = E[h_2(y - D_2^{1,2} - \xi_2[M_{T_2}(L_3 + r)]) \\ + g_1^A(s_1^A \wedge (y - D_2^{1,2} - \xi_2[M_{T_2}(L_3 + r)]), 0)] \\ = E[h_2(y - D_2^{1,2}) + g_1^A(s_1^A \wedge (y - D_2^{1,2}), 0)], \\ g_2^A(y, 1) = E[h_2(y - D_2^{1,2} - \xi_2[1]) + g_1^A((s_1^A \wedge (y - D_2^{1,2} - \xi_2[1])), 1)],$$

For B, $T_1 = 2, T_2 = 1, \bar{L}_2 = L_2, \bar{L}_3 = 0, r_2 = r, r_1 = r + 1$. Hence, $\xi_2[M_{T_2}(t)] = 0$ for any t . We have

$$g_1^B(y, 0) = E[h_1(y - D_1^1 - \xi_1[M_{T_1}(L_2 + 1)]) \\ + (b + h'_1)(y - D_1^1 - \xi_1[M_{T_1}(L_2 + 1)])^-], \\ g_1^B(y, 1) = E[h_1(y - D_1^1 - \xi_1[M_{T_1}(L_2 + 2)]) \\ + (b + h'_1)(y - D_1^1 - \xi_1[M_{T_1}(L_2 + 2)])^-], \\ g_2^B(y, 0) = E[h_2(y - D_2^{1,2} - \xi_1[M_{T_1}(L_2 + r_2)]) \\ + g_1^B((s_1^B \wedge (y - D_2^{1,2} - \xi_1[M_{T_1}(L_2 + r_2)])) + \xi_1[M_{T_1}(L_2 + r_1)]), 0)] \\ = E[h_2(y - D_2^{1,2}) + g_1^B(s_1^B \wedge (y - D_2^{1,2} + \xi_1[M_{T_1}(L_2 + 1)]), 0)], \\ g_2^B(y, 1) = E[h_2(y - D_2^{1,2} - \xi_1[1]) + g_1^B(s_1^B \wedge (y - D_2^{1,2} - \xi_1[1]) \\ + \xi_1[M_{T_1}(L_2 + 2)]), 1)].$$

Plugging these into (23), we get

$$\begin{aligned} & \mathbf{E} \left[h_2 (y - D_2^{12}) + \mathbf{E} [h_1 \left((s_1^B \wedge (y - D_2^{12} + \xi_1 [\mathbb{M}_2(L_2 + 1)])) \right) \right. \right. \\ & \quad \left. \left. - D_1^1 - \xi_1 [\mathbb{M}_2(L_2 + 1)] \right) + (b + h_1') \left((s_1^B \wedge (y - D_2^{12} + \xi_1 [\mathbb{M}_2(L_2 + 1)])) \right) \right. \\ & \quad \left. - D_1^1 - \xi_1 [\mathbb{M}_2(L_2 + 1)] \right) + \mathbf{E} [h_2 (y - D_2^{12} - \xi_1 [1]) \\ & \quad + \mathbf{E} [h_1 \left((s_1^B \wedge (y - D_2^{12} - \xi_1 [1] + \xi_1 [\mathbb{M}_2(L_2 + 2)])) \right) - D_1^1 \\ & \quad \left. - \xi_1 [\mathbb{M}_2(L_2 + 2)] \right) + (b + h_1') \left((s_1^B \wedge (y - D_2^{12} - \xi_1 [1] \right. \\ & \quad \left. + \xi_1 [\mathbb{M}_2(L_2 + 2)])) - D_1^1 - \xi_1 [\mathbb{M}_2(L_2 + 2)] \right)] \geq \mathbf{E} [h_2 (y - D_2^{12}) \\ & \quad + \mathbf{E} [h_1 \left((s_1^A \wedge (y - D_2^{12})) - D_1^1 \right) + (b + h_1') \left((s_1^A \wedge (y - D_2^{12})) - D_1^1 \right)] \\ & \quad + \mathbf{E} [h_2 (y - D_2^{12} - \xi_2 [1]) + \mathbf{E} [h_1 \left((s_1^A \wedge (y - D_2^{12} - \xi_2 [1])) - D_1^1 \right) \\ & \quad \left. + (b + h_1') \left((s_1^A \wedge (y - D_2^{12} - \xi_2 [1])) - D_1^1 \right)] \right]. \end{aligned}$$

Canceling the common terms from both sides and noting that (1) $\xi_2[1]$ and $\xi_1[1]$ follow the same distribution, (2) the two $\xi_1[\mathbb{M}_2(L_2 + 1)]$ terms in the first line and the second line and the two $\xi_1[\mathbb{M}_2(L_2 + 2)]$ terms in the third line and fourth line of the inequality come from the same time period (i.e., $t + L_2 + r_1$), we get the following:

$$\begin{aligned} & \mathbf{E} [\mathbf{E} [h_1 \left((s_1^B - \xi_1 [\mathbb{M}_2(L_2 + 1)]) \wedge (y - D_2^{12}) \right) - D_1^1 \\ & \quad + (b + h_1') \left((s_1^B - \xi_1 [\mathbb{M}_2(L_2 + 1)]) \wedge (y - D_2^{12}) \right) - D_1^1] \\ & \quad + \mathbf{E} [\mathbf{E} [h_1 \left((s_1^B - \xi_1 [\mathbb{M}_2(L_2 + 2)]) \wedge (y - D_2^{12} - \xi_1 [1]) \right) - D_1^1 \\ & \quad + (b + h_1') \left((s_1^B - \xi_1 [\mathbb{M}_2(L_2 + 2)]) \wedge (y - D_2^{12} - \xi_1 [1]) \right) - D_1^1] \\ & > \mathbf{E} [\mathbf{E} [h_1 \left((s_1^A \wedge (y - D_2^{12})) - D_1^1 \right) + (b + h_1') \left((s_1^A \wedge (y - D_2^{12})) - D_1^1 \right)] \\ & \quad + \mathbf{E} [\mathbf{E} [h_1 \left((s_1^A \wedge (y - D_2^{12} - \xi_2 [1])) - D_1^1 \right) \\ & \quad + (b + h_1') \left((s_1^A \wedge (y - D_2^{12} - \xi_2 [1])) - D_1^1 \right)] \right]. \end{aligned}$$

This is, in turn, equivalent to

$$\begin{aligned} & \mathbf{E} [g_1^A \left((s_1^B - \xi_1 [\mathbb{M}_2(L_2 + 1)]) \wedge (y - D_2^{12}), 0 \right) + \mathbf{E} [g_1^A \left((s_1^B - \xi_1 [\mathbb{M}_2(L_2 \right. \\ & \quad \left. + 2)]) \wedge (y - D_2^{12} - \xi_1 [1]), 0 \right)] \\ & > \mathbf{E} [g_1^A \left(s_1^A \wedge (y - D_2^{12}), 0 \right) + \mathbf{E} [g_1^A \left(s_1^A \wedge (y - D_2^{12} - \xi_2 [1]), 0 \right)]. \end{aligned}$$

Note that $s_1^A = \arg \min (g_1^A(y, 0) + g_1^A(y, 1)) = \arg \min (g_1^A(y, 0))$ because $g_1^A(y, 0) = g_1^A(y, 1)$. We also have that $s_1^B \geq s_1^A$ because $s_1^B = \arg \min (g_1^B(y, 0) + g_1^B(y, 1))$. For the first terms on both side of the inequality, fixing D_2^{12} and $\xi_1[\mathbb{M}_2(L_2 + 1)]$, we get $g_1^A(s_1^A \wedge y - D_2^{12}, 0) \leq g_1^A(s_1^B - \xi_1[\mathbb{M}_2(L_2 + 1)] \wedge y - D_2^{12}, 0)$, because $g_1^A(y, 0)$ is a decreasing function for $y < s_1^A$ and ξ_1 is nonnegative.

For the second terms on both side of the inequality, because $\xi_1[1]$ and $\xi_2[1]$ follow the same distribution and they are in separate expectations, we can compare the terms for fixed $\xi_1[1] = \xi_2[1]$. We have $g_1^A((s_1^B - \xi_1[\mathbb{M}_2(L_2 + 2)]) \wedge (y - D_2^{12} - \xi_1[1]), 0) \geq g_1^A(s_1^A \wedge (y - D_2^{12} - \xi_2[1]), 0)$, because $g_1^A(y, 0)$ is a decreasing function for $y < s_1^A$. This completes the proof. □

References

Atali, A., Lee, H. L., & zer, . (2005). If the inventory manager knew: Value of RFID under imperfect inventory information. Working paper. Stanford University.
 Bensoussan, A., akanyildırım, M., & Sethi, S. (2007). Partially observed inventory systems: The case of zero balance walk. *SIAM Journal of Control and Optimization*, 46, 176–209.
 Bollapragada, R., Rao, U., & Zhang, J. (2004). Managing inventory and supply performance in assembly systems with random supply capacity and demand. *Management Science*, 50, 1729–1743.

Bottani, E., Montanari, R., & Volpi, A. (2010). The impact of RFID and EPC network on the bullwhip effect in the Italian FMCG supply chain. *International Journal of Production Economics*, 124, 426–432.
 amdereli, A., & Swaminathan, J. M. (2009). Misplaced inventory and radio-frequency identification (RFID) technology: Information and coordination. *Production and Operation Management*, 19(1), 1–18.
 Chappell, G., Durdan, D., Gilbert, G., Ginsburg, L., Smith, J., & Tobolski, J. (2002). Auto-ID on delivery: The value of Auto-ID technology in the retail supply chain. *Accenture*.
 Chen, F. (1999). Decentralized supply chains subject to information delays. *Management Science*, 45, 1076–1090.
 Chen, L. (2013). Fixing phantom stockouts: Optimal data-driven shelf inspection policies. Working paper, Duke University.
 Chen, L., & Mersereau, A. (2013). Analytics for operational visibility in the retail store: The cases of censored demand and inventory record inaccuracy. Working Paper, Duke University.
 Chen, F., & Zheng, Y. S. (1994). Lower bounds for multi-echelon stochastic inventory systems. *Management Science*, 40, 1426–1443.
 Chopra, S., & Sodhi, M. (2007). Looking for the Bang from the RFID Buck. *Supply Chain Management Review*, May 34–41.
 Clark, A. J., & Scarf, H. (1960). Optimal policies for a multi-echelon inventory problem. *Management Science*, 6, 475–490.
 DeHoratius, N., Mersereau, A., & Schrage, L. (2008). Retail inventory management when records are inaccurate. *Manufacturing and Service Operations Management*, 10, 254–277.
 DeHoratius, N., & Raman, A. (2008). Inventory record inaccuracy: An empirical analysis. *Management Science*, 54, 627–641.
 Delen, D., Hardgrave, B. C., & Sharda, R. (2009). RFID for better supply-chain management through enhanced information visibility. *Production and Operation Management*, 16(5), 613–624.
 Deshpande, V., Schwarz, L. B., & Raju, V. (2009). Inventory management under product mis-identification/shipment errors. Working paper, Purdue University.
 ECR Europe (2001). Introducing a collaborative approach to reducing stock loss in the supply chain. Brussels, Belgium: ECR Europe.
 ECR Europe (2003). Shrinkage: A collaborative approach to reducing stock loss in the supply chain. Brussels, Belgium: ECR Europe.
 Fleisch, E., & Tellkamp, C. (2005). Inventory inaccuracy and supply chain performance: A simulation study of a retail supply chain. *International Journal of Production Economics*, 95, 373–385.
 Gaukler, G., zer, M. ., & Hausman, W. H. (2008). Order progress information: Improved dynamic emergency ordering policies. *Production and Operation Management*, 17(6), 599–613.
 Hardgrave, W., Aloysius, J., & Goyal, S. (2009). Does RFID improve inventory accuracy? A preliminary analysis. *International Journal of RF Technologies: Research and Applications*, 1(1), 44–56.
 Hardgrave, W., Aloysius, J., & Goyal, S. (2013). RFID-enabled visibility and retail inventory record inaccuracy: Experiments in the field. *Production and Operations Management*, 22(4), 843–856.
 Henig, M., & Gerchak, Y. (1990). The structure of periodic review policies in the presence of random yield. *Operations Research*, 38, 634–643.
 Hollinger, R. C. (2003). National retail security survey final report. University of Florida, Gainesville, FL.
 Iglehart, D. L., & Morey, R. C. (1972). Inventory systems with imperfect asset information. *Management Science*, 18, B388–B394.
 Jordan, H. (1994). *Cycle counting for record accuracy training aid*. The American Production and Inventory Control Society, Inc.
 Kang, Y., & Gershwin, S. B. (2005). Information inaccuracy in inventory systems – Stock loss and stockout. *IIE Transactions*, 37, 843–859.
 Kearney, A. T. (2004). Meeting the retail RFID mandate: A discussion of the issues facing CPG companies. Technical report, A.T. Kearney, Inc.
 Kim, C., Klabjan, D., & Simchi-Levi, D. (2007). Optimal expediting policies for an inventory system with stochastic lead time under radio frequency identification. Working paper. Massachusetts Institute of Technology.
 Kk, A. G., & Shang, K. (2007). Inspection and replenishment policies for systems with inventory record inaccuracy. *Manufacturing and Service Operations Management*, 9, 185–205.
 Lee, H. L., & zer, . (2007). Unlocking the value of RFID. *Production and Operation Management*, 16, 40–64.
 Lee, H. L., Padmanabhan, V., & Whang, S. (1997). Information distortion in a supply chain: The bullwhip effect. *Management Science*, 43, 546–558.
 Lee, H. L., & Yano, C. A. (1988). Production control in multistage systems with variable yield losses. *Operation Research*, 36, 269–278.
 Liu, S., So, R., & Zhang, F. (2010). The effect of supply reliability in a retail setting with joint marketing and inventory decisions. *Manufacturing and Service Operations Management*, 12(1), 19–32.
 Mersereau, A. J. (2012). Information-sensitive replenishment when inventory records are inaccurate. *Production and Operations Management*. <http://dx.doi.org/10.1111/j.1937-5956.2011.01305.x>.
 Pei, J., & Klabjan, D. (2010). Inventory control in serial systems under radio frequency identification. *International Journal of Production Economics*, 123, 118–136.
 Raman, A., & Ton, Z. (2003). Operational execution at Arrow Electronics. *Harvard Business School Case*, N9603-127.
 Reikik, Y., Sahin, E., Jemai, Z., & Dallery, Y. (2008). Execution errors in retail supply chains: Analysis of the case of misplaced products. *International Journal of Systems Science*, 39, 727–740.

- Sahin, E., Buzacott, J., & Dallery, Y. (2008). Analysis of a news vendor which has errors in inventory data records. *European Journal of Operational Research*, 188, 370–389.
- Sahin, E., & Dallery, Y. (2009). Assessing the impact of inventory inaccuracies within a newsvendor framework. *European Journal of Operational Research*, 197, 1108–1118.
- Sheppard, G. M., & Brown, K. A. (1993). Predicting inventory record-keeping errors with discriminant analysis: A field experiment. *International Journal of Production Economics*, 32, 39–51.
- Zipkin, P. (2000). *Foundations of inventory management*. New York: McGraw-Hill.
- Zipkin, P. (2006). The best things in life were free: On the technology of transactions. *Manufacturing and Service Operations Management*, 8, 321–329.