# MATH 106: Calculus

Midterm 2 - Fall 2009

Duration : 90 minutes

NAME ____________________

STUDENT ID ____________________

SIGNATURE ____________________

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|---|---|---|
| #1 | 20 | |
| #2 | 15 | |
| #3 | 20 | |
| #4 | 15 | |
| #5 | 15 | |
| #6 | 15 | |
| $\Sigma$ | 100 | |

- Put your name, student ID and signature in the boxes above.
- No calculators or any other electronic devices are allowed.
- This is a closed-book and closed-notes exam.
- Show all of your work; full credit will not be given for unsupported answers.
- Write your solutions clearly; no credit will be given for unreadable solutions.
- Mark your section below.

  SECTION 1 (SULTAN ERDOĞAN DEMIR, MW 11:30-13:20) ____________________

  SECTION 2 (SULTAN ERDOĞAN DEMIR, MW 14:30-16:20) ____________________

  SECTION 3 (EMRE MENGI, MW 9:30-11:20) ____________________

  SECTION 4 (EMRE MENGI, MW 14:30-16:20) ____________________

  SECTION 5 (KAZIM BÜYÜKBODUK, TuTh 11:30-13:20) ____________________

  SECTION 6 (KAZIM BÜYÜKBODUK, TuTh 14:30-16:20) ____________________

**Question 1.** Evaluate the limit in each part. Show the details of your work.

**(a)** $\lim_{x \to 0} \dfrac{\cos x - 1}{x^2}$

**(b)** $\lim_{x \to 1} \dfrac{(x-1)^2}{\arcsin x}$

**(c)** $\lim_{x \to 0^+} \left( \dfrac{1}{\ln(x+1)} - \dfrac{1}{\sin x} \right)$

**(d)** $\lim_{\theta \to 0} (\cos \theta)^{1/\theta^2}$

**Question 2.**

**(a)** Find $\dfrac{dy}{dx}$ if $y = \cos^2(\ln x)$. Do not simplify your answer.

**(b)** Find $\dfrac{dy}{dx}$ if $y + \sec(xy) = 2x^3 + y^4$.

**(c)** Find $\dfrac{dy}{dx}$ at $(e, 0)$ if $x^y = \ln(x + y)$.

**Question 3.** Consider the function $f(x) = x^3 + 2x^2 + x$.

**(a)** Find the interval(s) on which $f$ is increasing and the interval(s) on which $f$ is decreasing.

**(b)** Find the interval(s) on which $f$ is concave up and the interval(s) on which $f$ is concave down.

**(c)** Find the critical points of $f$. Classify each of these critical points as a local minimum, local maximum or neither.

**(d)** Find the points over the interval $[-2, -\frac{1}{2}]$ at which $f$ has a global minimum and a global maximum.

**Question 4.** An object moves along the curve $x^4y^2 = 1$. If the rate of change of the $x$-coordinate of the object is constant and equal to $-1$ *units/s*, find the rate of change of the distance from the object to the origin when the object passes through the point $(x, y) = (1, -1)$.

**Question 5.** Consider a box with square base. In order to be sent through P.T.T., the height of the box and the perimeter of the base can add up to at most 120 $cm$. What is the maximum volume for such a box?

**Question 6.** Let $f(x)$ be a twice differentiable function with $f(-1) = 1$, $f(0) = 4$ and $f(1) = 2$.

**(a)** Show that there exist two points $c_1 \in (-1, 0)$ and $c_2 \in (0, 1)$ such that $f'(c_1) = 3$ and $f'(c_2) = -2$.

**(b)** Show that $f$ has a critical point on $(-1, 1)$.

**(c)** Show that $f''(c) < 0$ for some $c \in (-1, 1)$.