European Journal of Operational Research 240 (2015) 160–171

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Cooperation in assembly systems: The role of knowledge sharing networks

Fernando Bernstein [a], A. Gürhan Kök [b], Ana Meca [c,*]

[a] Fuqua School of Business, Duke University, Durham, NC 27708-0120, USA
[b] College of Administrative Sciences and Economics, Koç University, Sarıyer, 34450 Istanbul, Turkey
[c] Operations Research Center, Universidad Miguel Hernández, Elche, 03202 Alicante, Spain

ARTICLE INFO

Article history:
Received 31 October 2012
Accepted 16 June 2014
Available online 30 June 2014

Keywords:
Supply chain management
Game theory

ABSTRACT

Process improvement plays a significant role in reducing production costs over the life cycle of a product. We consider the role of process improvement in a decentralized assembly system in which a buyer purchases components from several first-tier suppliers. These components are assembled into a finished product, which is sold to the downstream market. The assembler faces a deterministic demand/production rate and the suppliers incur variable inventory costs and fixed setup production costs. In the first stage of the game, which is modeled as a non-cooperative game among suppliers, suppliers make investments in process improvement activities to reduce the fixed production costs. Upon establishing a relationship with the suppliers, the assembler establishes a knowledge sharing network – this network is implemented as a series of meetings among suppliers and also mutual visits to their factories. These meetings facilitate the exchange of best practices among suppliers with the expectation that suppliers will achieve reductions in their production costs from the experiences learned through knowledge sharing. We model this knowledge exchange as a cooperative game among suppliers in which, as a result of cooperation, all suppliers achieve reductions in their fixed costs. In the non-cooperative game, the suppliers anticipate the cost allocation that results from the cooperative game in the second stage by incorporating the effect of knowledge sharing in their cost functions. Based on this model, we investigate the benefits and challenges associated with establishing a knowledge sharing network. We identify and compare various cost allocation mechanisms that are feasible in the cooperative game and show that the system optimal investment levels can be achieved only when the most efficient supplier receives the incremental benefits of the cost reduction achieved by other suppliers due to the knowledge transfer.

1. Introduction

This paper considers knowledge transfer in an assembly system composed of one assembler and multiple component suppliers. Knowledge sharing networks have been proposed as a mechanism to increase learning in organizations. Dyer and Nobeoka (2000) describe how Toyota helps its suppliers learn faster by creating knowledge sharing networks. Toyota implements a knowledge sharing network with the objective of creating a strong network identity, motivating the suppliers to engage in knowledge sharing and process improvement, and preventing free riders. One approach followed by this car manufacturer is to organize meetings with its suppliers and share knowledge about best practices with them. Toyota does not charge for these seminars, but the company expects suppliers to reduce their wholesale prices to the company once they achieve lower production costs (so the gains are eventually shared). Another method for sharing knowledge consists of having groups of suppliers visit the facilities of one of the suppliers in the group and learn from this supplier's best practices and make suggestions for improvement. These activities create a collaborative environment to foster multi-directional knowledge transfer. These groups typically consists of networks of 5–8 suppliers.

We model the assembly network as follows. The assembler faces a deterministic production rate and sources components from a set of suppliers. Suppliers' components are complementary and the assembler requires parts from all suppliers for the final product (i.e., the assembler's parts are single-sourced[1]). Each supplier incurs inventory holding costs and a fixed production cost.

* Corresponding author. Tel.: +34 966658717.
E-mail addresses: fernando@duke.edu (F. Bernstein), gkok@ku.edu.tr (A. Gürhan Kök), ana.meca@umh.es (A. Meca).

[1] This is generally consistent with the practice of several companies, including Toyota.

http://dx.doi.org/10.1016/j.ejor.2014.06.013
0377-2217/

For a given set of cost and demand rates, each supplier determines its optimal production lot size. We model the sequence of decisions as a two-stage bi-form game (Brandenburger & Stuart, 2007). In the first stage, suppliers invest in process improvement activities, which have a direct impact on their fixed production costs. Investments in cost reduction activities are costly and this cost increases at an increasing marginal rate. We model the first stage as a non-cooperative game, in which suppliers determine the amount of investment in effort to reduce their fixed costs. In the second stage, the assembler establishes a knowledge sharing network to enable transfer of knowledge related to process improvement among suppliers. We model knowledge exchange as a cooperative game among suppliers, in which, as a result of cooperation, all suppliers achieve a level of fixed cost reduction equal to that of the most efficient supplier(s), i.e., the one(s) with the lowest fixed cost.[2] In the non-cooperative game that precedes knowledge sharing, the suppliers anticipate the cost allocation that results from the cooperative game in the second stage by incorporating the effect of knowledge sharing in their cost functions. Based on this model, we investigate the costs, benefits and challenges associated with establishing a knowledge sharing network.

We model process improvement by considering reductions in the fixed costs (similar to Porteus, 1985). All suppliers incur an ex-ante fixed cost associated with their production activities. The fixed cost may correspond, for example, to the time spent and the cost incurred to carry out set-ups for production. Set-up times and costs are usually listed as one of the seven forms of waste (muda) in Lean Production System principles, and are commonly the target of any process improvement initiatives under the umbrella of Lean and Six Sigma programs. Suppliers can individually reduce their fixed costs by investing in cost reduction initiatives. We model the cost of such investments as a direct function of the target fixed cost each supplier desires to achieve. We must note that process improvement may also be due to learning-by-doing, which suggests that costs decrease as a function of the total production volume. Hatch and Mowery (1998) show that learning in the early stages of manufacturing is a function of the engineering resources allocated rather than the production volume in their empirical analysis of the semiconductor industry. Papers studying other aspects of cost-reduction initiatives include Mekler (1993) and Denizel, Ereng, and Benson (1997).

We investigate the impact of knowledge sharing on the investment process and the resulting cost structure for the assembly system. As a result of knowledge sharing, all suppliers indirectly benefit from the process improvement initiatives instituted by the most efficient suppliers in the network. In particular, we explore the design of a cost-allocation mechanism that efficiently assigns the gains from knowledge transfer to all parties. To that end, we first compute the system optimal level of cost reduction, taking into account the benefits from knowledge sharing that are collectively accrued by all suppliers. An ideal allocation scheme should encourage participation of the suppliers and, at the same time, establish proper incentives for investments in the cost reduction activities prior to knowledge sharing. Specifically, we characterize (a) the conditions under which all suppliers participate in a knowledge sharing network and (b) how the total cost should be allocated to the members of this network (through the creation of the appropriate transfer payments) to achieve a level of cost reduction commensurate with that which arises in the centralized system. To this end, we model the transfer of knowledge among suppliers that takes place in the second stage as a cooperative game, which we refer to as the knowledge sharing game, or KS-game. We prove that KS-games are totally balanced – i.e., the core of every subgame is non-empty. We interpret a non-empty core as a setting in which knowledge sharing is feasible, in the sense that there are possible cost transfers that make all suppliers better off by cooperating (i.e., by sharing knowledge). In its implementation, each element of the core corresponds to a certain cost allocation mechanism among suppliers in the network. We identify various allocation mechanisms that may arise in the core of KS-games. In particular, we discuss two important classes of allocation mechanisms. A so-called *altruistic* allocation allows the non-efficient suppliers to keep the entire benefit of cost reduction achieved due to learning from the efficient supplier. The so-called *tute* allocation mechanism benefits a supplier by transferring the incremental benefit generated by the inclusion of an efficient supplier into the network. We find that the system optimal level of cost reduction is achieved in an extreme point of the core and, specifically, under the "tute" allocation rule. In other words, the "altruistic" allocation never achieves the system optimal minimum cost, even though it is always in the core of the KS-games. This suggests that the assembler needs to carefully design a system of transfer payments that rewards suppliers for their efforts to reduce cost. The "altruistic" mechanism – based on the suppliers' willingness to engage in knowledge sharing activities without a proper accompanying reward – will result in a higher cost structure for the entire assembly system.

Various aspects of cost reduction have been studied in the context of decentralized assembly networks and production systems. Gupta and Loulou (1998) study vertical integration decisions under endogenous cost reduction. Gilbert and Cvsa (2003) study outsourcing decisions in a similar setting. Heese and Swaminathan (2006) examine the impact of cost reduction on a product line with component commonality. Chang, Ouyang, Wu, and Ho (2006) study lead time and ordering cost reduction in a single-vendor single-buyer setting. Bernstein and Kök (2009) study dynamic supplier cost reduction decisions and compare alternative contract structures employed by assemblers. Kim and Netessine (2013) study incentives for collaborative cost reduction in supply chains with a single supplier. Iida (2012) studies cost-reduction coordinating contracts in a supply chain with a single manufacturer and multiple suppliers. These papers employ a non-cooperative game framework to study cost reduction in production systems. On the other hand, papers that study cooperative cost allocation games in decentralized networks include Guardiola, Meca, and Puerto (2008, 2009), Meca, García Jurado, and Borm (2003), Meca, Timmer, García Jurado, and Borm (2004) and Meca (2007). Meca and Sosic (2014) introduce a class of cost-coalitional problems based on a priori information about the cost faced by each agent in each group that it could belong to. They study the effects of giving and receiving when there are players whose participation in an alliance always contributes to the savings of all alliance members (benefactors), and there also exist players whose cost decrease in such an alliance (beneficiaries). This stream of papers takes the cost structure as given and does not consider the system externalities that arise when firms invest in cost reduction. The present paper incorporates both the non-cooperative aspects of cost reduction by modeling decisions related to process improvement and the cooperative nature of knowledge sharing networks in assembly systems. Papers using cooperative game theory to study decentralized networks are scarce. Nagarajan and Sosic (2008) review and extend the problem of bargaining and negotiations in supply chain relationships. Recent surveys of applications of cooperative game theory to supply chain management include Meca and Timmer (2008),Fiestras-Janeiro, García-Jurado, Meca, and Mosquera (2011), and Dror and Hartman (2011). For theoretical issues and a framework for more general supply chain networks we refer to the book by Slikker and van den Nouweland (2001).

[2] In Section 5 we discuss other functions describing the change in cost structure as a result of knowledge sharing.

2. Model

We consider an assembly system consisting of an assembler that procures components from n suppliers from the set $N = \{1, 2, \ldots, n\}$. Suppliers produce different, complementary components.[3] Each unit of the finished product requires, without loss of generality, one unit of the component produced by each supplier. Production of the final product is deterministic and occurs at a rate $\lambda > 0$. Because all suppliers produce components used in the end product, they are subject to the same demand level. Supplier i's holding cost per item per unit time is $h_i > 0$, its initial (ex-ante) setup production cost is $\overline{K}_i > 0$, and the fixed setup cost it incurs after investment in process improvement – a decision of supplier i – is K_i.

The sequence of events is as follows: (1) Each supplier decides its level of investment in process improvement activities – as a result, its fixed setup cost is reduced correspondingly; (2) suppliers participate in knowledge-sharing activities; (3) each supplier in a knowledge-sharing group adopts the best-practices shared within the group, reducing its cost to the level of the most efficient supplier in its group; and (4) suppliers produce and ship to the assembler, incurring the fixed setup cost that results from their investment in process improvement in the first stage and the learnings achieved through knowledge-sharing in the second stage.

In this paper, we assume perfect information regarding the suppliers' fixed costs. This assumption is, in fact, consistent with the practice in many knowledge sharing networks. For example, according to Dyer and Nobeoka (2000), one of the conditions to participate in the learning network at Toyota is "to openly share valuable knowledge with other network members." Further, this article goes on to say that "suppliers are motivated to participate [and openly share knowledge] because they quickly learn that participating in the collective learning processes is vastly superior to trying to isolate their proprietary knowledge." Moreover, many of the activities surrounding the process of knowledge sharing involve visits to suppliers' plants, so information is naturally revealed during those visits. In general, the repercussions of misreporting information in a knowledge-sharing network may be very costly, such as losing the assembler's business or not benefiting from future knowledge-sharing activities. Thus, truth telling is ensured as part of the long-term relationship.

In the first stage, suppliers may invest in process improvement to reduce their fixed cost down from $\overline{K}_i$ to any $0 < K_i \leqslant \overline{K}_i$. Suppliers follow the economic order quantity (EOQ) – see, e.g., Chapter 3 in Zipkin (2000) – to determine their cyclic production levels to fulfill production requests and minimize costs. Supplier i's production batch size is denoted by Q_i. The optimal batch size is $Q_i^* = \sqrt{\frac{2\lambda K_i}{h_i}}$ and the corresponding minimum cost is $C^i(Q_i^*) = \sqrt{2\lambda h_i K_i}$. Each supplier's initial production cost is given by $\sqrt{2\lambda h_i \overline{K}_i}$. In this stage, each supplier chooses the level of process improvement to minimize its total cost in anticipation of the result of the KS-game that follows in the second stage. Supplier i incurs a cost $g_i(K_i|\overline{K}_i)$ by investing to reduce its fixed cost to an amount K_i, with $g_i(\cdot|\overline{K}_i) : (0, \overline{K}_i] \to \Re^+$. We assume that $g_i(\cdot|\overline{K}_i)$ is continuous, decreasing, and convex, with $g_i(\overline{K}_i|\overline{K}_i) = 0$ and $\lim_{K_i \to 0^+} g_i(K_i|\overline{K}_i) = \infty$. This means that reducing costs is increasingly costly. Given a vector $\mathbf{K} = (K_1, \ldots, K_n)$, we let $x_i(\mathbf{K})$ denote supplier i's final production cost, which results from the allocation of total cost achieved through knowledge sharing activities in the KS-game. Thus, the first-stage cost for supplier i is given by

$$\pi_i(\mathbf{K}) = g_i(K_i|\overline{K}_i) + x_i(\mathbf{K}).$$

We will focus attention on functions $g_i(\cdot|\overline{K}_i)$ such that the resulting cost function $\pi_i(\mathbf{K})$ is (strictly) convex–concave in K_i. Formally, $\pi_i(\mathbf{K})$ is (strictly) convex–concave in K_i on the interval $(0, \overline{K}_i]$ if there exists $r \in (0, \overline{K}_i]$ such that $\pi_i(\mathbf{K})$ is (strictly) convex in K_i on the interval $(0, r]$ and (strictly) concave in K_i on the interval $(r, \overline{K}_i]$ (see Porteus, 1985). Various investment functions $g_i(\cdot|\overline{K}_i)$ will satisfy this condition – for example, $g_i(K_i|\overline{K}_i) = b \ln(\overline{K}_i/K_i)$. We refer to Porteus (1985) for further examples of investment functions that lead to strictly convex–concave cost functions in this setting.

For simplicity, we hereafter normalize all cost functions by setting $2\lambda = 1$.

3. Knowledge-sharing cooperative game

This section presents the analysis of the second stage KS-game. Suppliers make their investment in cost reduction decisions in the first stage and initiate knowledge sharing activities with fixed costs $(K_1, \ldots, K_n)$. We model the KS-game as a multiple-agent cooperative game where each supplier (agent) faces an EOQ inventory problem. All suppliers in a knowledge sharing group (coalition) learn from the most efficient supplier(s) in the coalition (i.e., the supplier(s) with the lowest fixed cost). As a result, all suppliers in the coalition reduce their fixed costs to a level that equals that of the most efficient supplier. Any supplier outside the knowledge sharing network does not benefit from this cost-reduction learning opportunity. Although we introduce all the game-theoretic concepts we use in this paper, we refer to González-Díaz, García-Jurado, and Fiestras-Janeiro (2010) for more details on cooperative and non-cooperative games.

We refer to the knowledge sharing network as a KS-network and denote it with $(N, \{h_i, K_i\}_{i \in N})$. For every coalition $S \subset N$, the optimal production batch size for supplier $i \in S$ is $Q_i^S = \sqrt{\frac{K_S}{h_i}}$, where $K_S := \min_{i \in S}\{K_i\}$, and the corresponding optimal aggregate cost is $\sum_{i \in S} \sqrt{h_i K_S}$.

We associate a cost game to every KS-network. A cost game is a pair (N, c), where N is the finite set of agents and $c : 2^N \longrightarrow \mathbb{R}$ is the so-called characteristic function of the game, which assigns to each subset $S \subset N$ the cost $c(S)$ that is incurred if agents in S cooperate. By convention, $c(\emptyset) = 0$. The *cost of supplier i in coalition* $S \subseteq N$ is given by $c^S(\{i\}) = \sqrt{h_i K_S}$. (This cost can be interpreted as supplier i's cost after participating in a KS-network together with the suppliers in S.) Thus, in a KS-network $(N, \{h_i, K_i\}_{i \in N})$, the cost function is given by

$$c(S) := \sum_{i \in S} c^S(\{i\}) = \sum_{i \in S} \sqrt{h_i K_S}.$$

The class of KS-games has some similarities with the class of generalized holding cost games introduced by Meca (2007) and also with the class of inventory games studied in Meca (2000).

A cost game is said to be subadditive if it is never beneficial for a coalition to split into several smaller disjointed coalitions. Formally, for each $S, T \subset N$ such that $S \cap T = \emptyset$, it holds that $c(S) + c(T) \geqslant c(S \cup T)$. It is clear that KS-games are subadditive. This implies that the system cost is minimized if all suppliers participate in a knowledge sharing coalition. We next study how the total cost $c(N)$ is allocated to the suppliers. To that end, we define the core of the KS-game as the set

$$C(N, c) = \left\{ x \in \mathbb{R}^N \Big| \sum_{i \in N} x_i = c(N), \sum_{i \in S} x_i \leqslant c(S) \text{ for each } S \subset N \right\}.$$

The cost allocations in the core are those that are efficient (i.e., $\sum_{i \in N} x_i = c(N)$) and for which no group of suppliers has an incentive to deviate and form a separate coalition. In other words, if an

[3] According to Dyer and Nobeoka (2000), "direct competitors are not in the same [knowledge sharing] group."

allocation vector x belongs to the core and a group S forms a separate coalition, then the cost $c(S)$ incurred by this coalition is greater than or equal to the cost $\sum_{i\in S} x_i$ allocated to the members of the coalition. Cost allocations in the core satisfy a set of coalitional stability conditions and therefore they are usually called stable cost allocations. An element of the core identifies an allocation of total cost among the suppliers under which the grand coalition N is stable.

The core is a convex and compact polyhedron in $\Re^n$. For any vector x in the core, each supplier i's allocation must satisfy $c(N) - c(N\setminus\{i\}) \leqslant x_i \leqslant c(\{i\})$ – the second inequality follows by taking $S = \{i\}$ in the definition in the core, while the first inequality follows from the efficiency condition and by taking $S = N\setminus\{i\}$ in the definition of the core. That is, no supplier incurs a cost higher than its initial cost before cooperation and the lowest cost a supplier may incur after cooperation equals its marginal contribution to the grand coalition. If the core is non-empty then, because it is a polyhedron, it has a finite number of extreme points (and any point in the core can be described as a convex combination of its extreme points).

The following result shows that KS-games have a non-empty core (i.e., KS-games are balanced). Actually, KS-games are totally balanced (i.e., the core of every subgame is non-empty). We interpret a non-empty core as corresponding to a setting where knowledge sharing is feasible, in the sense that there are possible cost transfers that make all suppliers better off (or, at least, not worse off) by sharing knowledge. The totally balanced property suggests that knowledge sharing is consistent, i.e., for every group of suppliers knowledge sharing is always feasible.

Theorem 1. *Consider a KS-game (N, c). The core $C(N, c)$ is non-empty. Moreover, every KS-game (N, c) is totally balanced.*

To understand the logic behind this result, we denote by $\Pi(N)$ the set of all orderings in N. Formally, every $\sigma \in \Pi(N)$ is a one-to-one map which associates to every element of N a natural number in $\{1, 2, \ldots, n\}$. For every $i \in N$, the set of predecessors of i with respect to $\sigma \in \Pi(N)$ is $P_i^\sigma = \{j \in N \mid \sigma(j) < \sigma(i)\}$. The marginal vector associated with $\sigma \in \Pi(N)$ is defined as $m^\sigma(N, c) = (m^{\sigma_i}(N, c))_{i\in N}$, where $m^{\sigma_i}(N, c) = c(P_i^\sigma \cup \{i\}) - c(P_i^\sigma)$ for each $i \in N$. Notice that for every marginal vector m^σ, we have that $\sum_{i\in N} m_i^\sigma (N, c) = c(N)$. Hence, a marginal vector of (N, c) is an allocation of $c(N)$ which allocates to every supplier i its contribution to its predecessors according to a particular ordering. Drieseen (1988) shows that if a marginal vector is in the core, then it is an extreme point of the core. The proof of Theorem 1 is based on the fact that there exists a marginal vector that belongs to the core.

Consider a KS-game (N, c) associated to a KS-network $(N, \{h_i, K_i\}_{i\in N})$. We say that supplier $i \in N$ is efficient if $K_i = K_N$, i.e., if its setup production cost is smaller than or equal to the setup production cost of all the other suppliers in the system. We denote by $E(N, \mathbf{K})$ the set of all efficient suppliers and introduce the following definitions:

- A multiple KS-game is characterized by $|E(N, \mathbf{K})| \geqslant 2$. That is, there exists $m \geqslant 2$ such that $K_{i_1} = K_{i_2} = \cdots = K_{i_m} = K_N$, for $i_1 \neq i_2 \neq \cdots \neq i_m$.
- A simple KS-game is characterized by $|E(N, \mathbf{K})| = 1$. In such a case, there exists a supplier $j \in N$ such that $K_j < K_i$ for all $i \neq j$.

We later show that efficient suppliers play an important role in the characterization of the core.

We define the *altruistic* cost allocation as $x_i^a = c^N(\{i\})$, for all $i \in N$. As suggested by its name, this allocation is based on an *altruistic* behavior on the part of the suppliers, in the sense that every efficient supplier pays its own individual cost, while the other (inefficient) suppliers pay their corresponding reduced costs achieved from knowledge sharing in the grand coalition. Hence, inefficient suppliers keep all the benefit associated with cost reduction (based on knowledge transfer from the efficient suppliers) to themselves. The efficient suppliers do not see any benefit by sharing their knowledge with the network. Notice that this allocation coincides with the rule that allocates the value of the grand coalition proportional to the cost of each player in the grand coalition. One can show that the altruistic allocation is an extreme point of the core of the KS-game. (In a setting with $K_1 \leqslant K_2 \leqslant \cdots \leqslant K_n$, the altruistic allocation corresponds to setting $\sigma(i) = i$, for $i = 1, \ldots, n$.)

To further characterize the core of a KS-game, we now define the cost coalitional distance for supplier $i \in N$ as a function $d_i : 2^N \times 2^N \to \mathbb{R}_+$ such that

$$d_i(S, T) = \begin{cases} c^S(\{i\}) - c^T(\{i\}) & \text{if } S \subset T, i \in S \\ 0 & \text{otherwise.} \end{cases}$$

This distance satisfies the following properties: (1) Non-negativity, i.e., $\forall i \in N$ and $\forall S, T \subset N,\ d_i(S, T) \geqslant 0$; (2) Additivity, i.e., $\forall S \subset T \subset K \subset N$ and $\forall i \in S,\ d_i(S, T) + d_i(T, K) = d_i(S, K)$; (3) Monotonicity, i.e., $\forall S \subset T \subset N$ and $\forall i \in S,\ d_i(T, N) \leqslant d_i(S, N)$. Given a set of agents N and a vector of costs $\mathbf{K}$, we define a set of allocations of the KS-game, $D(N, \mathbf{K})$, obtained by making monetary transfers from non-efficient suppliers to an efficient supplier j. That is, letting $j \in N$ be an efficient supplier with respect to the vector $\mathbf{K}$, we define $D(N, \mathbf{K}) = \{x \in \mathbb{R}^n :$ for a given supplier $j \in E(N, \mathbf{K})$, there exists t in $\mathbb{R}^n$ such that $x_j = c(\{j\}) - \sum_{i\in N\setminus\{j\}} t_i,\ x_i = c^N(\{i\}) + t_i, t_i \geqslant 0, \forall i \in N\setminus\{j\};\ \sum_{i\in S} t_i \leqslant \sum_{i\in S} d_i(S, N),\ \forall S \subset N\setminus\{j\}\}$. The set $D(N, \mathbf{K})$ is non-empty – the altruistic cost allocation always belongs to $D(N, \mathbf{K})$. The elements of $D(N, \mathbf{K})$ represent different ways of allocating cost in the grand coalition. Using the altruistic allocation as the starting point, one can increase or decrease allocations to specific suppliers as long as the allocations remain efficient and do not increase the distance between the grand coalition and any of its subsets that exclude supplier j. The next result shows that the core $C(N, c)$ coincides with $D(N, \mathbf{K})$.[4]

Proposition 1. *The core of a KS-game is given by $C(N, c) = D(N, \mathbf{K})$.*

This characterization suggests a relatively simple way of computing the elements of the core. We will use Proposition 1 in the analysis of the core of the KS-game. The next result shows that the core of a multiple KS-game reduces to the altruistic cost allocation.

Theorem 2. *The core of a multiple KS-game is $C(N, c) = \{x^a\}$.*

As discussed above, the altruistic cost allocation is always an extreme point of the core of a KS-game. In a multiple KS-game, the efficient suppliers compete away their cost allocation to establish their position as efficient suppliers in a way that leaves them with no additional cost savings beyond those achieved by their investments in the first stage. This is not the case in a simple KS-game: the altruistic cost allocation can be modified in such a way that the cost of every non-efficient supplier increases by $t_i \geqslant 0$, and the unique efficient supplier benefits from a cost reduction equal to the sum of those amounts. That is, a non-efficient supplier i incurs the cost $c^N(\{i\}) + t_i$ and an efficient supplier j incurs the cost $c(\{j\}) - \sum_{i\in N\setminus\{j\}} t_i$. Setting $t_i = c^{N\setminus\{j\}}(\{i\}) - c^N(\{i\}) = d_i(N\setminus\{j\}, N)$ in a simple KS-game leads to another extreme point of the core, which we refer to as the "tute" cost allocation.[5] Under

[4] All proofs are relegated to the appendix.

[5] Tute is a Spanish card game of Italian origin. In the variant most widely played in Argentina and Uruguay (Tute Cabrero), players collect points and the players with the most and least points win the round, while the second player with the most points loses the round. See http://es.wikipedia.org/wiki/Tute_cabrero.

the tute cost allocation, the allocation to an efficient supplier j is $x_j^t = c(N) - c(N\backslash\{j\})$ and the allocation to a non-efficient supplier i is $x_i^t = c^{N\backslash\{j\}}(\{i\})$. (In a setting with $K_1 \leqslant K_2 \leqslant \cdots \leqslant K_n$, the tute cost allocation is associated to the order under which $\sigma(1) = n$, $\sigma(2) = 1, \sigma(i) = i - 1, i = 3, \ldots, n$.) For example, for a three-player network with $K_1 < K_2 < K_3$, under tute allocation, supplier 2's final cost remains equal to that before knowledge sharing (i.e., as if its fixed cost remained at K_2), supplier 3 benefits because its final cost is equal to K_2, and supplier 1 gets all the remaining benefit from reducing suppliers 2 and 3's fixed costs down to K_1.

The next two examples illustrate the altruistic and tute core allocations, and the result in Theorem 2. Moreover, both examples show that the Shapley value for a KS-network may be not a stable cost allocation.

Example 1. Consider a KS-network with $N = \{1,2,3\}, \lambda = 1/2$, $h_i = 1$, and $K_1 = 1/4, K_2 = 1$, and $K_3 = 25$. There is only one efficient supplier, $E(N, \mathbf{K}) = \{1\}$, and the associated KS-game is defined by $c(\emptyset) = 0, c(\{1\}) = 1/2, c(\{2\}) = 1, c(\{3\}) = 5, c(\{1,2\}) = 1$, $c(\{1,3\}) = 1, c(\{2,3\}) = 2$, and $c(N) = 3/2$. The altruistic allocation in this game is $x^a = (1/2, 1/2, 1/2) \in C(N,c)$. The tute cost allocation is given by $x^t = (-1/2, 1, 1) \in C(N,c)$. Under this allocation, supplier 2 is the only one that does not benefit from knowledge sharing. Supplier 2 incurs the cost $c(\{2\})$, while supplier 1 pays its minimum cost $c(N) - c(N\backslash 1)$. The Shapley value is $\Phi(N,c) = (-2/3, 1/12, 25/12) \notin C(N,c)$, since the core condition fails for coalition $S = \{1,3\}$.

Notice that the above KS-network has only one efficient supplier and the core of the associated KS-game involves multiple stable cost allocations, because all the inefficient suppliers are willing to learn from the single efficient supplier. However, the next example shows quite a different behavior for the case of a multiple KS-game. In this example, the KS-network has two efficient suppliers and the core of the associated KS-game reduces to a single point – the altruistic cost allocation – as demonstrated in Theorem 2.

Example 2. Consider a KS-network with $N = \{1,2,3\}, \lambda = 1/2, h_i = 1$, and $K_1 = 1, K_2 = 1$, and $K_3 = 9$. In this example, there are two efficient agents, $E(N, \mathbf{K}) = \{1,2\}$, and the associated KS-game is defined by $c(\emptyset) = 0, c(\{1\}) = 1, c(\{2\}) = 1, c(\{3\}) = 3, c(\{1,2\}) = 2$, $c(\{1,3\}) = 2, c(\{2,3\}) = 2$, and $c(N) = 3$. The *altruistic* allocation in this game is $x^a = (1,1,1) \in C(N,c)$. Moreover, $C(N,c) = \{(1,1,1)\}$. Again, the Shapley lies outside the core since $\Phi(N,c) = (2/3, 2/3, 5/3)$.

4. Analysis of the first-stage investment game

In this section, we analyze the non-cooperative game that arises in the first stage. Suppliers decide the extent to which they invest in reduction of their fixed costs in anticipation of the allocation of cost savings that results from the second stage KS-game. To simplify notation and analysis, we focus on the case of symmetric suppliers (in Section 5 we comment on how results might change for the case of asymmetric suppliers). In settings with symmetric suppliers, all holding and initial fixed costs are equal across suppliers, i.e., $h_1 = \cdots = h_N = h, \overline{K}_1 = \cdots = \overline{K}_N = \overline{K}$ and, for a given individual fixed cost value $K, g_1(K, \overline{K}) = \cdots = g_N(K, \overline{K}) = g(K, \overline{K})$. We first study the centralized system.

4.1. Centralized system

Before proceeding to the analysis of the equilibrium of the first-stage game, we characterize the first-best or centralized solution, denoted with a "$*$". In this setting, we assume that all suppliers jointly select their investment levels in the first stage to minimize their total cost, in anticipation of the ensuing knowledge-sharing activities. As a result of cooperation in the second stage, each supplier i will be allocated a portion of the cost $x_i(\mathbf{K})$, with the vector $x(\mathbf{K}) = (x_1(\mathbf{K}), \cdots, x_n(\mathbf{K})) \in C(N,c)$. (The allocation of cost in the second stage depends on the realization of fixed cost investments in the first stage.) Supplier i's total cost in the first stage is given by $\pi_i(\mathbf{K}) = x_i(\mathbf{K}) + g(K_i|\overline{K})$. The total system cost in the first stage is therefore given by

$$\sum_{i=1}^{n}\pi_i(\mathbf{K}) = \sum_{i=1}^{n}x_i(\mathbf{K}) + \sum_{i=1}^{n}g(K_i|\overline{K}) = c(N) + \sum_{i=1}^{n}g(K_i|\overline{K})$$
$$= n\sqrt{hK_N} + \sum_{i=1}^{n}g(K_i|\overline{K}),$$

where the second equality follows because x_i are elements of the core. Let us define K_n^* to be the solution to

$$\frac{\partial}{\partial K}\left[n\sqrt{hK}\right] + g'(K|\overline{K}) = 0. \tag{1}$$

The optimal level of fixed cost reduction depends on the number of suppliers n in the system – we highlight this dependence by using a subindex n. If one supplier invests to reduce its fixed cost to the level K_n^*, then it is optimal to set the fixed costs of all other suppliers at $\overline{K}$ because $g'(K_n^*|\overline{K}) \leqslant 0$. That is, the first-best solution is given by a vector of fixed costs in which one supplier invests to decrease its fixed cost up to the level K_n^*, while none of the other suppliers makes any investment, i.e., their fixed costs remain at $\overline{K}$. (There are n such vectors that minimize the system-wide cost.) As a result of knowledge sharing, all suppliers reduce their fixed costs up to the level K_n^* in the centralized system.

Next, we present our analysis of the non-cooperative game.

4.2. Equilibrium analysis

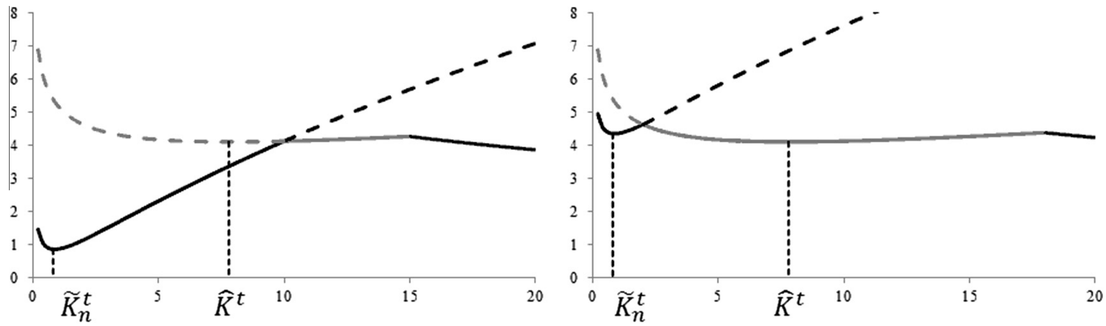
We begin our study of the first-stage non-cooperative game by formally defining the elements of the game. The set of players (suppliers) is $N = \{1, \ldots, n\}$ with cost functions $\pi_i(\mathbf{K}) = x_i(\mathbf{K}) + g(K_i|\overline{K}), i \in N$, where $x_i(\mathbf{K})$ is the cost allocation that arises from the second-stage cooperative game. Each supplier's feasible action space is $(0, \overline{K}]$. We assume perfect information (see Section 2 for a discussion regarding this assumption) and consider equilibria in pure strategies. We first establish an important result which states that the largest cost reduction (which corresponds to the lowest attainable fixed cost) that can be achieved in an equilibrium of the first-stage game arises from an extreme allocation of the core of the associated second-stage cooperative game.

Theorem 3. *The lowest equilibrium fixed cost that arises in the first-stage investment game can always be achieved from an extreme allocation of the core of the second-stage KS-game.*

Because our goal is to demonstrate that there exists a core allocation that induces a level of investment equal to that which arises in the centralized system, Theorem 3 suggests that it is sufficient to focus on the extreme allocations of the core. As discussed in Section 3, the *altruistic* and *tute* allocations are always extreme points of the core. We therefore first study the non-cooperative game that arises under each one of these two extreme core allocations.

4.2.1. Altruistic allocation

For a given vector of fixed costs $\mathbf{K}$, the *altruistic* allocation rule is given by $x_i^a(\mathbf{K}) = c^N(\{i\}) = \sqrt{K_N h}$. Under the altruistic allocation, the efficient supplier gives away knowledge for free – all benefits from knowledge-sharing activities are gained by the inefficient suppliers. Based on the altruistic allocation rule and for given fixed-cost investment decisions of the other suppliers, supplier i's cost in the first stage is given by



(a) $K_{j1} \geq t_2$ (b) $K_{j1} \leq t_1$ and $K_{j2} \geq t_3$

Fig. 1. Supplier i's cost in the first-stage game as a function of K_i, for fixed $K_{j1} \leqslant K_{j2}$.

$$\pi_i^a(\mathbf{K}) = \begin{cases} \sqrt{hK_i} + g(K_i|\overline{K}), & K_i \leqslant K_{N\setminus\{i\}} \\ \sqrt{hK_{N\setminus\{i\}}} + g(K_i|\overline{K}), & K_i > K_{N\setminus\{i\}}. \end{cases} \tag{2}$$

(We use the superscript a to denote the functions and relevant equilibrium solutions under the altruistic allocation.) We define $\tilde{\pi}^a(K) = \sqrt{Kh} + g(K|\overline{K})$ and $\hat{\pi}_i^a(\mathbf{K}) = \sqrt{K_{N\setminus\{i\}}h} + g(K_i|\overline{K})$. We hereafter use the "$\sim$" symbol to denote a supplier's cost function (and associated optimal decision) when it is the efficient supplier in the system (i.e., when its fixed cost is smaller than that of the other suppliers), and the symbol "$\wedge$" otherwise. In particular, $\tilde{\pi}^a$ is the relevant cost function for supplier i if it is the efficient supplier in the system, and $\hat{\pi}_i^a$ otherwise.

Let $\widetilde{K}^a$ be the minimum of $\tilde{\pi}^a(\cdot)$ over $(0, \overline{K}]$. The existence of $\widetilde{K}^a$ is guaranteed because $\tilde{\pi}^a(\cdot)$ is continuous and $\lim_{K \to 0^+} \tilde{\pi}^a(K) = \infty$. The cost function $\tilde{\pi}^a$ is strictly convex–concave with a unique local minimum (see Porteus, 1985). The capacity value $\widetilde{K}^a$ is the optimal cost for a supplier i if that is the efficient (lowest-cost) supplier in the ensuing KS-game. The function $\hat{\pi}_i^a$ is decreasing in K_i and it is therefore minimized at $\overline{K}$ for any vector of fixed costs of the other suppliers K_{-i}. (Note that the function $\pi_i^a(\mathbf{K})$ is continuous.) We next derive the best-response correspondence for each supplier.

Lemma 1. *The best-response correspondence of supplier i in stage 1 is given by*

$$BR_i(K_{-i}) = BR_i(K_{N\setminus\{i\}}) = \begin{cases} \widetilde{K}^a, & K_{N\setminus\{i\}} \geqslant t, \\ \overline{K}, & K_{N\setminus\{i\}} \leqslant t, \end{cases}$$

where the threshold t is defined as the unique value that satisfies $\tilde{\pi}^a\left(\widetilde{K}^a\right) = \sqrt{ht}$.

The best-response correspondence for supplier i takes a unique value on $(0, t) \cup (t, \overline{K})$, while $BR_i(t) = \{\widetilde{K}^a, \overline{K}\}$. Lemma 1 states that supplier i's best response to relatively low investments in cost reduction by the other firms (i.e., $K_{N\setminus\{i\}} \geqslant t$) is to reduce its fixed cost as far down as possible (while remaining cost-efficient) to the value $\widetilde{K}^a$. In contrast, when another supplier's own fixed cost is relatively low (implying a high investment in fixed cost reduction by that firm), supplier i does not invest in cost reduction and its cost remains at $\overline{K}$. Having established the suppliers' best-response correspondence, we now characterize the equilibria of the first-stage investment game.

Proposition 2. *There are n possible equilibria under the altruistic allocation. Under the i-th equilibrium, supplier i invests in cost reduction to achieve a level* $\widetilde{K}^a$ *in the first stage and none of the other suppliers invest (i.e., their fixed costs remain at* $\overline{K}$*). Moreover,* $\widetilde{K}^a > K_n^*$*, i.e., the altruistic allocation does not achieve the same level of investment as that which arises in the centralized system.*

Under the altruistic allocation, the efficient supplier gives away all benefits associated with knowledge sharing. As a result, only one supplier invests in equilibrium and that level of investment is not enough to achieve the level of system efficiency that arises under centralized control. We next turn attention to the *tute* allocation.

4.2.2. Tute allocation

Consider a given vector of fixed costs $\mathbf{K}$. Recall that K_N denotes the lowest fixed cost value in this vector and that $E(N, \mathbf{K})$ is the set of all efficient suppliers. If the vector $\mathbf{K}$ is such that multiple suppliers achieve the lowest fixed cost – i.e., $|E(N, \mathbf{K})| \geqslant 2$ – then, by Theorem 2, the only allocation in the core is the altruistic allocation. As a result, each supplier's cost function under such a vector is given as in (2). Suppose now that the vector $\mathbf{K}$ is such that there is a single supplier that achieves the lowest cost, i.e., $E(N, \mathbf{K}) = \{i\}$. Then, under the *tute* allocation, $x_i^t = c(N) - c(N\setminus\{i\}) = n\sqrt{hK_i} - (n-1)\sqrt{hK_{N\setminus\{i\}}}$ and $x_j^t = c^{N\setminus\{i\}}(\{j\}) = \sqrt{hK_{N\setminus\{i\}}}$ for all other suppliers $j \neq i$.

We now derive the cost function for an arbitrary supplier i and vector $\mathbf{K}$. Let $j \in E(N, \mathbf{K})$ be an arbitrary supplier within the set of efficient suppliers. Then,

$$\pi_i^t(\mathbf{K}) = \begin{cases} n\sqrt{hK_i} - (n-1)\sqrt{hK_{N\setminus\{i\}}} + g(K_i|\overline{K}), & \text{if } K_i \leqslant K_{N\setminus\{i\}} \\ \sqrt{hK_{N\setminus\{j\}}} + g(K_i|\overline{K}), & \text{if } K_i > K_{N\setminus\{i\}} \end{cases}$$

(We use the superscript t to denote the cost functions and relevant equilibrium solutions under the tute allocation.) Note that the cost functions are continuous and, when the vector $\mathbf{K}$ results in multiple efficient suppliers, the function coincides with the one associated to the altruistic allocation.

Let $\tilde{\pi}^t(K, \kappa) = n\sqrt{hK} - (n-1)\sqrt{h\kappa} + g(K|\overline{K})$ and let $\widetilde{K}_n^t$ be the largest minimum of $\tilde{\pi}^t(\cdot, \kappa)$ over $(0, \overline{K}]$. Note that $\widetilde{K}_n^t$ is independent of κ. The quantity $\widetilde{K}_n^t$ is the optimal fixed cost for a supplier i if that is the only efficient supplier in the system. On the other hand, let $\hat{\pi}(K) = \sqrt{hK} + g(K|\overline{K})$ and let $\widehat{K}^t$ be its minimum over $(0, \overline{K}]$. (Note that $\widehat{K}^t = \widetilde{K}^a$, the fixed cost achieved under the altruistic allocation.) The value $\widehat{K}^t$ is a given supplier i's optimal fixed cost if there are multiple efficient suppliers in the system, including supplier i, or if there is a single efficient supplier j and $K_i = K_{N\setminus\{j\}}$, i.e., supplier i incurs the second smallest fixed cost in the system. We note that $\widetilde{K}_n^t$ is decreasing in n and that $\widetilde{K}_n^t \leqslant \widehat{K}^t$ for all n. We next derive each supplier's best-response correspondence under the tute allocation.

Lemma 2. *Let* $i \in N$ *and let* K_{-i} *be the vector of fixed costs for all suppliers except i. Suppose that* $K_{j1} \leqslant K_{j2} \leqslant K_j$ *for all other* $j \neq i, j_1, j_2$*. Then, there exist unique thresholds* $t_1 < t_2 < t_3$ *such that supplier i's best response is given by*

$$BR_i(K_{-i}) = BR_i(K_{j1}, K_{j2}) = \begin{cases} \widetilde{K}^t_n, & \text{for } K_{j1} \in (t_1, t_2) \text{ and } K_{j2} \geqslant \overline{K}(K_{j1}) \\ \widetilde{K}^t_n, & \text{for } K_{j1} \geqslant t_2 \\ \widehat{K}^t, & \text{for } K_{j1} \leqslant t_1 \text{ and } K_{j2} \geqslant t_3 \\ \overline{K}, & \text{for } K_{j1} \leqslant t_1 \text{ and } K_{j2} \leqslant t_3 \\ \overline{K}, & \text{for } K_{j1} \in (t_1, t_2) \text{ and } K_{j2} \leqslant \overline{K}(K_{j1}) \end{cases}$$

As shown in Lemma 2, each supplier's best-response correspondence only depends on the fixed cost values of the two most efficient remaining suppliers. Fig. 1 illustrates the shape of a supplier's cost function. The graph shows the cost function of supplier i for different values of $K_{j1} \leqslant K_{j2}$. The solid curve in each graph corresponds to the function $\pi^t_i(\mathbf{K})$.

The best-response correspondence of any supplier results in one of three possible values for the equilibrium fixed cost: $\widetilde{K}^t_n, \widehat{K}^t$, or $\overline{K}$. As a result, the following are the relevant best-response values for an arbitrary supplier i (the best-response is indicated as a function of the two lowest fixed costs among the suppliers in $N \backslash \{i\}$):

$$BR_i(\overline{K}, \overline{K}) = \widetilde{K}^t_n;\ \ BR_i\left(\widehat{K}^t, \overline{K}\right) = \widetilde{K}^t_n;\ \ BR_i\left(\widetilde{K}^t_n, \overline{K}\right) = \widehat{K}^t;$$

$$BR_i\left(\widetilde{K}^t_n, \widehat{K}^t\right) = \overline{K};\ \ BR_i\left(\widetilde{K}^t_n, \widetilde{K}^t_n\right) = \overline{K};\ \ BR_i\left(\widehat{K}^t, \widehat{K}^t\right) = \begin{cases} \widetilde{K}^t_n, & \text{if } \widehat{K}^t \geqslant t_2 \\ \overline{K}, & \text{if } \widehat{K}^t < t_2, \end{cases}$$

where t_2 is the threshold identified in Lemma 2. Because $BR_i\left(\widehat{K}^t, \overline{K}\right) = \widetilde{K}^t_n$, $BR_i\left(\widetilde{K}^t_n, \overline{K}\right) = \widehat{K}^t$, and $BR_i\left(\widetilde{K}^t_n, \widehat{K}^t\right) = \overline{K}$, we have that all vectors in which $K_i = \widetilde{K}^t_n$ for one supplier $i, K_j = \widehat{K}^t$ for another supplier $j \neq i$, and $K_l = \overline{K}$ for all other suppliers $l \neq i, j$, are equilibria of the first-stage game. These are, in fact, all the possible equilibria of the two stage game under the tute allocation. We prove this in the following result.

Proposition 3. *There are $n(n-1)$ possible equilibria under the tute allocation. These equilibria include all vectors $\mathbf{K}$ in which $K_i = \widetilde{K}^t_n$ for one supplier i, $K_j = \widehat{K}^t$ for another supplier $j \neq i$, and $K_l = \overline{K}$ for all other suppliers $l \neq i, j$. Moreover, $\widetilde{K}^t_n = K^*_n$, i.e., the tute allocation induces the same level of cost reduction as in the centralized system.*

According to Proposition 3, in any equilibrium of the first-stage game under the tute allocation exactly two suppliers invest in process improvement. Let us examine this result. Under the tute allocation, the efficient supplier receives all the benefit from knowledge sharing – that is, its allocation is equal to its marginal contribution to the grand coalition. The remaining suppliers receive an allocation equal to the cost incurred by the second most efficient supplier, i.e., $x^t_j = \sqrt{hK_{N\backslash\{i\}}}$, where i is the efficient supplier. In equilibrium, it is optimal for that second most efficient supplier to invest in cost reduction up to $\widehat{K}^t$. To see this, we explore the suppliers' best response functions. If $n-2$ suppliers decide not to invest and one supplier invests a moderate amount in cost reduction, then the remaining supplier has all the incentive to invest as much as economically optimal to reduce its fixed cost. In that way, the latter supplier benefits substantially by receiving an allocation equal to its marginal contribution to the network. If two suppliers invest to reach fixed cost levels $\widetilde{K}^t_n$ and $\widehat{K}^t$, respectively, then each of the remaining $n-2$ suppliers has clearly no incentive to invest. The crucial (and more interesting) result involves the second most efficient supplier. Suppose indeed that $n-2$ suppliers do not invest in cost reduction and another supplier invests a substantial amount lowering its fixed cost down to $\widetilde{K}^t_n$. The remaining supplier will certainly not invest as much as the efficient supplier – in that case, there would be two efficient suppliers and the tute allocation reduces to the altruistic allocation, under which the two most efficient suppliers fail to reap any of benefits in the ensuing cooperative game. If the remaining supplier does not invest (like the other $n-2$ suppliers), then none of those $n-1$ suppliers will experience any gains in the cooperative game (recall that the allocation to those $n-1$ suppliers is given by $\sqrt{hK_{N\backslash\{i\}}}$, where i is the efficient supplier). Therefore, it is in the best interest of that remaining supplier to invest, even some moderate amount, in cost reduction. In that way, all other $n-1$ suppliers also benefit from knowledge sharing activities.

To conclude, the *tute* allocation rule induces an equilibrium of the two-stage game in which one supplier invests in process improvement to achieve a fixed cost equal to $\widetilde{K}^t_n$. This fixed cost is, in turn, replicated by all other suppliers as a result of the second-stage knowledge-sharing network. This level of cost reduction coincides with the optimal level achieved in the centralized system, although the equilibrium investment vector is not equal to the first-best solution. Indeed, in the equilibrium derived from the tute allocation, two suppliers invest in equilibrium (one achieving a fixed cost $\widetilde{K}^t_n$ and another one achieving a fixed cost $\widehat{K}^t$), while only one supplier invests in the centralized solution. Knowledge-sharing activities become more effective with a larger number of suppliers – as shown earlier, the larger the number of suppliers participating in knowledge-sharing activities, the higher the level of fixed cost reduction that results in an equilibrium under the tute allocation (and, also, under the centralized solution) since $\widetilde{K}^t_n$ is decreasing in n. This is because, under the tute allocation, the efficient supplier earns its marginal contribution to the grand coalition in the KS-game. As a result, the more suppliers in the system, the higher is the efficient supplier's contribution to the network (as more suppliers are able to reduce their fixed cost through the learning activities in the second stage). A similar dynamic applies in the centralized system.

4.3. Special cases: networks with two and three suppliers

We have so far examined the first-stage investment game under two extreme points of the KS-game, namely the altruistic and tute allocations. The number of extreme points for the general case of n suppliers can be exceedingly large. Our goal in this paper is to show that there exists a core allocation of the KS-game that induces the centralized optimal level of investment in cost reduction – we have shown that this is achieved under the tute allocation. We conclude this section by providing comments on the cases of $n = 2$ and $n = 3$ – in these cases, we can fully characterize all the extreme points of the core. To begin, for the case of two suppliers, the core of the second-stage cooperative game has two extreme points, the altruistic and the tute core allocations. From the result in Theorem 3 and the preceding analysis of the altruistic and tute allocations, we conclude that for the case of $n = 2$, the centralized optimal level of investment is only achieved under the tute allocation. We next explore the case of three suppliers. The following result provides a characterization of the core of the KS-game for the case of three suppliers.

Lemma 3. *Let (N, c) be a KS-game with $N = \{1, 2, 3\}$ and suppose, without loss of generality, that $K_1 < K_2 \leqslant K_3$. Consider the following allocation rules:*

- *Altruistic:* $x^a_1(\mathbf{K}) = c(\{1\}), x^a_2(\mathbf{K}) = c^N(\{2\}), x^a_3(\mathbf{K}) = c^N(\{3\})$.
- *Tute:* $x^t_1(\mathbf{K}) = c(N) - c(\{2,3\})$, $x^t_2(\mathbf{K}) = c(\{2\})$, $x^t_3(\mathbf{K}) = c(\{2,3\}) - c(\{2\})$.
- *E3:* $x^{E3}_1(\mathbf{K}) = c(\{1\}) - [c(\{2\}) - c^N(\{2\})], x^{E3}_2(\mathbf{K}) = c(\{2\}), x^{E3}_3(\mathbf{K}) = c^N(\{3\})$.
- *E4:* $x^{E4}_1(\mathbf{K}) = c(N) - c(\{2,3\})$, $x^{E4}_2(\mathbf{K}) = c^N(\{2\})$, $x^{E4}_3(\mathbf{K}) = c(\{2,3\}) - c^N(\{2\})$.
- *E5:* $x^{E5}_1(\mathbf{K}) = c(N) - c(\{2,3\})$, $x^{E5}_2(\mathbf{K}) = c(\{2,3\}) - c(\{3\})$, $x^{E5}_3(\mathbf{K}) = c(\{3\})$

- *E6:* $x_1^{E6}(\mathbf{K}) = c(\{1\}) - [c(\{3\}) - c^N(\{3\})]$, $x_2^{E6}(\mathbf{K}) = c^N(\{2\})$, $x_3^{E6}(\mathbf{K}) = c(\{3\})$.

Then, the core is defined by the convex hull of the following extreme points:

- $\{Altruistic, Tute, E3, E4\}$ *if* $\sqrt{K_2} - \sqrt{K_1} \leqslant \sqrt{K_3} - \sqrt{K_2}$;
- $\{Altruistic, Tute, E3, E5, E6\}$ *if* $\sqrt{K_2} - \sqrt{K_1} > \sqrt{K_3} - \sqrt{K_2}$.

(Note that $E4 = E5 = E6$ *if* $\sqrt{K_2} - \sqrt{K_1} = \sqrt{K_3} - \sqrt{K_2}$.*)*

The preceding result fully characterizes the core of the KS-game for the case of $N = 3$ suppliers. We have already examined the first-stage investment games under the altruistic and tute allocations. The analysis for the core allocation $E3$ is similar to that of the tute allocation. While $\widetilde{K}^t_{n=3}$ arises as the minimum system cost achieved under the tute allocation rule, the quantity $\widetilde{K}^{E3}$ is the minimum cost achieved under the allocation given by $E3$. This quantity minimizes the function $2\sqrt{hK} + g(K, \overline{K})$, while $\widetilde{K}^t_{n=3}$ minimizes $3\sqrt{hK} + g(K, \overline{K})$. We therefore have that $\widetilde{K}^t_{n=3} < \widetilde{K}^{E3}$, so the tute allocation rule leads to a higher level of investment in cost reduction for the system than the level achieved under the allocation given by $E3$. (In turn, both allocations lead to more significant cost reductions than the altruistic allocation.) A similar analysis follows for the two remaining extreme core allocations in the case of three suppliers. These core allocations are $E4 - E5$ and $E4 - E6$, where costs are allocated according to $E4$ when $\sqrt{K_2} - \sqrt{K_1} \leqslant \sqrt{K_3} - \sqrt{K_2}$, and according to either $E5$ or $E6$ when $\sqrt{K_2} - \sqrt{K_1} > \sqrt{K_3} - \sqrt{K_2}$ (when $\sqrt{K_2} - \sqrt{K_1} = \sqrt{K_3} - \sqrt{K_2}$, these three core allocations coincide). The minimum fixed cost achieved under either pair of cost allocations is no smaller than $\widetilde{K}^t_{n=3}$. Following again the result of Theorem 3, we have that the minimum fixed cost reduction achieved in the case of three suppliers for any core allocation is given by $\widetilde{K}^t_{n=3}$, and this is achieved by the *tute* allocation mechanism. We next consider an example to illustrate our findings for the case $N = 3$.

Example 3. Consider a symmetric KS-network with $N = \{1, 2, 3\}, 2\lambda = 1, b = 1$, and $h_i = 1, \overline{K}_i = 16$, and $g_i(K_i/16) = \ln(16/K_i)$, for $i = 1, 2, 3$. Assume that there is a unique efficient supplier; otherwise, the core reduces to the altruistic allocation. The first-stage investment game is given by the cost functions $\pi_i(\mathbf{K}) = x_i(\mathbf{K}) + \ln(16/K_i)$ for $i = 1, 2, 3$, with action space $K_i \in (0, 16]$, where $x(\mathbf{K}) = (x_i(\mathbf{K}))_{i=1,2,3}$ is a core allocation arising from the second-stage cooperative game. In particular, for the altruistic allocation, $x_i^a = \sqrt{K_N}$, for $i = 1, 2, 3$. For the tute allocation, $x_m^t = 3\sqrt{K_N} - 2\sqrt{K_{N\setminus\{m\}}}$, where $m = \arg\min K_N$, and $x_i^t = \sqrt{K_{N\setminus\{m\}}}$ for $i \neq m$.

The set of optimal investment levels under the centralized solution is $\{(4/9, 16, 16), (16, 4/9, 16), (16, 16, 4/9)\}$. We next study the first-stage investment game under the altruistic allocation. Here, the set of equilibria is $\{(6, 16, 16), (16, 6, 16), (16, 16, 6)\}$. The set of equilibria under the tute allocation is $\{(4/9, 6, 16), (6, 4/9, 16), (16, 4/9, 6)\}$. Finally, under all other extreme points of the core, the equilibrium level of investment in cost reduction is no larger than that achieved under the tute allocation. Indeed, the equilibrium fixed cost achieved by the efficient supplier under the allocations $E3$ and $E6$ is equal to 1, while that achieved under allocations $E4$ and $E5$ is 4/9 (the same as under the tute allocation). Comparing the results, we conclude that the altruistic allocation is the least efficient among the extreme allocations of the core, while the tute allocation induces the same level of cost reduction as that achieved in the centralized system.

5. Implementation costs and other extensions

In the model analyzed in this paper, we assume that reducing the setup costs to the level of the most efficient supplier as a result of knowledge-sharing activities comes at no additional cost. In practice, there may be a positive cost associated with the implementation of changes to improve the process in the desired way. We suspect that this cost is negligible in many cases, as illustrated by two examples discussed in Dyer and Nobeoka (2000). The first alludes to the fact that Toyota provides free consulting hours to implement projects. The second example cites a manager from Kojima Press, a supplier of spoilers and body parts, stating that "Last year we were able to reduce our paint costs by 30%. This was possible due to a suggestion to lower the pressure on the paint sprayer and adjust the spray trajectory, thereby wasting less paint." (Ref. page 356). This is a simple and fairly inexpensive change that, at the same time, allowed the supplier to reduce paint costs. These examples suggest that the know-how (or consulting effort) is the major component of the cost associated with an operational improvement. This know-how is at the core of knowledge transfer in knowledge-sharing activities. Therefore, in many cases, the additional implementation cost of an operational improvement is relatively small and the results in the previous sections apply. Nevertheless, in this section, we demonstrate that our model can be generalized to explicitly incorporate implementation costs (when these are significant). We explore this generalization for the case of two suppliers.

Consider the following modification to the cost function in the KS-game. Suppose that, after the original cost reduction investments take place in the first stage, supplier 1 is the efficient supplier. The cost of reducing supplier 2's setup cost from K_2 to K_1 is $\alpha(\sqrt{K_2} - \sqrt{K_1})$ with $0 \leqslant \alpha < 1$. That is, the implementation cost depends on the relative cost reduction achieved through knowledge sharing. The total cost in the centralized system is given by

$$(2 - \alpha)\sqrt{K_1} + \alpha\sqrt{K_2} + g(K_1|\overline{K}) + g(K_2|\overline{K}).$$

The centralized investment level $K^*(\alpha)$ is given by the solution to the first-order condition $\frac{2-\alpha}{2\sqrt{K}} + g'(K|\overline{K}) = 0$. For $g(K|\overline{K}) = \ln(\overline{K}/K)$, it can be shown that $K_1^*(\alpha) = 4/(2-\alpha)^2$ and $K_2^* = \overline{K}$. Under the *altruistic* allocation, payoffs are given by $\pi_1(\mathbf{K}) = \sqrt{K_1} + g(K_1|\overline{K})$ and $\pi_2(\mathbf{K}) = (1-\alpha)\sqrt{K_1} + \alpha\sqrt{K_2} + g(K_2|\overline{K})$ (assuming that $K_1 < K_2$). Hence, similar to the analysis for the case without implementation costs, $BR_i(K_j) \in \{\widetilde{K}^a(\alpha), \overline{K}\}$ for $i, j = 1, 2, j \neq i$, where $\widetilde{K}^a(\alpha)$ is the largest minimum of $\pi_2(\mathbf{K})$, independent of K_1. The suppliers' best response and equilibrium strategies remain as in the original model without implementation costs. Importantly, the equilibrium fixed cost is higher than that achieved in the centralized solution, i.e., $\widetilde{K}^a(\alpha) > K^*$ for any $0 \leqslant \alpha < 1$. Under the *tute* allocation, payoffs are given by $\pi_1(\mathbf{K}) = (2-\alpha)\sqrt{K_1} + g(K_1) - (1-\alpha)\sqrt{K_2}$ and $\pi_2(\mathbf{K}) = \sqrt{K_2} + g(K_2)$ (assuming that $K_1 < K_2$). Hence, the best response and the equilibrium strategies remain similar to those in the model without implementation costs, although they now depend on the parameter α. In equilibrium, one supplier invests to lower its fixed cost down to $\widetilde{K}^t(\alpha)$, which is obtained by solving $\partial\pi_1/\partial K_1 = 0$. The other supplier invests to lower its fixed cost down to $\widehat{K}^t(\alpha)$, which is obtained by solving $\partial\pi_2/\partial K_2 = 0$. In this setting, one can also verify that the equilibrium investment level is the same as that which arises in the centralized solution, i.e., $\widetilde{K}^t(\alpha) = K^*(\alpha)$ for any α.

To summarize, the analysis for this specific function describing the cost of implementation suggests that when such costs are present, the investment dynamics are similar to those that arise in the original model (without implementation costs) – i.e., the tute allocation induces an equilibrium that matches the fixed cost performance of the centralized system.

On a different note, the analysis of the first-stage (non-cooperative) game in Section 4 has assumed symmetric suppliers. However, in general, some supplier characteristics may not be symmetric. It is reasonable to expect that the investment cost function $g(K)$ is similar across suppliers because the cost of investment mainly depends on consulting rates and engineering hours, and those are relatively consistent within firms in the same industry. Nevertheless, there may be cases in which the investment cost functions do differ across suppliers, as the suppliers may be at different stages of technological or managerial sophistication. On the other hand, the holding cost rate h_i is most likely different across suppliers as this cost is typically proportional to the value of a supplier's component. The generalization to suppliers with non-identical holding cost rates is a relatively straightforward extension to the analysis in Sections 3 and 4 – non-identical holding cost rates would only change the size of the benefits achieved from a particular setup cost vector. In such setting, the altruistic (tute) allocation also leads to an equilibrium under which only one firm (two firms) invest in cost reduction. Our finding regarding the performance of the first-stage equilibria under the altruistic and tute allocations vis-à-vis the optimal investment in the centralized system is similar to that under equal holding costs. Although we conjecture that the qualitative results would remain unchanged in a system with fully asymmetric supplier characteristics, a formal analysis is beyond the scope of this paper.

Finally, we note that we have not explicitly modeled the potential intangible benefits associated with the participation in knowledge sharing networks or those that result from being an efficient supplier. In practice, efficient suppliers may enjoy a better relationship with the assembler or receive more favorable contract terms. Similarly, suppliers that consistently fail to invest in process improvement stand the risk of receiving less favorable contract terms or altogether losing the contract with the assembler. Incorporating these and other long term benefits is outside the scope of the paper. In such cases, all suppliers would have stronger incentives to investment in process improvement and cost reduction initiatives. Therefore, the investment levels under all core allocations would lead to lower equilibrium fixed costs.

6. Managerial implications and concluding remarks

In this paper, we consider the implementation of knowledge-sharing activities in a decentralized assembly network. We propose a model in which component suppliers engage in a cooperative knowledge-sharing game following a non-cooperative game in which they invest in process improvement initiatives to reduce their own costs. Each component supplier incurs fixed and variable costs and determines the optimal production batch size according to the EOQ model. In the first-stage, non-cooperative game, suppliers can make costly investments to reduce their fixed production cost. In the cooperative game that follows (modeling knowledge-sharing activities), the knowledge generated as a result of investments in cost reduction is shared by all suppliers (i.e., the fixed costs of all component suppliers are reduced to the level of the most efficient – lowest cost – supplier). The cooperative game suggests ways in which the benefits of knowledge sharing are allocated among suppliers. We examine the dynamics of knowledge-sharing activities in the context of symmetric suppliers (i.e., suppliers have ex-ante identical cost structures), and then discuss the implications of this assumption.

In order to understand the incentives for investment induced by knowledge-sharing, we study the core of the cooperative knowledge-sharing game. The complete characterization of the core is, in general, difficult. We show that the lowest fixed-cost equilibrium is achieved in an extreme point of the core. We study two particular extreme points of the set of core allocations: the *altruistic* allocation and the *tute* allocation. Under the altruistic allocation, the least efficient suppliers benefit from knowledge-sharing while the efficient suppliers do not gain anything by participating in those activities. Under the tute allocation, the most efficient supplier receives the incremental benefit that it generates for the entire network. Based on the results of the cooperative game, we characterize the equilibrium investment levels for the supplier network, which depend on the allocation rule used in the cooperative stage. There are generally multiple equilibria in the first-stage investment game. We find that the tute allocation mechanism induces suppliers to achieve a level of cost reduction equal to that which arises in the centralized setting. This is not the case under the altruistic allocation. This suggests that the benefits associated with knowledge sharing activities need to be carefully allocated among suppliers in the network to induce the proper level of investment in cost reduction.

From an implementation perspective, the altruistic allocation is the simplest, as suppliers keep the benefit of the learning activities to themselves. Implementing the tute allocation, or any other similar allocation, requires the assembler to take a more active role and reward the efficient suppliers by instituting a set of transfer payments from the other suppliers. Indeed, an internal transfer mechanism is necessary to achieve a higher level of investment and involvement from suppliers. This is consistent with the practice at Toyota, as evidenced by the Dyer and Nobeoka article: "To encourage suppliers to participate and openly share knowledge, Toyota has heavily subsidized the network (with knowledge and resources) during the early stages of formation to ensure that suppliers realized substantial benefits from participation." In addition, there are intangibles and long term benefits from sharing process knowledge with other firms – such as earning new and larger contracts and receiving more free consulting projects. In sum, our results suggest that Toyota and its suppliers may benefit from implementing explicit reward/transfer mechanisms for suppliers that share knowledge – beyond the associated intangible benefits.

The size of the network has a significant impact on the outcome of knowledge-sharing activities. As we have demonstrated, the equilibrium investment levels in the centralized system and under the tute allocation increase with the network size (implying a reduction to lower levels of the suppliers' fixed costs). On the other hand, a larger network induces free riding from a larger subset of suppliers. In that respect, the number and size of networks need to balance these counteracting forces – higher efficiency versus the free rider problem. Indeed, Dyer and Nobeoka (2000) identify the free rider problem as one of the major dilemmas associated with knowledge sharing activities. The article states that knowledge-sharing groups at Toyota consist of five to eight suppliers. Toyota recognizes that effective learning takes place within relatively smaller learning activity-centered nests. They create these nests within the larger network to increase the learning effectiveness and to reduce free riding by the suppliers.

In this paper, we have proposed a stylized model of knowledge sharing – all suppliers can achieve the fixed cost of the most efficient suppliers through knowledge sharing activities. By analyzing a biform game, we explore the equilibrium behavior of suppliers and its implications on the design of appropriate incentives for knowledge-sharing. Our model uncovers the forces at work in the process of knowledge-sharing activities and evidences that the appropriate incentives have to be in place (e.g., in the form of transfer payments) to achieve levels of cost reduction commensurate with those that would be achieved under centralized control.

Acknowledgments

The authors are grateful to the reviewers for helpful comments that have helped to improve the paper. The authors also acknowledge the financial support of Ministerio de Ciencia e Innovación through Projects MTM2008-06778-C02-01 and MTM2011-23205, and the Generalitat Valenciana through Project ACOMP/2014.

Appendix A. Proofs

Proof of Theorem 1. Consider the KS-game (N,c) associated to a KS-network $(N,\{h_i,K_i\}_{i\in N})$. For the purpose of this proof, we assume w.l.o.g. that $K_1 \leqslant K_2 \leqslant \cdots \leqslant K_n$. Consider now a marginal vector $m^\sigma(N,c)$ such that σ satisfies that $\sigma^{-1}(1)$ is an efficient agent of the KS-network. We first prove that $m^\sigma(N,c)$ belongs to the core of (N,c). To that end, it suffices to show that for every non-empty coalition $S\subset N$, it holds that $\sum_{i\in S}m_i^\sigma(N,c)\leqslant c(S)$. We distinguish two cases.

1. S contains the efficient agent $\sigma^{-1}(1)$. Then,

$$\sum_{i\in S}m_i^\sigma(N,c)=c(\sigma^{-1}(1))+\sum_{j\in S\setminus\{\sigma^{-1}(1)\}}\left(c(P_j^\sigma\cup\{j\})-c(P_j^\sigma)\right)$$
$$=c(\sigma^{-1}(1))+\sum_{j\in S\setminus\{\sigma^{-1}(1)\}}\left(\sum_{i\in P_j^\sigma\cup\{j\}}\sqrt{h_ik_N}-\sum_{i\in P_j^\sigma}\sqrt{h_ik_N}\right)$$
$$=c(\sigma^{-1}(1))+\sum_{j\in S\setminus\{\sigma^{-1}(1)\}}\sqrt{h_jk_N}=\sum_{j\in S}\sqrt{h_jk_N}=\sum_{j\in S}\sqrt{h_jk_S}=c(S).$$

2. S does not contain the efficient agent $\sigma^{-1}(1)$. In this case, let $\overline{S}=S\cup\{\sigma^{-1}(1)\}$. Using the same proof above we conclude that $\sum_{i\in\overline{S}}m_i^\sigma(N,c)=c(\overline{S})$. Taking into account that $m^\sigma_{\sigma^{-1}(1)}(N,c)=c(\sigma^{-1}(1))$ and that c is subadditive, we have that $c(\sigma^{-1}(1))+\sum_{i\in S}m_i^\sigma(N,c)=\sum_{i\in\overline{S}}m_i^\sigma(N,c)=c(\overline{S})\leqslant c(\sigma^{-1}(1))+c(S)$, which implies that $\sum_{i\in S}m_i^\sigma(N,c)\leqslant c(S)$.

The proof of (N,c) being totally balanced follows from the fact that for all $S\subseteq N$, every subgame (S,c_S) of a KS-game (N,c) is a KS-game as well. □

Proof of Proposition 1. Consider a KS-game (N,c) associated to a KS-network $(N,\{h_i,K_i\}_{i\in N})$. For the purpose of this proof, we assume w.l.o.g. that $K_1\leqslant K_2\leqslant\cdots\leqslant K_n$. We have to prove that $C(N,c)\subseteq D(N,c)$ and $D(N,c)\subseteq C(N,c)$.

[1] $C(N,c)\subseteq D(N,c)$. Take $x\in C(N,c)$. Then, for all $i\neq 1$,

$$x_i-c^N(\{i\})=\sum_{j\in N\setminus\{i\}}[c^N(\{j\})-x_j]\geqslant\sum_{j\in N\setminus\{i\}}c^N(\{j\})-c(N\setminus\{i\})=0,$$

where the first equality follows from efficiency, the inequality from the coalitional stability of the core elements, and the last equality from the fact that $K_N=K_{N\setminus\{i\}}$ for all $i\neq 1$. Hence, taking $t_i=x_i-c^N(\{i\})$ we obtain that $x_i=c^N(\{i\})+t_i,\ \forall i\in N\setminus\{1\},\ t_i\geqslant 0$, and $x_1=c(\{1\})-\sum_{i\in N\setminus\{1\}}t_i$. Consider now $S\subset N\setminus\{1\}$. Then, $\sum_{i\in S}t_i\leqslant c(S)-\sum_{i\in S}c^N(\{i\})=\sum_{i\in S}d_i(S,N)$, where the inequality follows from the fact that $c(S)\geqslant\sum_{i\in S}x_i=\sum_{i\in S}[c^N(\{i\})+t_i]$.

[2] $D(N,c)\subseteq C(N,c)$. Take $x\in D(N,c)$. To prove that x belongs to the core of (N,c), it suffices to show that for every non-empty coalition $S\subset N$, we have that $\sum_{i\in S}x_i\leqslant c(S)$. We distinguish two cases. First, suppose that S contains the extreme agent 1. Then,

$$x_1+\sum_{i\in S\setminus\{1\}}x_i=c(\{1\})-\sum_{i\in N\setminus\{1\}}t_i+\sum_{i\in S\setminus\{1\}}c^N(\{i\})+\sum_{i\in S\setminus\{1\}}t_i=c(S)-\sum_{i\in N\setminus S}t_i\leqslant c(S).$$

Suppose now that S does not contain the extreme agent 1. In this case,

$$\sum_{i\in S}x_i=\sum_{i\in S}c^N(\{i\})+\sum_{i\in S}t_i\leqslant\sum_{i\in S}c^N(\{i\})+\sum_{i\in S}d_i(S,N)=\sum_{i\in S}c^S(\{i\})=c(S).\qquad\square$$

Proof of Theorem 2. Consider a cost game (N,c). Let $M_i(N,c):=c(N)-c(N\setminus\{i\})$ be the marginal contributions of player i to the grand coalition, and $M(N,c)=(M_i(N,c))_{i\in N}$ the collection of all those marginal contributions. It is easy to show that $C(N,c)=\{M(N,c)\}$ if the vector of marginal contributions to the grand coalition is efficient – i.e., $\sum_{i\in N}M_i(N,c)=c(N)$. Indeed, since any core allocation should satisfy $x_i\geqslant M_i(N,c)$ for all players $i\in N$, if any of those inequalities was strict, then we would have that $\sum_{i\in N}x_i=c(N)>\sum_{i\in N}M_i(N,c)$, which is a contradiction with the fact that $M(N,c)$ is efficient.

Consider a multiple KS-game (N,c) associated to a KS-network $(N,\{h_i,K_i\}_{i\in N})$. It is easy to prove that the vector of marginal contributions to the grand coalition is efficient here. Indeed, $M_i(N,c)=c^N(\{i\})=x_i^a$ since $k_N=k_{N\setminus\{i\}}$ for all $i\in N$. Hence, $C(N,c)=\{x^a\}$. □

Proof of Theorem 3. Let $x^{e_1}(\mathbf{K}),\ldots,x^{e_M}(\mathbf{K})$ be the extreme points of the core of the second-stage knowledge-sharing cooperative game as a function of the first-stage vector of fixed costs $\mathbf{K}$. From the definition of the core, we have that the allocation to supplier i from each extreme point m is given by $x_i^{e_m}(\mathbf{K})=\sum\beta_{i,r}^{e_m}c(S_r)=\sum\beta_{i,r}^{e_m}|S_r|\sqrt{hK_{S_r}}$, for certain coefficients $\beta_{i,r}^{e_m}$ (that depend on the specific extreme point e_m and supplier i) and subsets $S_r\subset N$. (This follows because an extreme point of the core is the solution to a set of linear equations with a right-hand side vector containing elements of the form $c(S_r)$.) Suppose now that there is an interior allocation $x^I(\mathbf{K})=\sum_{m=1}^M\alpha_mx^{e_m}(\mathbf{K})$ with $\sum_{m=1}^M\alpha_m=1$ and $0\leqslant\alpha_m\leqslant 1$, such that the largest cost reduction (i.e., lowest fixed cost) in the first stage arises under this interior allocation and not in any of the extreme points of the core. (That is, the interior allocation is strictly superior to the extreme allocations in the sense that it leads to a strictly larger reduction in the fixed cost compared to those achieved under the extreme points.) Let $\mathbf{K}^{MIN}$ be the equilibrium of the first stage that arises under the allocation x^I and, without loss of generality, suppose that the lowest fixed cost under this equilibrium is attained by supplier 1. Then,

$$0=\left.\frac{\partial[x_1^I(\mathbf{K})+g(K_1|\overline{K})]}{\partial K_1}\right|_{K=\mathbf{K}^{MIN}}=\sum_{m=1}^M\alpha_m\left.\frac{\partial[x_1^{e_m}(\mathbf{K})+g(K_1|\overline{K})]}{\partial K_1}\right|_{K=\mathbf{K}^{MIN}}.$$

Because the derivative is with respect to K_1 and the functions $x_1^{e_m}(\mathbf{K})$ are separable, we have that

$$\sum_{m=1}^M\alpha_m\left.\frac{\partial[x_1^{e_m}(\mathbf{K})+g(K_1|\overline{K})]}{\partial K_1}\right|_{K=\mathbf{K}^{MIN}}=\sum_{m=1}^M\alpha_m\left.\frac{\partial\left[\sum_{\{r:1\in S_r\}}\beta_{1,r}^{e_m}|S_r|\sqrt{hK_1}+g(K_1|\overline{K})\right]}{\partial K_1}\right|_{K=\mathbf{K}_1^{MIN}}.$$

Each function $\sum_{\{r:1\in S_r\}}\beta_{1,r}^{e_m}|S_r|\sqrt{hK_1}+g(K_1|\overline{K})$ is either (strictly) convex or (strictly) convex–concave, depending on the parameters $\beta_{1,r}^{e_m}$, with a minimum achieved at $K_1^{e_m}$. Because we are supposing that the lowest equilibrium fixed cost is achieved by x^I, and not by any of the extreme core allocations, we have that $K_1^{MIN}<K_1^{e_m}$. Thus,

$$\sum_{m=1}^{M}\alpha_m \frac{\partial\left[\sum_{\{r:1\in S_r\}}\beta_{1,r}^{e_m}|S_r|\sqrt{hK_1}+g(K_1|\overline{K})\right]}{\partial K_1}\Bigg|_{K=\mathbf{K}_1^{MIN}} < \sum_{m=1}^{M}\alpha_m \frac{\partial\left[\sum_{\{r:1\in S_r\}}\beta_{1,r}^{e_m}|S_r|\sqrt{hK_1}+g(K_1|\overline{K})\right]}{\partial K_1}\Bigg|_{K=\mathbf{K}_1^{e_m}} = 0,$$

because the function is strictly convex or strictly convex–concave, leading to a contradiction. This concludes the proof. □

Proof of Lemma 1. We prove this result by showing that the following properties hold: (i) The functions $\tilde{\pi}^a(K)$ and $\hat{\pi}_i^a(\mathbf{K})$ cross only once at $K_i = K_{N\setminus\{i\}}$. ($ii$) $\tilde{\pi}^a(K) \leqslant \hat{\pi}_i^a(\mathbf{K})$ if and only if $K_i \leqslant K_{N\setminus\{i\}}$, i.e., $\pi_i^a(\mathbf{K}) = \min\{\tilde{\pi}^a(K), \hat{\pi}_i^a(\mathbf{K})\}$. ($iii$) There exists a unique value $t \in (\widetilde{K}^a, \overline{K})$ such that $\tilde{\pi}^a(\widetilde{K}^a) = \sqrt{ht}$.

Properties (i)-(ii) follow from the derivatives of $\tilde{\pi}^a(K)$ and $\hat{\pi}_i^a(\mathbf{K})$ with respect to K_i, i.e.,

$$\frac{\partial\tilde{\pi}^a}{\partial K} = \frac{\sqrt{h}}{2\sqrt{K}} + g'(K,\overline{K}) > g'(K,\overline{K}) = \frac{\partial\hat{\pi}_i^a}{\partial K_i}.$$

If the functions $\tilde{\pi}^a(K)$ and $\hat{\pi}_i^a(\mathbf{K})$ crossed at more than one point, then their derivatives should also cross at least once, which cannot happen. To see part (iii), note that

$$\tilde{\pi}^a\left(\widetilde{K}^a\right) = \sqrt{h\widetilde{K}^a} + g\left(\widetilde{K}^a,\overline{K}\right) > \sqrt{h\widetilde{K}^a} = \sqrt{h\widetilde{K}^a} + g(\overline{K},\overline{K}) = \hat{\pi}_i^a\left(K_i = \overline{K}, K_{N\setminus\{i\}} = \widetilde{K}^a\right).$$

At the same time, because $\widetilde{K}^a$ minimizes $\tilde{\pi}^a(\cdot)$, we have that

$$\tilde{\pi}^a(\widetilde{K}^a) < \tilde{\pi}^a(\overline{K}) = \sqrt{h\overline{K}} + g(\overline{K},\overline{K}) = \sqrt{h\overline{K}} = \hat{\pi}_i^a(K_i = \overline{K}, K_{N\setminus\{i\}} = \overline{K}).$$

Therefore, there exists a unique $t \in (\widetilde{K}^a, \overline{K})$ such that

$$\tilde{\pi}^a(\widetilde{K}^a) = \sqrt{ht} + g(\overline{K},\overline{K}) = \hat{\pi}_i^a(K_i = \overline{K}, K_{N\setminus\{i\}} = t),$$

leading to the desired result. □

Proof of Proposition 2. First, if all suppliers except for supplier j select a fixed cost equal to $\overline{K}$ in the first stage, then $K_{N\setminus\{j\}} = \overline{K} \geqslant t$, implying that supplier j's best response is to select the fixed cost $\widetilde{K}^a$. Conversely, for all other suppliers $i \neq j, K_{N\setminus\{i\}} = \widetilde{K}^a \leqslant t$, so their best response is to choose $\overline{K}$. It follows that the n vectors in which one supplier chooses $\widetilde{K}^a$ and the others choose $\overline{K}$ are equilibria of the first-stage game under the altruistic allocation. Suppose now that there is an equilibrium in which suppliers $j1$ and $j2$ (and, possibly, others), choose a fixed cost equal to $\widetilde{K}^a$. Then, $K_{N\setminus\{j1\}} = \widetilde{K}^a \leqslant t$, which would imply that supplier $j2$'s best response is $\overline{K}$. Therefore, a vector of fixed costs with more than one value equal to $\widetilde{K}^a$ cannot be equilibria of the first-stage game. We conclude that the n equilibria described above are the only equilibria of the first-stage game. It finally follows that $\widetilde{K}^a > K_n^*$ from the shape of the function $\tilde{\pi}^a(K)$ and the first-order condition in the centralized system given in (1). □

Proof of Lemma 2. We first show that the following properties hold: (i) There exists a unique value $t_1 \in \left(\widetilde{K}_n^t, \widehat{K}^t\right)$ such that $\tilde{\pi}^t\left(\widetilde{K}_n^t, t_1\right) = \hat{\pi}^t\left(\widehat{K}^t\right)$. ($ii$) There exists a unique value $t_3 \in \left(\widehat{K}^t, \overline{K}\right)$ such that $\hat{\pi}^t\left(\widehat{K}^t\right) = \sqrt{ht_3}$. ($iii$) There exists a unique value $t_2 \in (t_1, t_3)$ such that $\tilde{\pi}^t\left(\widetilde{K}_n^t, t_2\right) = \sqrt{ht_2}$. ($iv$) For $\kappa \in [t_1, t_2]$, there exists a unique $\overline{K}(\kappa) > t_2$ such that $\tilde{\pi}^t\left(\widetilde{K}_n^t, \kappa\right) = \sqrt{h\overline{K}(\kappa)}$.

To see part (i), first note that

$$\tilde{\pi}^t\left(\widetilde{K}_n^t, \widetilde{K}_n^t\right) = \sqrt{h\widetilde{K}_n^t} + g\left(\widetilde{K}_n^t|\overline{K}\right) > \sqrt{h\widehat{K}^t} + g(\widehat{K}^t|\overline{K}) = \hat{\pi}^t(\widehat{K}^t) \text{ and}$$

$$\tilde{\pi}^t\left(\widetilde{K}_n^t, \widehat{K}^t\right) = n\sqrt{h\widetilde{K}_n^t} - (n-1)\sqrt{h\widehat{K}^t} + g\left(\widetilde{K}_n^t|\overline{K}\right) < \sqrt{h\widehat{K}^t} + g(\widehat{K}^t|\overline{K}),$$

where the first inequality follows because $\widehat{K}^t$ minimizes $\hat{\pi}^t(\cdot)$ and the second inequality follows because $\widetilde{K}_n^t$ minimizes $\tilde{\pi}^t(\cdot,\kappa)$ for any κ. This implies that the two functions cross at least once on the interval $\left(\widetilde{K}_n^t, \widehat{K}^t\right)$. Note also that

$$\frac{\partial\tilde{\pi}^t(K,\kappa)}{\partial K} = \frac{n\sqrt{h}}{2\sqrt{K}} + g'(K|\overline{K}) > \frac{\sqrt{h}}{2\sqrt{K}} + g'(K|\overline{K}) = \frac{\partial\hat{\pi}^t(K)}{\partial K},$$

so the two functions cannot cross more than once (otherwise, their derivatives would cross at least once). This proves (i). We similarly prove (ii) by noting that $\hat{\pi}^t\left(\widehat{K}^t\right) > \sqrt{h\widehat{K}^t}, \hat{\pi}^t\left(\widehat{K}^t\right) < \sqrt{h\overline{K}}$ (because $\widehat{K}^t$ minimizes $\hat{\pi}^t(\cdot)$), and that $\frac{\partial\hat{\pi}^t(K)}{\partial K} < \frac{\partial\sqrt{hK}}{\partial K}$. Part ($iii$) also follows from a similar argument. Note that $\tilde{\pi}^t\left(\widetilde{K}_n^t, t_1\right) = \hat{\pi}^t\left(\widehat{K}^t\right) > \sqrt{ht_1}$ $\left(\text{because } t_1 < \widehat{K}^t\right)$ and that $\tilde{\pi}^t\left(\widetilde{K}_n^t, t_3\right) < \tilde{\pi}^t\left(\widetilde{K}_n^t, \widehat{K}^t\right) < \hat{\pi}^t(\widehat{K}^t) = \sqrt{ht_3}$ $\left(\text{because } \widetilde{K}_n^t \text{ minimizes } \tilde{\pi}^t(\cdot,\kappa) \text{ for any } \kappa\right)$. Moreover, $\tilde{\pi}^t(K,\kappa)$ is decreasing in κ while $\sqrt{h\kappa}$ is increasing in κ, so the two functions can cross at most once. Finally, for part (iv), note that $\sqrt{ht_2} = \tilde{\pi}^t\left(\widetilde{K}_n^t, t_2\right) < \tilde{\pi}^t\left(\widetilde{K}_n^t, \kappa\right) < \tilde{\pi}^t\left(\widetilde{K}_n^t, t_1\right) = \hat{\pi}^t\left(\widehat{K}^t\right) = \sqrt{ht_3}$ for all $\kappa \in (t_1, t_2)$, following the results in parts (i), (ii) and (iii). Because $\tilde{\pi}^t\left(\widetilde{K}_n^t, \kappa\right)$ is decreasing for $\kappa \in (t_1, t_2)$ and $\sqrt{hK}$ is increasing for $K \in (t_2, t_3)$, we obtain the desired result.

We now characterize supplier i's best response to a vector $\mathbf{K}_{-i}$ with $K_{ji} \leqslant K_{j2} \leqslant K_j$ for all $j \neq i, j1, j2$. Consider first the case with $K_{j1} \leqslant t_1$ and $K_{j2} \geqslant t_3$. We have that $\tilde{\pi}^t\left(\widetilde{K}_n^t, K_{j1}\right) \geqslant \tilde{\pi}^t\left(\widetilde{K}_1^t, t_1\right) = \hat{\pi}^t\left(\widehat{K}^t\right)$ and $\sqrt{h\overline{K}} \geqslant \sqrt{hK_{j2}} \geqslant \sqrt{ht_3} = \hat{\pi}^t\left(\widehat{K}^t\right)$. Therefore, $K = \widehat{K}^t$ is supplier i's best-response in that case. When $K_{j1} \leqslant t_1$ *and* $K_{j2} \leqslant t_3$, we have $\tilde{\pi}^t\left(\widetilde{K}_n^t, K_{j1}\right) \geqslant \tilde{\pi}^t\left(\widetilde{K}_1^t, t_1\right) = \hat{\pi}^t\left(\widehat{K}^t\right) = \sqrt{ht_3} \geqslant \sqrt{hK_{j2}}$, so $\overline{K}$ is optimal. If $K_{j1} \geqslant t_2$, then $K_{j2} > K_{j1} \geqslant t_2 > t_1$, which implies that $\tilde{\pi}^t\left(\widetilde{K}_n^t, K_{j1}\right) < \tilde{\pi}^t\left(\widetilde{K}_n^t, t_1\right) = \hat{\pi}^t\left(\widehat{K}^t\right)$ and $\tilde{\pi}^t\left(\widetilde{K}_n^t, K_{j1}\right) < \tilde{\pi}^t\left(\widetilde{K}_n^t, t_2\right) = \sqrt{ht_2} < \sqrt{hK_{j2}} \leqslant \sqrt{h\overline{K}}$. Therefore, $\widetilde{K}_n^t$ is optimal. Finally, consider $K_{j1} \in (t_1, t_2)$. In that case, we have $\tilde{\pi}^t\left(\widetilde{K}_n^t, K_{j1}\right) < \tilde{\pi}^t\left(\widetilde{K}_n^t, t_1\right) = \hat{\pi}^t\left(\widehat{K}^t\right)$, so $\widehat{K}^t$ cannot be supplier i's best-response. Whether $\overline{K}$ or $\widetilde{K}_n^t$ is supplier i's best-response in this case follows directly from part (iv) of the properties shown above. □

Proof of Proposition 3. We show that the equilibria described in the statement of the result are all the possible equilibria under the tute allocation. Indeed, suppose that in an equilibrium of the two-stage game, no supplier chose $\widetilde{K}_n^t$. If all suppliers choose $\overline{K}$, then $BR_i(\overline{K},\overline{K}) = \widetilde{K}_n^t$, which is a contradiction. If one supplier chooses $\widehat{K}^t$, then $BR_i\left(\widehat{K}^t,\overline{K}\right) = \widetilde{K}_n^t$, again a contradiction. If two or more suppliers choose $\widehat{K}^t$, then either $BR_i\left(\widehat{K}^t,\widehat{K}^t\right) = \widetilde{K}_n^t$, which is a contradiction, or $BR_i\left(\widehat{K}^t,\widehat{K}^t\right) = \overline{K}$. The latter scenario could only occur if the equilibrium was given by the vector $\left(\widehat{K}^t,\widehat{K}^t,\overline{K},\cdots,\overline{K}\right)$. But in this case, the first supplier's best-response would be

$BR_1\left(\widehat{K}^t, \overline{K}\right) = \widetilde{K}^t_n$, again a contradiction. If exactly two suppliers selected $\widetilde{K}^t_n$, then for the same reason all others should select $\overline{K}$, which is not possible since for the first supplier we would have $BR_1\left(\widetilde{K}^t_n, \overline{K}\right) = \widehat{K}^t$. We conclude that exactly one supplier invests to reduce its fixed cost to $\widetilde{K}^t_n$ in equilibrium. Since $BR_i\left(\widetilde{K}^t_n, \widehat{K}^t\right) = \overline{K}$ and $BR_i\left(\widetilde{K}^t_n, \overline{K}\right) = \widehat{K}^t$ we must have that exactly one other supplier chooses $\widehat{K}^t$ and all others choose $\overline{K}$. □

Proof of Lemma 3. For the purpose of this proof, we assume w.l.o.g. that $K_1 < K_2 \leqslant K_3$. We know by Proposition 1 that $x \in C(N,c)$ implies that $x_1 = c(1) - t_2 - t_3$ with $t_2 + t_3 \leqslant c(23) - c^N(2) - c^N(3), x_2 = c^N(2) + t_2$ with $0 \leqslant t_2 \leqslant c(2) - c^N(2)$, and $x_3 = c^N(3) + t_3$ with $0 \leqslant t_3 \leqslant c(3) - c^N(3)$. Then, the candidates to be extreme points for the above polyhedron are the altruistic (setting $t_2 = t_3 = 0$) and the tute (setting $t_2 + t_3 = c(23) - c^N(2) - c^N(3), t_2 = c(2) - c^N(2)$) allocations, $E3$ (setting $t_2 = c(2) - c^N(2), t_3 = 0$), $E4$ (setting $t_2 + t_3 = c(23) - c^N(2) - c^N(3), t_2 = 0$), $E5$ (setting $t_2 + t_3 = c(23) - c^N(2) - c^N(3), t_3 = c(3) - c^N(3)$), $E6$ (setting $t_2 = 0, t_3 = c(3) - c^N(3)$), $N1$ (setting $t_2 + t_3 = c(23) - c^N(2) - c^N(3), t_3 = 0$), and $N2$ (setting $t_2 = c(2) - c^N(2), t_3 = c(3) - c^N(3)$), where $N1 = (c(N) - c(23), c(23) - c^N(3), c^N(3))$ and $N2 = (c(1) - c(2) - c(3), c(2), c(3))$.

We already know that the altruistic and the tute allocations are extreme points of the core. It is easy to check that $E3$ is a marginal vector (associated to the order which $\sigma(1) = 2, \sigma(2) = 1, \sigma(3) = 3$) that belongs to the core. Hence, $E3$ is always an extreme point for the core. Let us prove now that E4 is an extreme point of the core if $\sqrt{K_2} - \sqrt{K_1} \leqslant \sqrt{K_3} - \sqrt{K_2}$, $E5$ and $E6$ are extremes points of the core if $\sqrt{K_2} - \sqrt{K_1} \geqslant \sqrt{K_3} - \sqrt{K_2}$, and neither $N1$ nor $N2$ are core allocations.

Considering the reasoning above, we have that $C(N,c) = conv\{altruistic, tute, E3, E4\}$ if $\sqrt{K_2} - \sqrt{K_1} \leqslant \sqrt{K_3} - \sqrt{K_2}$, and $C(N,c) = conv\{altruistic, tute, E3, E5, E6\}$ if $\sqrt{K_2} - \sqrt{K_1} \leqslant \sqrt{K_3} - \sqrt{K_2}$. Indeed, first we prove that $E4$ belongs to the core under the first condition. Note that $x_1^{E4}(K) + x_2^{E4}(K) = c(N) - c(23) + c^N(2) = c(12) - [c(2) - c^N(2)] - [c^{23}(3) - c^N(3)] < c(12)$, since $c(2) - c^N(2) > 0$ and $c^{23}(3) - c^N(3) > 0$ because $K_1 < K_2$. Also, $x_1^{E4}(K) + x_3^{E4}(K) = c(13), x_2^{E4}(K) + x_3^{E4}(K) = c(23), x_1^{E4}(K) = c(N) - c(23) = c(1) - [c(2) - c^N(2)] - [c^{23}(3) - c^N(3)] < c(1)$, and $x_2^{E4}(K) = c^N(2) < c(2)$ because $K_1 < K_2$. In addition, $x_3^{E4}(K) = c(23) - c^N(2) = c(2) + c^{23}(3) - c^N(2) = c(3)$ if and only if $c(2) - c^N(2) = c(3) - c^{23}(3)$, which is equivalent to $\sqrt{K_2} - \sqrt{K_1} = \sqrt{K_3} - \sqrt{K_2}$. Notice then that $E4$ satisfies $(n-1) = 2$ restrictions as equalities: $x_1^{E4}(K) + x_3^{E4}(K) = c(13)$ and $x_2^{E4}(K) + x_3^{E4}(K) = c(23)$. Hence, $E4$ belongs to the core. We now prove that $E5$ is not always a core allocation. Indeed, $x_1^{E5}(K) < c(1), x_2^{E5}(K) < c(2), x_3^{E5}(K) = c(3)$, $x_1^{E5}(K) + x_2^{E5}(K) < c(12)$, and $x_2^{E5}(K) + x_3^{E5}(K) = c(23)$. However, if $c(2) - c^N(2) < c(3) - c^{23}(3)$, which is equivalent to $\sqrt{K_2} - \sqrt{K_1} < \sqrt{K_3} - \sqrt{K_2}$, then $x_1^{E5}(K) + x_3^{E5}(K) = c(13) + [c(3) - c^{23}(3)] - [c(2) - c^N(2)] > c(13)$. Next, we prove that $E6$ is also an extreme point under the same condition. Indeed, $x_1^{E6}(K) < c(1), x_2^{E6}(K) < c(2)$, $x_3^{E6}(K) = c(3), x_1^{E6}(K) + x_2^{E6}(K) < c(12)$, and $x_1^{E6}(K) + x_3^{E6}(K) = c(13)$. However, if $c(2) - c^N(2) < c(3) - c^{23}(3)$, which is equivalent to $\sqrt{K_2} - \sqrt{K_1} \leqslant \sqrt{K_3} - \sqrt{K_2}$, then $x_2^{E6}(K) + x_3^{E6}(K) = c^N(2) + c(3) > c(23)$. Hence, we conclude that $E5$ and $E6$ are extremes points of the core if $\sqrt{K_2} - \sqrt{K_1} \geqslant \sqrt{K_3} - \sqrt{K_2}$.

Finally, we prove that neither $N1$ nor $N2$ are in the core. Indeed, $N1$ is not a core allocation since $x_2^{N1}(K) = c(23) - c^N(3) = c(2) + c^{23}(3) - c^N(3) > c(2)$ because $c^{23}(3) - c^N(3) > 0$ since $K_1 < K_2$. On the other hand, if $t_2 = c(2) - c^N(2), t_3 = c(3) - c^N(3)$, then $t_2 + t_3 > c(23) - c^N(2) - c^N(3)$. Hence, $N2$ is not a core allocation. □

References

Bernstein, F., & Kök, A. G. (2009). Dynamic cost reduction through process improvement in assembly networks. *Management Science, 55*(Issue 4), 552–567.

Brandenburger, A., & Stuart, H. (2007). Biform games. *Management Science, 53*(4), 537–549.

Chang, H-C., Ouyang, L.-Y., Wu, K.-S., & Ho, C.-H. (2006). Integrated vendorbuyer cooperative inventory models with controllable lead time and ordering cost reduction. *European Journal of Operational Research, 170*(2), 481–495.

Denizel, M., Ereng, S., & Benson, H. P. (1997). Dynamic lot-sizing with setup cost reduction. *European Journal of Operational Research, 100*(3), 537–549.

Drieseen, T. H. S. (1988). *Cooperative games, solutions and applications*. Dordretch, The Netherlands: Kluwer Academic Publisher.

Dror, M., & Hartman, B. C. (2011). Survey of cooperative inventory games and extensions. *Journal of the Operational Research Society, 62*, 565–580.

Dyer, J. H., & Nobeoka, K. (2000). Creating and managing a high-performance knowledge-sharing network: The Toyota case. *Strategic Management Journal, 21*, 345–367.

Fiestras-Janeiro, M. G., García-Jurado, I., Meca, A., & Mosquera, M. A. (2011). Cooperative game theory and inventory management. *European Journal of Operational Research, 210*, 459–466 (Invited Review).

Gilbert, S., & Cvsa, V. (2003). Strategic supply chain contracting to stimulate downstream process innovation. *European Journal of Operational Research, 150*(Issue 3), 617–639.

González-Díaz, J., García-Jurado, I., & Fiestras-Janeiro, M. G. (2010). An introductory course on mathematical game theory. *Graduate studies in mathematics* (Vol. 115). American Mathematical Society.

Guardiola, L. A., Meca, A., & Puerto, J. (2008). PI-games and PMAS games: Characterizations of the Owen point. *Mathematical Social Science, 56*(Issue 1), 96–108.

Guardiola, L. A., Meca, A., & Puerto, J. (2009). Production-inventory games: A new class of totally balanced combinatorial optimization games. *Games and Economic Behavior, 65*, 205–219.

Gupta, S., & Loulou, R. (1998). Process innovation, product differentiation, and channel structure: Strategic incentives in a duopoly. *Marketing Science, 17*(4), 301–316.

Hatch, N., & Mowery, D. (1998). Process innovation and learning by doing in semiconductor manufacturing. *Management Science, 44*(11), 1461–1477.

Heese, H., & Swaminathan, J. (2006). Product line design with component commonality and cost-reduction effort. *Manufacturing & Service Operations Management, 8*(2), 206–219.

Iida, T. (2012). Coordination of cooperative cost-reduction efforts in a supply chain partnership. *European Journal of Operational Research, 222*(Issue 2), 180–190.

Kim, S., & Netessine, S. (2013). Collaborative cost reduction and component procurement under information asymmetry. *Management Science, 59*(1), 189–206.

Meca, A. (2000). *Juegos Cooperativos Asociados a Problemas de Inventario*. PhD Thesis. Spain: University Miguel Hernández of Elche.

Meca, A. (2007). A core-allocation family for generalized holding cost games. *Mathematical Methods of Operations Research, 65*(Issue 3), 499–517.

Meca, A., García Jurado, I., & Borm, P. (2003). Cooperation and competition in inventory games. *Mathematical Methods of Operations Research, 57*(3), 481–493.

Meca, A., & Sosic, G. (2014). Benefactors and beneficiaries: The effects of giving and receiving on cost-coalitional problems. *Production and Operations Management*. http://dx.doi.org/10.1111/poms.12170, <http://onlinelibrary.wiley.com/doi/10.1111/poms.12170/abstract>.

Meca, A., & Timmer, J. (2008). Supply chain collaboration. In V. Kordic (Ed.), *Supply chain, theory and applications* (pp. 1–18). Vienna, Austria: I-Tech Education and Publishing.

Meca, A., Timmer, J., García Jurado, I., & Borm, P. (2004). Inventory games. *European Journal of Operational Research, 156*(Issue 1), 127–139.

Mekler, V. A. (1993). Setup cost reduction in the dynamic lot-size model. *Journal of Operations Management, 11*, 35–43.

Nagarajan, M., & Sosic, G. (2008). Game-theoretic analysis of cooperation among supply chain agents: Review and extensions. *European Journal of Operational Research., 187*, 719–745.

Porteus, E. L. (1985). Investing in reduced setups in the EOQ model. *Management Science, 31*(8), 998–1010.

Slikker, M., & van den Nouweland, A. (2001). *Social economic networks in cooperative game theory*. Boston, MA: Kluwer Academic Publishers.

Zipkin, P. (2000). *Foundations of inventory management*. New York: McGraw-Hill.