Math 107: Introduction to Linear Algebra

Midterm 2, Part 1 - Spring 2020

#1	35	
#2	40	
#3	25	
$\sum$	100	

- The following time-frames are reserved for the questions.
  - Question 1, 19:00 - 19:30
  - Question 2, 19:30 - 20:00
  - Question 3, 20:30 - 21:00

- You must return your solution to each question by the end of the time-frame reserved for the question.

Question 1.
This question has three parts. The third part is unrelated to the first two.

(a) **(20 points)** Find a set of three linearly independent eigenvectors for the matrix

$$A = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 0 & -1 \\ 0 & 1 & -4 \end{bmatrix} \tag{1}$$

if one of its eigenvectors is $\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$, and two of its eigenvalues are -2 and -3.

(b) **(8 points)** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix in (1).

What is the matrix for T relative to the basis composed of the eigenvectors in part **(a)**?

(c) **(7 points)** What is the dimension of the null space of $I - \lambda I$, where I is the $n \times n$ identity matrix, and λ is one of the eigenvalues of I.

What are the possible values for the dimension of the null space of $A - \lambda I$, where A is an $n \times n$ matrix, and λ is one of its eigenvalues? Explain.

Question 2.
Let $T : \mathbb{P}_2 \to \mathbb{P}_1$ be the linear transformation such that

$$T(\mathbf{p}) \;=\; \frac{d\mathbf{p}}{dt} - \frac{d^2\mathbf{p}}{dt^2}t.$$

In each part, indicate your reasoning.

(a) **(10 points)** Find a basis for Kernel(T).

(b) **(10 points)** What is the dimension of Range(T)?

(c) **(20 points)** Find the matrix for T relative to the bases $B = \{t+t^2, 2+2t, 1-t\}$ for $\mathbb{P}_2$ and $C = \{t, 1+t\}$ for $\mathbb{P}_1$.

Question 3. (25 points)
A matrix A is called symmetric if $A^T = A$. Moreover, the trace of a matrix is defined as the sum of its diagonal entries. Let S be the set of all 3×3 real symmetric matrices with trace equal to zero.

Indicate whether S is a vector space (over the set of real numbers, that is with the set of scalars equal to the set of real numbers).

- If S is a vector space, write down
  (i) a basis for S, and
  (ii) an isomorphism from S to $\mathbb{R}^{\kappa}$, where κ is the dimension of S.
- If S is not a vector space, explain why it is not.