Math 107, Week 6 Questions to Practice

(1) Let $H = \text{span}\{1-x, x-x^2, x^2-1, x\}$ over $\mathbb{F} = \mathbb{R}$.

(a) Find a basis for H.

(b) Show that $H = \mathbb{P}_2$.

(2) Let S_1, S_2 be two subspaces of a vector space V. Prove or disprove that the following sets are subspaces.

(a) $S_1 \cap S_2$

(b) $S_1 \cup S_2$

(c) $S_1 + S_2 := \{v_1 + v_2 \mid v_1 \in S_1,\ v_2 \in S_2\}$

(d) $S_1 \setminus S_2$

(3) Are the following subsets of the vector space

$$M_{2\times 2} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \right\}$$

consisting of 2×2 matrices with real entries (over $\mathbb{F} = \mathbb{R}$) subspaces? If the subset is a subspace, find also a basis for the subspace.

(a) The set of 2×2 upper triangular matrices given by

$$\left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in \mathbb{R} \right\}$$

(b) The set of 2×2 orthogonal matrices given by

$$\{Q \in M_{2\times 2} \mid Q^{-1} = Q^T\}$$

4. Let V be a vector space with an addition $+$ and a multiplication with scalar $\cdot$ over the scalars $\mathbb{F}$. Prove the following using the basic axioms of a vector space.

(a) $-\mathbf{v} \in V$ is unique for every $\mathbf{v} \in V$.

(b) $(-1)\mathbf{v} = -\mathbf{v}$ for every $\mathbf{v} \in V$.

(c) $c\mathbf{0} = \mathbf{0}$ for every $c \in \mathbb{F}$.

Solve the following questions from the orange textbook by Lay, Lay and McDonald.
4.1 : 3, 6, 7, 11, 19, 23, 24
4.2 : 3, 15, 17, 25
4.3 : 3, 12, 24, 26