Math 107, Week 7 Questions to Practice

(1) Let V, W be vector spaces (over $\mathbb{R}$) with bases $B = \{\mathbf{b}_1, \ldots, \mathbf{b}_k\}$, $\widetilde{B} = \{\tilde{\mathbf{b}}_1, \ldots, \tilde{\mathbf{b}}_n\}$, respectively. Furthermore, suppose $T : V \to W$ is a linear transformation.

It can be shown that the transformation $\widetilde{T} : \mathbb{R}^k \to \mathbb{R}^n$ defined by $\widetilde{T}([\mathbf{v}]_B) = [\mathbf{w}]_{\widetilde{B}}$ whenever $T(\mathbf{v}) = \mathbf{w}$ is linear. Show that T is one-to-one if and only if the set of columns of the standard matrix A of $\widetilde{T}$ is linearly independent.

(2) Let $T : \mathbb{P}_2 \to \mathbb{P}_2$ be the linear transformation such that

$$T(1) = 1 + x^2, \quad T(x) = 1 + 2x - x^2, \quad \text{and} \quad T(x^2) = 2 - 2x + 4x^2$$

Using the result of question (1) determine whether T is one-to-one or not.

(3) Find an isomorphism between the vector spaces

$$\widetilde{M}_{2\times 2} := \left\{ \begin{bmatrix} a & b \\ b & c \end{bmatrix} \mid a, b, c \in \mathbb{R} \right\}, \text{ and}$$

$$\mathbb{P}_4^E := \text{ The set of even polynomials } p : \mathbb{R} \to \mathbb{R} \text{ of degree at most four,}$$

hence show that these two vector spaces are isomorphic.

Solve the following questions from the orange textbook by Lay, Lay and McDonald.
4.2 : 31, 33, 34, 35
4.3 : 9, 13, 15, 32, 33
4.4 : 3, 7, 13, 21, 25, 27