Math 107, Week 3 Questions to Practice

(1) Find the standard matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ that reflects about the plane $x_1 = x_2$.

(2) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and $\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_k \in \mathbb{R}^n$ be given vectors. Show that if $\{T(\mathbf{a}_1), T(\mathbf{a}_2), \ldots, T(\mathbf{a}_k)\}$ is linearly independent, then $\{\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_k\}$ is also linearly independent.

(3) Let us consider an $m \times n$ linear system $A\mathbf{x} = \mathbf{b}$ with two free variables. Furthermore assume $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3 \in \mathbb{R}^n$ are particular solutions satisfying $A\mathbf{p}_1 = A\mathbf{p}_2 = A\mathbf{p}_3 = \mathbf{b}$ such that $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ is linearly independent.

Write down the solution set of $A\mathbf{x} = \mathbf{b}$ in terms of $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$.

Solve the following questions from the orange textbook by Lay, Lay and McDonald
1.5 : 6, 13, 18, 30
1.7 : 6, 7, 14, 29, 39
1.8 : 15, 17, 24, 32, 35
1.9 : 2, 9, 26