MATH 504: Numerical Methods - I

Midterm2 - Fall 2011

Duration : 90 minutes

#1	20	
#2	20	
#3	20	
#4	20	
#5	20	
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NAME ____________________

STUDENT ID ____________________

SIGNATURE ____________________

- Put your name and student ID in the boxes above.
- No calculators or any other electronic devices are allowed.
- This is a closed-book and closed-notes exam.
- Show all of your work; full credit will not be given for unsupported answers.

Question 1. Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix}$$

(a) Find a left inverse A^L of A.
(b) Find the projector onto Range(A) along Null(A^L).
(c) Find the pseudoinverse B^+ of B.
(d) Find the orthogonal projector onto Range(B).

Question 2. Suppose that the product $y = Ax$ is performed in the IEEE floating point arithmetic where $x \in \mathbb{C}^n$ and $A \in \mathbb{C}^{n\times n}$

Find a tight upper bound for the relative forward error

$$\frac{\|\tilde{y} - y\|_1}{\|y\|_1}$$

where $\tilde{y}$ is the computed value of y.
(Note: Tight means for some matrices the upper bound becomes an exact equality.)

Question 3. Every matrix $A \in \mathbb{C}^{m\times n}$ has a factorization of the form

$$A = QL$$

where $Q \in \mathbb{C}^{m\times m}$ is unitary and $L \in \mathbb{C}^{m\times n}$ is lower triangular.

Devise an algorithm to calculate the QL factorization of $A \in \mathbb{C}^{m\times n}$ based on Householder reflectors. Write down a pseudocode for your algorithm and calculate the total number of flops required.

Question 4. The life expectancy in Canada is listed at various years in the table below.

t (year)	$s = (t - 1950)/5$	y (life expectancy at birth)
1965	$s_1 = 3$	$y_1 = 72$
1970	$s_2 = 4$	$y_2 = 73$
2010	$s_3 = 12$	$y_3 = 82$

(a) Pose the problem of finding the line $\ell(s) = x_1 s + x_0$ minimizing

$$\sqrt{\sum_{i=1}^{3} (\ell(s_i) - y_i)^2}$$

with respect to the unknows $x_0, x_1 \in \mathbb{R}$ as a least-squares problem of the form

$$\text{minimize}_{x\in\mathbb{R}^2} \|Ax - b\|_2.$$

(b) Solve the least squares problem in **(a)**, consequently determine the line that best represents the life expectancy in Canade in the least-squares sense, by using the normal equations. Is the solution to the least squares problem unique?

Question 5. This question concerns the LU factorization for tridiagonal matrices. A matrix is $T \in \mathbb{C}^{n\times n}$ is tridiagonal if $t_{ij} = 0$ for all i, j such that $|i - j| > 1$.

(a) Calculate an LU factorization for the tridiagonal matrix

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix}.$$

(b) Suppose that a tridiagonal matrix T is row-reducible to an upper triangular matrix by only applying row-replace operations. Show that $T = LU$ for some lower triangular matrix L and upper triangular matrix U with the property that $\ell_{ij} = u_{ij} = 0$ for all i, j such that $|i - j| > 1$.