Math 107, Week 10 Questions to Practice

(1) Prove by induction that the determinant of an $n \times n$ matrix with two identical rows is equal to zero.

(2) Consider an $n \times n$ matrix

$$A = \begin{bmatrix} q_{11}(s) & q_{12}(s) & \dots & q_{1n}(s) \\ q_{21}(s) & q_{22}(s) & & q_{2n}(s) \\ \vdots & & \ddots & \\ q_{n1}(s) & q_{n2}(s) & & q_{nn}(s) \end{bmatrix}$$

that depends on a real parameter s, where $q_{ij} : \mathbb{R} \to \mathbb{R}$ is a polynomial of degree two in s for $i, j = 1, \dots, n$. Now consider those s such that A is invertible with the inverse

$$A^{-1} = \begin{bmatrix} r_{11}(s) & r_{12}(s) & \dots & r_{1n}(s) \\ r_{21}(s) & r_{22}(s) & & r_{2n}(s) \\ \vdots & & \ddots & \\ r_{n1}(s) & r_{n2}(s) & & r_{nn}(s) \end{bmatrix}.$$

What kind of functions are $r_{ij} : \mathbb{R} \to \mathbb{R}$ for $i, j = 1, \dots, n$? For instance, are they polynomials, if yes what are their degree? Or, are they rational functions, if yes what are the degrees of the polynomials that appear in the numerators and the denominators of the rational functions?

3. Prove that the determinant of an $n \times n$ matrix A is the product of all of its eigenvalues.
(Hint: $\det A = p(0)$ where $p(\lambda) := \det(A - \lambda I_n)$.)

Solve the following questions from the orange textbook by Lay, Lay and McDonald.
3.1 : 3, 5, 9, 41
3.2 : 5, 9, 21, 39
3.3 : 3, 7, 11
5.1 : 6, 11, 15, 23, 25
5.2 : 3, 5, 15