

MATH 107: Introduction to Linear Algebra

Midterm 2, Part 2 - Spring 2020

#1	50	
#2	50	
Σ	100	

- The following time-frames are reserved for the questions.
 - Question 1, 19:00 - 19:35
 - Question 2, 19:40 - 20:15
- You must return your solution to each question by the end of the time-frame reserved for the question.

Question 1.

Let $A = \begin{bmatrix} a & 1 & 2 \\ b & 3 & 4 \\ c & 5 & 6 \end{bmatrix}$ for some $a, b, c \in \mathbb{R}$ such that $\det A = 12$.

(a) **(9 points)** Determine the dimension of the column space of A . (That is determine the rank of A .) Justify your answer.

(b) **(8 points)** Calculate the determinant of $\begin{bmatrix} a-1 & 1 & 2 \\ b-2 & 3 & 4 \\ c-3 & 5 & 6 \end{bmatrix}$.

(c) **(8 points)** Calculate the determinant of $\begin{bmatrix} a & 3 & 3 \\ b & 7 & 6 \\ c & 11 & 9 \end{bmatrix}$.

(c) **(25 points)** Calculate the first row of A^{-1} .

Question 2.

Let

$$A = \begin{bmatrix} 3 & -3 \\ 3 & 5 \\ 3 & 5 \\ 3 & -3 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ 0 \\ 1 \end{bmatrix}.$$

Moreover, let \mathbf{a}_1 and \mathbf{a}_2 be the first and the second columns of A , respectively.

- (a) **(30 points)** Find the orthogonal projection of \mathbf{x} onto $W = \text{span}\{\mathbf{a}_1, \mathbf{a}_2\}$.
- (b) **(20 points)** Find a QR factorization of A .