Math 107, Week 4 Questions to Practice

(1) Prove the following for a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$:

(i) If $m > n$, then T cannot be onto;

(ii) If $n > m$, then T cannot be one-to-one.

(2) Let us consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined as follows: $T(\mathbf{x})$ is obtained from $\mathbf{x} = (x_1, x_2, x_3)$ by rotating the vector of its first two components (x_1, x_2) by an angle of θ in the counter clock-wise direction leaving x_3 unchanged, followed by a reflection about the plane $x_3 = 0$. Find the standard matrix of T.

(3) Let $T_1 : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation such that

$$T_1(1,0,0) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \quad T_1(0,1,0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad T_1(0,0,1) = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix},$$

and $T_2 : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation such that

$$T_2(1,0,0) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \quad T_2(0,1,0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad T_2(0,0,1) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}.$$

(a) Find the standard matrix of $T_2(T_1(\mathbf{x}))$.

(b) Find the standard matrix of $T_1^{-1}(\mathbf{x})$.

(4) Let A be an $m \times n$ matrix and B be an $n \times p$ matrix. Prove that

$$(AB)^T = B^T A^T.$$

Solve the following questions from the orange textbook by Lay, Lay and McDonald
1.9 : 11, 22, 23, 24
2.1 : 5, 11, 15, 17, 22, 27, 28
2.2 : 3, 7, 11, 13, 20, 31, 35