MATH 107: Introduction to Linear Algebra

Midterm 2 - Spring 2017

Duration : 120 minutes

#1	15	
#2	15	
#3	10	
#4	15	
#5	10	
#6	15	
#7	10	
#8	10	
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NAME & LAST NAME __________________

STUDENT ID __________________

SIGNATURE __________________

- Put your name, student ID and signature in the space provided above.
- No calculators or any other electronic devices are allowed.
- This is a closed-book and closed-notes exam.
- Show all of your work; full credit will not be given for unsupported answers.
- Write your solutions clearly; no credit will be given for unreadable solutions.
- Mark your section below.

  SECTION 1 (EMRE MENGI TUTH 11:30-12:45) __________________

  SECTION 2 (EMRE MENGI, TUTH 8:30-9:45) __________________

  SECTION 3 (EMRE MENGI, MW 13:00-14:15) __________________

  SECTION 4 (DOĞAN BILGE, MW 14:30-15:45) __________________

Problem 1. (15 points) Find bases for the column space and row space of the following matrix. Show your work.

$$A = \begin{bmatrix} 1 & -1 & 2 & 6 \\ 2 & -2 & 1 & 0 \\ -2 & 2 & -3 & -8 \end{bmatrix}$$

Problem 2. In parts **(a)** and **(b)** indicate whether S is a subspace of V. Justify your answer.

(a) (8 points)

$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \mid x_1 + 2x_2 + 3x_3 = 0 \right\} \quad \text{and} \quad V = \mathbb{R}^3.$$

(b) (7 points)

$$\begin{aligned} S &= \{f : \mathbb{R} \to \mathbb{R} \mid f(x) \leq 1 \text{ for all } x \in [0,1]\} \text{ and} \\ V &= \text{The vector space of functions } f : \mathbb{R} \to \mathbb{R}. \end{aligned}$$

Problem 3. (10 points) Consider the polynomials

$$\mathbf{p}_1(x) = 1 - x + x^2, \quad \mathbf{p}_2(x) = 1 + 2x + 3x^2 \quad \text{and} \quad \mathbf{p}_3(x) = -1 + 7x + 3x^2$$

in $\mathbb{P}_2$. Determine whether the set $\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ is linearly independent or linearly dependent. Show your work.

Problem 4. Consider a linear system $A\mathbf{x} = \mathbf{b}$ where A is 13×17. In parts **(a)** and **(b)** below, justify your answer.

(a) (8 points) What is the smallest dimension possible for Nul A?

(b) (7 points) Suppose now that the linear system has a solution $\mathbf{x}$ for every right-hand side $\mathbf{b}$. Determine the rank of A.

Problem 5. (10 points) Write down an isomorphism from

$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; x_1 + x_3 = x_2 \right\}$$

to $\mathbb{R}^2$.

Problem 6. (15 points) Both of the sets $B = \{1 - x, x - x^2, x^2 - 2\}$ and $C = \{1, 1 + x, 1 + x + x^2\}$ are bases for $\mathbb{P}_2$. Find the change-of-coordinates matrix from B to C, that is find the matrix $P_{C \leftarrow B}$ such that for every $\mathbf{p} \in \mathbb{P}_2$,

$$[\mathbf{p}]_C \; = \; P_{C \leftarrow B} \, [\mathbf{p}]_B .$$

Problem 7. (10 points) Let $T : \mathbb{P}_6 \to \mathbb{R}^7$ be the linear transformation defined by

$$T(\mathbf{p}) := \begin{bmatrix} \mathbf{p}(-3) \\ \mathbf{p}(-2) \\ \mathbf{p}(-1) \\ \mathbf{p}(0) \\ \mathbf{p}(1) \\ \mathbf{p}(2) \\ \mathbf{p}(3) \end{bmatrix}.$$

Show that $\text{Range}(T) = \mathbb{R}^7$.

Problem 8. (10 points) Let B be an $m \times m$ invertible matrix. Furthermore, let A be an $m \times n$ matrix. Prove that

$$\text{Rank } BA \quad = \quad \text{Rank } A.$$