

Math 107, Week 5 Questions to Practice

(1) Let A, B be $n \times n$ matrices. Prove that AB is invertible if and only if both A and B are invertible.

Furthermore, show that if A and B are invertible, then $(AB)^{-1} = B^{-1}A^{-1}$.

(2) Let A be an $n \times n$ matrix. Prove that A is invertible if and only if A^T is invertible.

Furthermore, show that if A is invertible, then $(A^T)^{-1} = (A^{-1})^T$.

(3) Find the 3×3 elementary matrices E_1, E_2, E_3 such that

$$E_3 E_2 E_1 \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} = I_3.$$

4. Let A and X be $n \times n$ matrices such that $AX = I_n$. Show that A is invertible.

(Hint: Show that the columns of A span \mathbb{R}^n .)

Solve the following questions from the orange textbook by Lay, Lay and McDonald.

2.2 : 24

2.3 : 3, 5, 13, 17, 34