Math 107, Week 9 Questions to Practice

(1) Recall that one way of obtaining the change-of-coordinates matrix $P_{C \leftarrow B}$ from $B = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$ to $C = \{\mathbf{c}_1, \dots, \mathbf{c}_n\}$ is performing the row reduction

$$\begin{bmatrix} P_C & P_B \end{bmatrix} \rightsquigarrow \begin{bmatrix} I_n & P_{C \leftarrow B} \end{bmatrix},$$

where

$$P_C = \begin{bmatrix} \mathbf{c}_1 & \dots & \mathbf{c}_n \end{bmatrix} \quad \text{and} \quad P_B = \begin{bmatrix} \mathbf{b}_1 & \dots & \mathbf{b}_n \end{bmatrix}.$$

Show in general that if F and G are $n \times n$ matrices and G is invertible, then the row reduction

$$\begin{bmatrix} G & F \end{bmatrix} \rightsquigarrow \begin{bmatrix} I_n & X \end{bmatrix}$$

yields $X = G^{-1}F$. This is turn shows that $P_{C \leftarrow B} = P_C^{-1} P_B$.

Solve the following questions from the orange textbook by Lay, Lay and McDonald.
4.7 : 1, 7, 13, 15, 16, 20(a)