M&SOM MANUFACTURING & SERVICE OPERATIONS MANAGEMENT
Vol. 13, No. 2, Spring 2011, pp. 261–270
ISSN 1523-4614 | EISSN 1526-5498 | 11 | 1302 | 0261
informs®
DOI 10.1287/msom.1100.0317

The Role of Component Commonality in Product Assortment Decisions

Fernando Bernstein, A. Gürhan Kök, Lei Xie
Fuqua School of Business, Duke University, Durham, North Carolina 27708
{fernando@duke.edu, gurhan.kok@duke.edu, lei.xie@duke.edu}

We consider a firm that produces multiple variants of a product. Products are assembled using a combination of common and dedicated components. We characterize the optimal assortment and derive the optimal inventory levels for the common and dedicated components under various bill-of-material configurations. We investigate the effect of commonality on product variety and compare its benefits under different demand characteristics. Commonality always leads to increased profits, but its effect on the level of product variety depends on the type of commonality. If all common components are used for the production of the entire set of products, then the optimal variety level increases relative to the system with no commonality. However, if the common components are used by a subset of the final products, then the optimal variety level may decrease with commonality. We find that the effects of commonality on profit and variety level are stronger under a demand model in which product demands are more variable and exhibit pairwise negative correlation relative to a model with independent demands.

Key words: supply chain management; OM-marketing interface; product variety; consumer choice; inventory management
History: Received: August 6, 2009; accepted: August 8, 2010. Published online in *Articles in Advance* January 21, 2011.

1. Introduction

Providing a wide assortment of variants in a product category is critical for most manufacturers. A higher variety level tends to attract more customers, especially in a competitive environment, as it allows them to find items that more closely match their preferences. In turn, this helps retain customers for future purchases. However, variety is costly from an operational perspective. Higher variety leads to increased manufacturing complexity, for example, in the form of more frequent switch-overs in production, and to a more fragmented product line. That is, adding more variants to the assortment tends to reduce the demand for individual products and, at the same time, increase their relative demand variability. This reduces the extent to which companies can take advantage of economies of scale. Thus, higher inventory levels are required to maintain the same service level.

Companies employ different strategies to support a high level of variety, such as using common components, investing in flexible capacity, and redesigning products and processes to benefit from delayed differentiation—all of these are different forms of operational flexibility. This generates a higher demand volume for the common parts and therefore results in lower operational costs. The benefits of implementing some form of flexibility for a given line of products are well documented in the literature; this paper explores the effect of commonality on assortment decisions. There are three forces at play in our model: (1) Higher variety leads to increased total demand; (2) this also leads to a higher relative demand variability for each product; and (3) at the same time, the demand aggregation effect for the common components mitigates the increased costs associated with higher demand variability. In view of these three effects, we investigate how the choice of an assembly configuration (or bill of materials) affects a firm's assortment and stocking decisions.

In particular, we consider an assemble-to-order system in which a manufacturer produces a number of variants in one product category. Each variant is assembled from several components. There are product-specific (i.e., dedicated) components and components that are common to a subset of (or all) the product variants. We consider two demand models, both introduced in van Ryzin and Mahajan (1999). Under the independent population (IP) model, all consumers make purchasing decisions independently of each other. Under the trend-following (TF) demand model, consumers follow the choice made by the early buyers, which leads to a higher demand variance for each product and to negative correlation between products, yielding the largest risk pooling benefit from a common component. In these settings, we characterize the optimal component inventory levels and the structure of the optimal assortment for some special cases of the bill of materials. Based on these results, we compare a system in which all variants are produced using only dedicated components to systems that incorporate flexibility in the form of component commonality.

We find that the effect of commonality on product assortment decisions depends on how these components are integrated in the bill of materials. If each common component used in manufacturing is shared by all product variants, then the optimal level of variety increases with commonality. In this case, the demand pooling effect results in high enough savings that the firm can introduce more variants. However, this result does not necessarily hold for more general bills of materials. In fact, when some of the common components are shared by a subset of the product variants, the optimal assortment may decrease relative to that in a setting without commonality. This reduction in the optimal assortment occurs when removing a variant that does not use the common component leads to a sufficiently increased scale effect on the variants that do share that component, making it more attractive to reduce the assortment in a way that maximizes the demand pooling effect. In these cases, it may be optimal to rationalize the product line to shift demand to products that share common resources. In this respect, we explore conditions that lead to increased or reduced assortment offerings in general systems. When commonality does increase the depth of the optimal product assortment, we find that the extent of this increase depends on the characteristics of market demand. Specifically, the TF model, which exhibits higher demand variance and pair-wise negative correlation between products, leads to higher increases in the set of products offered relative to the IP model.

Our results are relevant for product line managers (who decide which products to offer) and supply chain managers and product designers (who make procurement or design decisions that affect the bill-of-material configuration). If common components are used to produce all (or most of) the products, then a wide range of variants can be offered. If design limitations preclude the use of common components across the entire product line, then it may be optimal to restrict the product offering to benefit from an increased scale effect for the products that use the common components, leading to the exclusion of some products that do not share those common components. Hence, although introducing commonality between a given set of products increases profitability, the resulting optimal assortment may be smaller, leading to a lower level of variety and market share. Our results also suggest that there may be opportunities to increase variety inexpensively by exploring new product designs that utilize as much component commonality as feasible with the most popular products.

This paper builds on two streams of research: retail assortment planning and component commonality/resource flexibility. The literature on retail assortment planning generally focuses on the consumer choice aspect of assortment decisions and works with the simplest production structure, in which each variant is produced using a dedicated component. Van Ryzin and Mahajan (1999) consider such a model and demonstrate that the optimal assortment consists of a certain number of the most popular products. Other papers in this area include Smith and Agrawal (2000), Cachon et al. (2005), Gaur and Honhon (2006), and Kök and Fisher (2007); see Kök et al. (2008) for a detailed review of this stream of work. The literature on component commonality (e.g., van Mieghem 1998, 2004; Bernstein et al. 2007) and other forms of operational flexibility (e.g., Fine and Freund 1990, Lee and Tang 1997) studies inventory and common-component allocation decisions and explores the resulting value of flexibility for a given fixed assortment of products. Similarly, the literature on assemble-to-order systems investigates optimal inventory policies for a given assortment of products (Song and Zipkin 2003, Song and Zhao 2009). In this paper, we consider a setting in which the set of products offered is an endogenous decision based on the structure of the bill of materials that includes flexible resources in the form of common components and on a consumer choice model like those considered in the assortment planning literature.

Product variety has also been widely investigated in the marketing literature. For example, Kahn (1998) provides a review of papers that explore how product variety influences the revenue potential of a product line. The use of common components in manufacturing may lead to lower costs, but also a to lower degree of product differentiation from the consumers' perspective. This trade-off is investigated in the product line design literature (e.g., Desai et al. 2001, Heese and Swaminathan 2006). A number of papers focus on how to create and implement product variety in manufacturing settings (e.g., Ramdas et al. 2003). Hopp and Xu (2005) consider a product selection problem and show that modularity always leads to higher variety.

The rest of the paper is organized as follows. Section 2 describes the model set-up and §3 presents the results for the dedicated system. Section 4 presents the analysis of systems with common and dedicated components, but in which the common components are used in the production of all product variants. Section 5 contains an analysis of systems with more general bills of materials. Section 6 concludes the paper. All proofs and supplementary material are provided in an electronic companion (Appendix A) and in Appendices B and C, which are available from the authors upon request (Appendices B and C are also available at the authors' websites).

2. The Model

We consider a manufacturer that makes product line decisions regarding the set of product variants to offer in the market. The manufacturer also decides the capacity or inventory levels for the components used in production. Each product variant may represent, for example, a color/size/design combination of a garment or a specific configuration of a personal computer. The set of all variants is denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$, and the firm offers a subset $S \subset \mathcal{N}$. Each variant is produced according to a bill of materials that dictates the components used in its fabrication. There are product-specific (or dedicated) components and components that are common to a subset of (or all) the variants. For example, Nike's recently designed "LunarLite" foam is used in several of the company's shoes, while a specific color may be common to a subset of the sneakers, and a certain pattern may be specific to one product variant. Similar examples apply to apparel and modular products, such as a personal computer. The firm holds inventory of components, and final production/assembly time is negligible. This model structure applies to assemble-to-order systems, such as Dell's personal computers, where the final step of production takes place after customers place their orders for specific product configurations. In the case of Nike, the company has recently introduced an online system that links Nike manufacturing partners to reduce lead times and reduce the percentage of shoes it orders on speculation from 30% to 3%, essentially creating a make-to-order system for the manufacturers (Holmes 2003). The model may also apply to settings in which the manufacturer produces according to orders from retailers, provided that final production or assembly takes place after the retailers' orders are received.

We model demand using a consumer choice model similar to that in van Ryzin and Mahajan (1999). The choice of a product variant within the offered assortment is based on the multinomial logit model. A consumer chooses a product in the assortment or selects the no-purchase option (denoted by 0) to maximize her utility. The expected utility derived from option $i \in S \cup \{0\}$ is given by u_i. The probability of a customer choosing option i is given by

$$q_i^S = \frac{\theta_i}{\sum_{j\in S} \theta_j + \theta_0}, \quad i \in S \cup \{0\},$$

where $\theta_i = e^{u_i}$. (We refer to van Ryzin and Mahajan (1999) for details on the multinomial logit model.) For simplicity, we refer to θ_i as the utility for variant i and let Θ denote the vector $(\theta_1, \theta_2, \ldots, \theta_N)$. We assume that products are indexed in descending order of their popularity (as measured by their utility parameters), i.e., $\theta_1 \geq \theta_2 \cdots \geq \theta_N$.

We assume that consumers choose a variant based on the offered assortment S and that if the selected product is not available, then the sale is backordered or lost.[1] This assumption implies a static or assortment-based substitution, therefore ignoring the dynamics of product substitution that are based on the availability of products at the time of the customer arrival. This model is particularly suitable for catalog retailers and for manufacturers selling customized products. In those settings, consumers choose from a menu of products in a catalog or on a website. Depending on the availability of components, the firm delivers that product immediately or the sale is backordered or lost. The same is the case with apparel manufacturers that receive orders from retailers based on the offered assortment.

Demand arrives over a single selling season. We consider two models of consumer demand, both introduced in van Ryzin and Mahajan (1999). Under the IP model, the total number of customers interested in the product category follows a Poisson distribution with rate λ, and each customer selects a variant independently of the choices that all other customers make. Thus, demand for variant i follows a Poisson distribution process, with rate $\lambda_i = q_i^S \lambda$. We approximate this distribution with a Normal distribution with mean λ_i and standard deviation $\sigma_i = \sigma \lambda_i^\beta$, with $\sigma > 0$ and $0 < \beta < 1$.

We also consider the TF model. The TF model is a stylized model construct that describes a rather extreme representation of customer behavior for products where the trend is established by the leader (early buyers, buyers with influence, reputable consumer reports, etc.), and all other customers follow the trend. This model is appropriate for fashion goods, where customers tend to follow trends set early in the season. Thus, for each variant i, demand D_i is either zero (with probability $1 - q_i^S$) or equal to total market demand (with probability q_i^S). Total market demand is itself a Normal random variable with mean λ and standard deviation σ. We assume that λ is sufficiently larger than σ so that the probability of a negative demand realization is negligible. The trend-following

[1] Because we consider a single-period setting, backorders and lost sales are equivalent from a modeling point of view and would lead to the same profit expression (except for a constant) if the backorder penalty cost equals the profit margin. It is possible to include a salvage value and a shortage penalty cost by a simple redefinition of parameters.

model exhibits pair-wise negative correlation between product demands and a higher demand variance at the individual product level relative to the IP model.[2] In our setting, demand variance and correlation influence not only the capacity decisions and the system's profit, but also the assortment decision.

The bill of materials consists of a set of components $\mathcal{M} = \{1, 2, \ldots, M\}$ and a production matrix that matches components with end products. We assume, without loss of generality, that a finished product requires one unit of each of its components. We investigate special forms of the production matrix. In a dedicated system (denoted D), all components are product specific. We then study a setting (denoted C) in which each variant is produced using a dedicated component and a component that is common to all products. (Although we assume that every product requires two components—dedicated or common—any one of these components may represent a kit of parts or sub-assemblies.) We finally consider settings with more general bills of materials.

As in van Ryzin and Mahajan (1999), we assume identical total production costs and prices across all variants for tractability. The selling price of each variant is p. In system D, the costs of the two dedicated components for each product are k_d and k_c, respectively. In system C, the cost of each dedicated component is k_d and the cost of the common component is k_c (this component replaces all the dedicated components with cost k_c in system D). Under other bills of material, the cost of components is set so that the total unit production cost is $k_d + k_c$ for all products.

The firm's objective is to determine the assortment (which products to offer) and the stocking levels of all components to maximize profit. We define a *popular* set as the set of the n most popular products and denote it by $A_n = \{1, 2, \ldots, n\}$. We denote the optimal profit of the firm when it offers assortment S by Π_S and define $\Pi_n \stackrel{\text{def}}{=} \Pi_{A_n}$. We denote the structure of the bill of materials (D or C) and the demand model (IP or TF) in the superscript of the relevant variables.

3. Preliminaries

We first explore the optimal assortment structure in the dedicated system. In this setting, each product is manufactured using only dedicated components. Because a product requires one unit of each component, the results here essentially assume a single dedicated component per product, with an aggregate cost of $k = k_d + k_c$ per unit.

Given an assortment S, we let $S(\delta) = S \cup \{m\}$ denote the set of variants in S plus an additional variant $m \notin S$ with utility δ. The resulting optimal profit is $\Pi_S^D(\delta) \stackrel{\text{def}}{=} \Pi_{S(\delta)}^D$. Van Ryzin and Mahajan (1999) show that $\Pi_S^D(\delta)$ is a quasi-convex function of δ (implying that, given an assortment S, it is optimal to either add the next most popular product not in S or not add any other variant). The paper also shows that the optimal assortment is a popular set. (We actually prove the result for a more general formulation of the TF model, in which the total market demand is random, whereas this is a constant in van Ryzin and Mahajan 1999. A detailed analysis of this case and the proofs of these results are presented in Appendix A.) This result provides a useful characterization of the optimal assortment, as it reduces the number of sets to be considered for optimization from 2^N to N. Note that the optimal assortment is not necessarily equal to the set of all possible products $\mathcal{N}$. Although adding a variant increases total demand, the relative demand variability for each variant in the assortment also increases, leading to higher inventory costs.

We next present and discuss two technical conditions that we impose throughout the rest of the paper. Because $\Pi_S^D(\delta)$ first decreases and then increases in δ, let δ_n denote the value of $\delta > 0$ such that $\Pi_n^D(0) = \Pi_n^D(\delta_n)$. If the utility of the next most popular product is greater than δ_n (i.e., $\theta_{n+1} > \delta_n$), then it is profitable to include variant $n+1$ in the assortment. The quantity δ_n is a function of the utility vector and of the cost and price parameters. If $\Pi_n^D(\delta)$ is increasing, then we let $\delta_n = 0$. If $\Pi_n^D(\delta) < \Pi_n^D(0)$ for all δ, then we let $\delta_n = \infty$. The conditions are as follows.

Condition 1. *Given a set of utilities $\{\theta_1, \ldots, \theta_N\}$, the following holds for all $n = 1, \ldots, N-1$: If $\delta_n \le \theta_n$, then $\delta_{n-1} - \delta_n \le \theta_n - \theta_{n+1}$.*

Condition 2. *δ_n is increasing in k as long as $\delta_n < \theta_n$.*

Condition 1 implies that the profit function in the dedicated system is quasi-concave in n (see Theorem A.1 in Appendix A). This result, together with the optimality of a popular set, implies that the optimal assortment can be found iteratively by adding one product variant at a time (in a sequence given by their popularity), starting with the empty set. Condition 2 implies that adding a product to the assortment becomes less attractive as production costs increase. Both conditions depend only on the parameters of the model. In our numerical experiments, covering a wide range of parameter values, these conditions are always satisfied. The conditions enable us to compare the size of the optimal assortment across various bill-of-material configurations, but they are not required for the results on the structure of the optimal assortment under any of the system configurations.

[2] Consider an assortment S. Under the TF model, $\mathrm{Cov}(D_i, D_j) = -\lambda^2 q_i^S q_j^S < 0$. Also, setting $\sigma = 1$ and $\beta = 1/2$ in the IP model and $\sigma = \lambda^{1/2}$ in the TF model (both corresponding to the Normal approximation to a Poisson random variable), $\mathrm{var}(D_i) = \lambda q_i^S$ under the IP model and $\mathrm{var}(D_i) = \lambda q_i^S + \lambda^2 q_i^S(1 - q_i^S)$ under the TF model.

4. Systems with Component Commonality

In this section we explore settings in which the bill of materials includes dedicated as well as common components and in which the common component is shared by all offered product variants. This production structure features what we call universal commonality. In its more general form, this system consists of a common component, which can represent a kit of common components, with cost k_c, and a set of dedicated components, such that each product variant is produced using the common component and a dedicated component (possibly representing a kit of dedicated components), with cost k_d. This setting is comparable to the dedicated system, in the sense that the total cost associated with the production of a variant is equal to $k = k_c + k_d$. Figure 1(a) illustrates a system with this bill-of-material configuration.

4.1. IP Model

We begin with the IP demand model. For a given assortment S, let y_i be the stocking level for the dedicated component corresponding to variant i, $i \in S$, and y_c the stocking level for the common component. The corresponding profit is

$$\Pi_S^{C,\,\mathrm{IP}}(\{y_i\}_{i\in S}, y_c) = pE\left[\min\left\{y_c, \sum_{i\in S}\min\{y_i, D_i\}\right\}\right] - \sum_{i\in S} k_d y_i - k_c y_c.$$

As demonstrated in Van Mieghem (1998), $\Pi_S^{C,\,\mathrm{IP}}$ is jointly concave in $(\{y_i\}_{i\in S}, y_c)$.

It is difficult to analytically compare the size of the optimal assortment between systems D and C under the IP demand model, primarily because of the lack of closed-form expressions for the optimal stocking levels in system C. In the dedicated system, each component experiences the same demand as its corresponding finished product. In contrast, in system C, the dedicated components face the same demand stream as in system D, whereas the common component faces a demand stream that is equal to the aggregate demand for all variants. As a result, the profit function is not separable in the stocking quantities. By considering a simplified version of the demand distribution under the IP model, the following result shows that universal commonality increases the size of the optimal assortment.

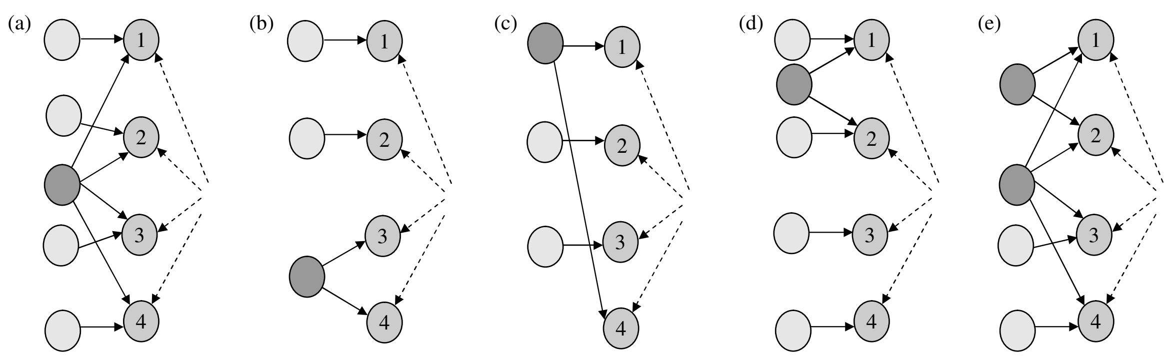
Theorem 1. *Consider a setting with N identical products (i.e., $\theta_i = \theta$ for $i = 1, \ldots, N$ and, for any assortment S with $n = |S|$, $q_i^S = q^S = \theta/(n\theta + \theta_0)$ for any $i \in S$) for sufficiently large N. Furthermore, consider the following two-point demand distribution, defined for a given assortment S:*

$$D_i^S = \begin{cases} \lambda q^S + \sigma^2 & \text{with probability } \dfrac{\lambda q^S}{\lambda q^S + \sigma^2}, \\ 0 & \text{otherwise.} \end{cases}$$

(This distribution preserves the mean and standard deviation of the original Normal distribution, with $\beta = 1/2$.) Then $\Pi_{n+1}^{D,\,\mathrm{IP}} - \Pi_n^{D,\,\mathrm{IP}} \le \Pi_{n+1}^{C,\,\mathrm{IP}} - \Pi_n^{C,\,\mathrm{IP}}$, which implies that the optimal assortment in the system with universal commonality contains at least as many variants as the optimal assortment in the dedicated system.

Theorem 1 states that if the marginal gain from adding a new variant in the dedicated system is positive, then that new variant should be included in the system with commonality as well. This, together with the quasi-concavity of the profit function in the dedicated system as a function of n (see Theorem A.1 in Appendix A), allows us to conclude that replacing a set of dedicated components with a component that is common to all product variants results in an increase in the size of the optimal assortment. Universal commonality leads to a broader product offering because pooling resources for one component of each product

Figure 1 Sample Structures of the Bill of Materials Involving Commonality



reduces the realized overage and underage costs for all products. This mitigates the cost of having a high level of product variety.

For the original model with Normal demand distributions, an extensive numerical study suggests that the optimal assortment in the system with commonality is a popular set and that it is indeed no smaller than the optimal assortment in the dedicated system. The study consists of 360 experiments, all with 10 possible variants and considering two possible demand rates, $\lambda = 200$ and $\lambda = 400$ (further details are presented in Appendix A). Comparing the number of variants in the optimal assortments in systems D and C, we find that component commonality leads to an increase in the level of variety from an average of 6.67 variants to an average of 7.81 variants when $\lambda = 200$ and from an average of 7.65 variants to an average of 8.74 variants when $\lambda = 400$. The impact of commonality is larger at low demand levels ($\lambda = 200$) because demand is more variable in those settings and inventory costs represent a larger portion of total profit. We also find that the effect of commonality on the level of product variety is larger in settings with a relatively high utility of the no-purchase option and in settings in which product utilities have relatively similar values.[3] In all cases, universal component commonality effectively reduces inventory costs, leading to higher variety levels. At the same time, in settings with larger θ_0 or with similar values of the products' utilities, the gains in market share from adding another product to an existing assortment are relatively larger. Hence, the firm has a stronger incentive to increase variety in those settings, therefore amplifying the effect of commonality on both variety level and profit. Finally, we find that when the cost of the common component is higher than the corresponding cost in the dedicated system, the impact of commonality in the increase of product variety may be less significant.

4.2. TF Demand Model

We now focus on the TF demand model. Under this model, all customers purchase the product variant selected by the first customer. (Recall that, at the time of making assortment and component capacity/stocking decisions, there is uncertainty regarding which product variant will be the preferred choice in the market.) The profit function for the firm is given by

$$\Pi_S^{C,\,\mathrm{TF}}(\{y_i\}_{i\in S}, y_c) = \sum_{i\in S} q_i^S pE\min\{y_c, \min\{y_i, D\}\} - \sum_{i\in S} k_d y_i - k_c y_c.$$

Because it is optimal to stock fewer of the dedicated components than of the common component, that is, $y_i \le y_c$, we have that $\Pi_S^{C,\,\mathrm{TF}}(\{y_i\}_{i\in S}, y_c) = \sum_{i\in S}(q_i^S pE\min\{y_i, D\} - k_d y_i) - k_c y_c$. This function is jointly concave in $(\{y_i\}_{i\in S}, y_c)$. Proposition A.1 in Appendix A derives the optimal stocking quantities and the resulting optimal profit in this setting. The result indicates that it is optimal to stock an equal amount of the dedicated components corresponding to a subset of the most popular variants and that this amount is itself equal to the stocking quantity of the common component. The stocking quantities of the remaining variants decrease in order of their popularity. Using the form of the optimal profit function and optimal stocking levels, we characterize the effect of adding one more variant to an existing assortment, which leads to the following result.

Theorem 2. (1) *The function $\Pi_S^{C,\,\mathrm{TF}}(\delta)$ is quasi-convex in δ. Therefore, the optimal assortment is a popular set.* (2) *Moreover, $\Pi_{n+1}^{D,\,\mathrm{TF}} - \Pi_n^{D,\,\mathrm{TF}} \le \Pi_{n+1}^{C,\,\mathrm{TF}} - \Pi_n^{C,\,\mathrm{TF}}$, which implies that the optimal assortment in the system with universal commonality is no smaller than that in the dedicated system.*

This result shows that, under the TF model, the structure of the optimal assortment is preserved with the introduction of universal commonality, but the optimal level of product variety increases (weakly) relative to the dedicated system.

4.3. Comparison of IP and TF Models

We have shown that, under both demand models, replacing a set of dedicated components with a common component shared by all product variants results in a larger optimal assortment. At the same time, for a fixed set of products, the effect of commonality on profit is more significant when the difference between the sum of the individual products' demand variabilities and the aggregate demand variability is relatively larger, as is the case under the TF model, compared with the IP model. In our setting, however, changes in the demand model affect not only the benefits of risk pooling but also—possibly—the size of the optimal assortment. In this section, we report the results of a numerical study that examines the effect of commonality on the optimal variety level under the two demand models.

The study for the TF model is based on the same 360 experiments reported in §4.1, with the exception

[3] Consider the following example with $\lambda = 200$, $p = 10$, $k_d = k_c = 2$, $\sigma = 1$, $\beta = 0.5$, and $\theta_0 = 10$. For $\Theta = (20, 18, 16, 14, 12, 10, 8, 6, 4, 2)$, the optimal variety level increases from 6 in the dedicated system to 7 in the system with commonality, accompanied by a 1.7% increase in optimal profit (from 953.9 to 970.1). In contrast, for $\Theta = (20, 19, 18, 17, 16, 15, 14, 13, 12, 11)$, the optimal variety level increases from 7 in the dedicated system to 9 in the system with commonality, leading to a 2.1% increase in optimal profit (from 968.4 to 989.0).

that each of the 9 possible cost vectors is adjusted to ensure that the average number of variants (across all other 40 parameter combinations) in the dedicated system is the same as that in the IP model. For each instance, we compute the optimal assortment and profit in system D and in system C. Table 2 in Appendix A summarizes the results. The increase in the optimal level of variety is generally higher in terms of frequency and average absolute magnitude in the TF model than in the IP model. Under the IP model, the level of variety increases by an average of 1.12 units. In contrast, in the TF model, the level of variety increases by an average of 2.84 units. Similarly, the percentage profit improvement due to commonality is higher in the TF model (4.15% versus 3.55%). Based on these findings, we conclude that the effect of commonality on the level of variety and on profit is larger under the TF model, even when assortment decisions are endogenous.

4.4. Effect of Cost of Common Component

We have so far assumed that the per unit cost of a common component is the same as its dedicated-component counterpart in the dedicated system (both equal to k_c). In many cases, designing or purchasing a common component may be more costly than designing or purchasing a dedicated component. Proposition B.2 in Appendix B shows that, in a system with dedicated components, the level of variety decreases as unit costs increase. Hence, the addition of commonality (weakly) increases the level of variety if the common component is equal in cost or somewhat more expensive than the dedicated component it replaces. However, the level of variety may decrease in the system with commonality, relative to the dedicated system, if the common component is significantly more expensive. Based on our numerical study, we find that the optimal number of product variants is larger with commonality, even for increases of up to 30% in the cost of the common components.

5. General Systems

In this section we explore systems with general bills of materials. In particular, we consider systems in which common components may be used to produce some, but not all, of the finished products. We consider a few representative structures of the bill of materials, all of which are illustrated in Figure 1. The configuration in (a) represents the system with universal commonality studied in §4. In (b) and (c), there is only one common component shared by two products, whereas the other products are built using dedicated components only. In (d), two products use both dedicated and common components, and the other two only use dedicated components. In (e), all products share a common component and require an additional component, which is a dedicated component each for products 3 and 4 and a common component for products 1 and 2. We refer to the systems in (b)–(e) as systems with partial component commonality. The results obtained in this section for the settings depicted in Figure 1(b)–(e) allow us to derive insights regarding the effect of commonality in product assortment decisions in systems with general bills of materials.

We start by considering a simple setting with N product variants and $M = N - 1$ components, in which two variants, i and j, share a common component and all other variants use dedicated components (as illustrated in Figure 1(b) and (c)). We compare the optimal assortment in this system with that in the system with only dedicated components. To make the comparison meaningful, we keep the cost of producing each variant equal in both systems. That is, in the system with dedicated components, the cost of each dedicated component is given by $k_d + k_c$, whereas in the system with partial commonality the cost of the common component is $k_d + k_c$, and all other products continue to use the same dedicated components as in the original system. (Recall that a popular set is defined according to the order given by the utilities in the vector Θ.)

Proposition 1. *Consider a system with partial component commonality in which variants i and j are produced using a common component and all other variants use dedicated components. For both demand models, the optimal assortment is either a popular set or it consists of variants i and j plus a popular subset of the remaining variants.*

Proposition 1 implies that in a system with a general bill of materials, the optimal assortment may not necessarily be a popular set. For example, in Figure 1(c), under the IP model with parameters $p = 10$, $k_d = k_c = 3$, $\lambda = 150$, $\sigma = 1$, $\beta = 0.5$, $\theta_0 = 5$, and $\Theta = (16, 15, 8, 1)$, the optimal assortment is $\{1, 2, 4\}$. In contrast, under the same parameter values, but in the setting in which all variants use dedicated components, the optimal assortment is $\{1, 2, 3\}$. That is, pooling the resources used by products 1 and 4 makes it more profitable to include the least popular variant in the assortment, rather than keeping variant 3, to benefit from the scale generated by aggregating the demands for these products. We next analyze the size of the optimal assortment in settings with commonality, relative to the dedicated system.

Proposition 2. *Consider either the IP or the TF demand models. Let A^{i+j} be the optimal assortment of a system in which variants i and j $(i < j)$ are produced using a common component and all other variants use dedicated* *components. Let A_n be the optimal assortment in the corresponding dedicated system in which all variants are produced using dedicated components.*

(1) *If $i, j \leq n$, then there exists a threshold t_1 such that A^{i+j} is a strict subset of A_n containing variants i and j if $\theta_n < t_1$, and $A^{i+j} = A_n$ otherwise.*

(2) *If $i < n$ and $j > n$, then there exists a threshold t_2 such that A^{i+j} is a strict subset of A_n containing variants i and j if $\theta_n < t_2$, and $A^{i+j} = A_n \cup \{j\}$ otherwise.*

(3) *If $i, j > n$, then, there exist thresholds $t_3 > t_4$ such that*

(a) $A^{i+j} = A_n \cup \{i, j\}$, *if* $t_3 < \theta_i + \theta_j \leq \theta_n$ *or* $t_4 \leq \theta_n < \theta_i + \theta_j$;

(b) $A^{i+j} = A_n$, *if* $\theta_i + \theta_j < \min(t_3, \theta_n)$;

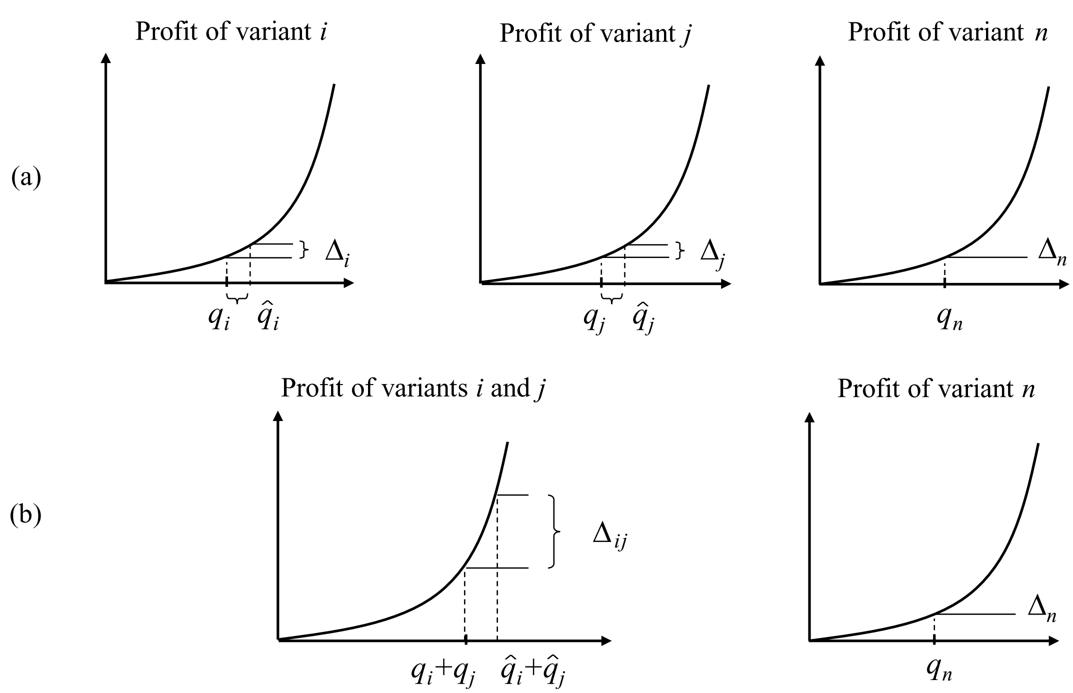
(c) $A^{i+j} = A' \cup \{i, j\}$, *where* A' *is a strict subset of* A_n, *if* $\theta_n < \min(t_4, \theta_i + \theta_j)$.

This result provides a complete characterization of the optimal assortment in the system with a common component for variants i and j. The optimal assortment actually depends on the relative "popularity" of variants i and j (given that $i < j$). In particular, if either variant i or both variants i and j are in the optimal assortment of the original dedicated system, then both variants are in the optimal assortment in the system with partial commonality as well (Proposition 2, parts 1 and 2). However, in this setting, the optimal assortment may be smaller than that in the dedicated system. If neither variant i nor variant j is in the optimal assortment of the dedicated system, then it is optimal to offer these two variants in the system with commonality only if the sum of their utilities is large enough (Proposition 2, part 3). At the same time, if that is the case, then one or more variants from the optimal assortment of the dedicated system may be dropped in the optimal assortment of the system with partial commonality (Proposition 2, part 3(*c*)).

Along the lines of the system studied in Proposition 2, if a product designer had the liberty to choose the two products that should share a common component, then the profit-maximizing solution would be achieved by pooling the components of the two most popular products (see Proposition B.6 in Appendix B). At the same time, Proposition 2 suggests that this may result in a smaller optimal assortment. With partial commonality, the extent of product variety can only increase if commonality is used with at least one product outside the optimal assortment in the dedicated system. In this way, new variants may be introduced to the assortment at a relatively low cost, benefiting from the operational scale of the products already in the assortment.

We conclude that, in sharp contrast to the results in §4, replacing a strict subset of dedicated components with a common component may result in a reduced number of variants in the optimal assortment, and this set may no longer be a popular set. To understand why partial commonality may reduce the optimal assortment, let us consider the case with $i < j < n$. A setting with $n = 3$, $i = 1$, and $j = 2$ is illustrated in Figure 2. The graphs in (a) show the profit of each variant in the dedicated system, whereas the graphs in (b) show the combined profit associated with variants i and j and the profit for variant n in the

Figure 2 **Profit of Variants in the Dedicated and Partial-Commonality Systems**



system with partial commonality. Variant n is in the optimal assortment of the dedicated system because the profit associated with this variant is higher than the aggregate increase in profit that the other products in the assortment would experience without variant n (recall that the choice probabilities—and therefore the expected demand rates—of the other products in the assortment increase if variant n is removed from the assortment, in this case from q_i to $\hat{q}_i$ and from q_j to $\hat{q}_j$). That is, $\Delta_n > \Delta_i + \Delta_j$ in Figure 2(a). In contrast, in the system with partial commonality, the common component shared by variants i and j experiences a larger demand rate (and therefore a more pronounced scale effect) than the dedicated component for variant i does in the dedicated system. Note that the profit function of an individual product variant is convex increasing in its demand rate (because of the economies of scale associated with inventory costs). Hence, in the system with partial commonality, the profit that variant n generates may no longer be larger than the profit impact of the demand spillover effect (in particular, onto the common component for variants i and j) that occurs in the absence of variant n. In reference to Figure 2(b), we may have that $\Delta_n < \Delta_{ij}$. In that case, variant n would be excluded from the assortment.

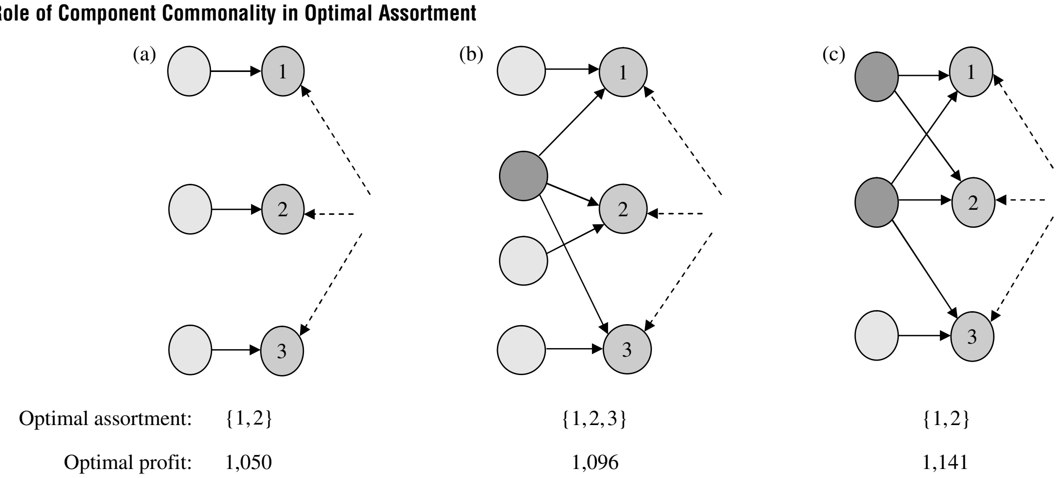
We next consider two generalizations of the system, represented in Figures 1(d) and (e). In Figure 1(d), variants i and j are assembled from one common component and one dedicated component each (with costs k_c and k_d, respectively); in the corresponding dedicated system, these variants are assembled using only dedicated components (each with cost $k_d + k_c$). In Figure 1(e), there is a component common to all products and another component that is just common products i and j. All other variants require, in addition, a dedicated component. The results for the system in Figure 1(d) are similar to those in Proposition 2. That is, the optimal assortment in this setting is either a popular set or a popular set plus variants $\{i, j\}$. Also, the number of variants in the optimal assortment associated with the system with partial commonality may decrease, remain the same, or increase relative to the dedicated system. Regarding the system in Figure 1(e), we compare it to the system in Figure 1(a) to examine the effect of introducing an additional common component to a subset of the variants in a system with universal commonality. The optimal assortment in the system shown in Figure 1(a) is itself no smaller than that in the related system with only dedicated components. Here too, the optimal assortment in the system with partial commonality shown in Figure 1(e) may be larger, equal to, or smaller than that in the system with universal commonality. The existence of a common component shared by a strict subset of the product variants again creates an imbalance in terms of the scale advantage generated by demand aggregation—this benefit is higher for those products that use a larger number of common components. As a result, it may be optimal to exclude a product that makes less use of commonality to drive more demand to those that use more common components and therefore increase the benefits associated with demand pooling. (Propositions B.7 and B.8 in Appendix B contain formal statements and proofs of these results.)

To close, although introducing commonality to a manufacturing system always increases total profit (as long as the costs of the components do not change), the effect on the optimal variety level is not necessarily monotone in the amount of commonality. Figure 3 shows an example based on the TF model with $p = 10$, $k_d = k_c = 0.3$, $\lambda = 150$, $\theta_0 = 5$, and $\Theta = (12, 11, 5.8)$.

6. Conclusion

This paper considers the role of component commonality in product assortment and component inventory decisions in assemble-to-order systems. We character-

Figure 3 **Role of Component Commonality in Optimal Assortment**



ize and compare the structure of the optimal assortment and the optimal inventory levels for various bill-of-material configurations. Although component commonality increases total system profit, we find that the effect of commonality on the optimal assortment depends on the specific configuration of the bill of materials. With universal commonality (a common component is shared by all variants), the optimal assortment is (weakly) larger than that in the system without commonality. Furthermore, the assortment consists of a subset of the most popular variants (as in the case without commonality). However, these results do not hold for more general bill-of-material configurations. If the common component is shared by a strict subset of the variants, then the optimal assortment may no longer be a popular set, and the optimal level of variety may increase or decrease relative to the system without commonality. In particular, the level of product variety (weakly) decreases when commonality is introduced for products that are already in the optimal assortment corresponding to the dedicated system (i.e., products with high demand volumes). This result may hold even when commonality is introduced in a system that already involves other components shared by all product variants. These findings indicate that product line and supply chain managers must be aware of the effect that introducing commonality in manufacturing may have on optimal product line decisions, such as the removal of products with relatively small demand. This is particularly important if market share and market coverage (level of product variety) are critical to the competitive position of a firm. Finally, we find that the effect of commonality on both profit and variety level is more pronounced when demands are more variable and exhibit pair-wise negative correlation. In an alternative interpretation of our model, the results apply to the use of flexible resources and the related capacity decisions (rather than common components and the corresponding inventory decisions).

Electronic Companion

An electronic companion to this paper is available on the *Manufacturing & Service Operations Management* website (http://msom.pubs.informs.org/ecompanion.html).

Acknowledgments

The authors are grateful to the editor-in-chief, associate editor, and the referees for helpful comments that have helped improve the paper.

References

Bernstein, F., G. DeCroix, Y. Wang. 2007. Incentives and commonality in a decentralized multiproduct assembly system. *Oper. Res.* **55**(4) 630–646.

Cachon, G. P., C. Terwiesch, Y. Xu. 2005. Retail assortment planning in the presence of consumer search. *Manufacturing Service Oper. Management* **7**(4) 330–346.

Desai, P., S. Kekre, S. Radhakrishnan, K. Srinivasan. 2001. Product differentiation and commonality in design: Balancing revenue and cost drivers. *Management Sci.* **47**(1) 37–51.

Fine, C., R. Freund. 1990. Optimal investment in product-flexible manufacturing capacity. *Management Sci.* **36**(4) 449–466.

Gaur, V., D. Honhon. 2006. Product variety and inventory decisions under a locational consumer choice model. *Management Sci.* **52**(10) 1528–1543.

Heese, S., J. Swaminathan. 2006. Product line design with component commonality and cost reduction effort. *Manufacturing Service Oper. Management* **8**(2) 206–219.

Holmes, S. 2003. Nike. *BusinessWeek Online*. http://zdnet.businessweek.com/magazine/content/03_47/b3859634.htm.

Hopp, W., X. Xu. 2005. Product line selection and pricing with modularity in design. *Manufacturing Service Oper. Management* **7**(3) 172–187.

Kahn, B. 1998. Dynamic relationships with customers: High-variety strategies. *J. Acad. Marketing Sci.* **26**(1) 45–53.

Kök, A. G., M. Fisher. 2007. Demand estimation and assortment optimization under substitution: Methodology and application. *Oper. Res.* **55**(6) 1001–1021.

Kök, A. G., M. Fisher, R. Vaidyanathan. 2008. Assortment planning: Review of literature and industry practice. N. Agrawal, S. A. Smith, eds. *Retail Supply Chain Management*. Springer, New York.

Lee, H., C. Tang. 1997. Modeling the costs and benefits of delayed production differentiation. *Management Sci.* **43**(1) 40–53.

Ramdas, K., M. Fisher, K. Ulrich. 2003. Managing variety for assembled products: Modeling component systems sharing. *Manufacturing Service Oper. Management* **5**(2) 142–156.

Smith, S., N. Agrawal. 2000. Management of multi-item retail inventory systems with demand substitution. *Oper. Res.* **48**(1) 50–64.

Song, J-S., Y. Zhao. 2009. The value of commonality in a dynamic inventory system with lead times. *Manufacturing Service Oper. Management* **11**(3) 493–508.

Song, J-S., P. Zipkin. 2003. Supply chain operations: Assemble-to-order systems. A. G. de Kok, S. Graves, eds. *Handbooks in Operations Research and Management Science*, Vol. 11. *Supply Chain Management*. Elsevier, Amsterdam, The Netherlands, 561–596.

Van Mieghem, J. 1998. Investment strategies for flexible resources. *Management Sci.* **44**(8) 1071–1078.

Van Mieghem, J. 2004. Commonality strategies: Value drivers and equivalence with flexible capacity and inventory substitution. *Management Sci.* **50**(3) 419–424.

van Ryzin, G., S. Mahajan. 1999. On the relationship between inventory costs and variety benefits in retail assortment. *Management Sci.* **45**(11) 1496–1509.