Math 304 (Spring 2010) - Numerical Methods
Syllabus

Instructor
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Lecture Hours and Location
Monday, Wednesday between 15:30-16:45 at SOS Z27

Textbook
Applied Numerical Analysis Using Matlab, 2nd Ed by Laurene V. Fausett

The textbook touches on almost all topics that will be covered in class. However, it is a light book theoretically. We will occasionally depart from the textbook to analyze the convergence and accuracy of the numerical algorithms (or at the very least to justify why the algorithm works). Therefore a wise student should attend the lectures. Lectures will be compatible with the textbook, however it will go deeper than the textbook on certain topics.

Supplementary Books

- Numerical Mathematics by Alfio Quarteroni, Riccardo Sacco and Fausto Saleri
- Fundamentals of Matrix Computations, 2nd Ed by David S. Watkins

An electronic copy of the book by Quarteroni, Sacco and Saleri can be accessed through library (visit the website `http://libunix.ku.edu.tr/` and search for this book). The book by Watson is available at the reserve desk in the library.

Prerequisites
Math 203 with a grade D or better

Course Webpage
`http://home.ku.edu.tr/~emengi/teaching/math304/math304.html`

Grading
Homeworks will be assigned weekly. There will be two midterms and a final. Your overall grade will be determined based on following scheme.

Total Score = %20 (Homework Score) + %20 (Midterm 1) + %25 (Midterm 2) + %35 (Final)

Keep in mind that there will be a curve in the end. Your letter grade will depend on your rank in the class.

Midterm
The midterms will be held during the regular lecture hour on the following dates.

- Midterm 1 - March 24, Wednesday
- Midterm 2 - May 5, Wednesday

The midterms will be closed-book exams. Everything covered in class until the end of the previous week is included in each midterm.

Final
All of the topics covered in class throughout the semester are included. Date, time and location of the final will be announced later towards the end of the semester. The final will be a closed-book exam.

Homeworks
The homeworks will be assigned weekly on Fridays. Each homework is due following week on Friday by 4pm. Your homework score will be the average of the homeworks. Each homework will consist of 3-4 questions and should not require more than a few hours. Half of the homework questions will be conceptual. The remaining half will be computational and require performing computations in Matlab. You can reach a freely available Matlab manual from the website `http://www.math.mtu.edu/~msgocken/intro/intro.html`

Attendance
Attendance will not be taken in class. We will occasionally go beyond the textbook. Therefore I highly recommend to attend classes regularly. On random dates I will challenge you with questions. If you can solve it on the board, you will be awarded one bonus point overall.

Description
This course covers some of the most fundamental topics in numerical analysis. The first part concerns numerical linear algebra. We will introduce numerical algorithms for the solutions of linear systems, linear least squares problems (best approximate solution for an inconsistent linear system) and eigenvalue problems. In each case we will analyze the efficiency and accuracy of the algorithm in the presence of rounding errors. Special emphasis will be put on matrix factorizations, in particular LU and QR factorizations.

In the second part we will learn how to solve nonlinear systems mainly by using Newton's method. As a special application we will consider unconstrained optimization. Then we will spend some time on interpolation and numerical integration. Finally we will focus on the numerical solution of differential equations.

There will be a fine balance between theoretical and computational issues. Convergence of the iterative algorithms, for instance for eigenvalue problems and nonlinear systems, will be analyzed. At the very least we will justify why these algorithms converge to the actual solution and how quickly they converge. In the homeworks you will apply these numerical algorithms to real world problems.

Course Calendar

The section from the textbook corresponding to each topic is written next to the topic in paranthesis.

- **(Week 1)** Fundamental Problems (1.1), Fundamentals of Numerical Computation, Rounding Errors (1.3.1), Linear Systems - Basics
- **(Week 2)** Gaussian Elimination (3.1), LU factorization (4.1)
- **(Week 3)** Householder Transformations (4.2.1), QR Factorization (4.3)
- **(Week 4)** Eigenvalues and Eigenvectors - Basics, Power Iteration and Rayleigh Quotient (5.1)
- **(Week 5)** Inverse Iteration and Rayleigh Quotient Iteration (5.2), The QR Algorithm (5.3)
- **(Week 6)** Nonlinear Systems, Bisection Method (2.1), Newton's Method for Univariate Functions (2.3), Secant Method (2.2)
  Midterm 1 - March 24, Wed (Covers weeks 1-5)
- **(Week 7)** Newton's Method for Multivariate Functions (7.1.1), Convergence Analysis of Newton's Method
- **(Week 8)** Unconstrained Optimization (7.2), Steepest Descent and Newton's Method for Unconstrained Optimization (7.2), Least Squares Problem (9.1)
- **(Week 9)** Numerical Solution of Least Squares Using QR Factorization, Interpolation, Lagrange Interpolation (8.1.1), Hermite Interpolation (8.2)
- **(Week 10)** Numerical Integration (11.2), Orthogonal Polynomials in particular Legendre and Chebyshev Polynomials (9.2.2-4), Gaussian Quadrature (11.3)
- **(Week 11)** Numerical Solution of Ordinary Differential Equations, Euler's Method (12.1)
  Midterm 2 - May 5, Wed (Covers weeks 1-10 with emphasis on weeks 6-10)
- **(Week 12)** Multistep Methods (12.3), Runge-Kutta Methods (12.2)
- **(Week 13)** Systems of ODEs (13.1), Stiff ODEs and their Numerical Solution (13.2)
- **(Week 14)** Partial Differential Equations, Poisson Equation and its Solution via Finite Difference Schemes (15.3)