Math 171B: Introduction to Numerical Optimization

Midterm 1

Name ____________________

#1	30	
#2	40	
#3	30	
Total	100	

- Duration: 50 minutes.
- No calculators.
- Show your work.
- Put your name in the box above.

Question 1. Consider the function $f : \mathbf{R} \to \mathbf{R}$ defined as $f(x) = x^p$ where $p \geq 2$ is a fixed integer.

(a) (10 points.) Find the sequence $\{x_k\}$ generated by Newton's method. In particular write down the relation between two consecutive iterates x_{k+1} and x_k of Newton's method. (Note: Your answer must depend on p.)

(b) (20 points.) Assume that the sequence $\{x_k\}$ in part **(a)** converges to the unique root zero. Derive the order of convergence for the sequence $\{x_k\}$.

Question 2. Consider the vector-valued function $F : \mathbf{R}^2 \to \mathbf{R}^2$ such that

$$F(x) = \begin{bmatrix} x_1^2 + x_2^2 - 1 \\ (x_1 - 1)^2 + (x_2 - 1)^2 - 1 \end{bmatrix}.$$

(a) (20 points) Find the linear model $L(x)$ for $F(x)$ about $x_0 = (1/2, 3/2)$ such that

$$L(x_0) = F(x_0) \text{ and } L'(x_0) = F'(x_0).$$

(b) (10 points) Write down the definition of a Lipschitz continuously differentiable function from $\mathbf{R}^n \to \mathbf{R}^m$. Is $F(x)$ Lipschitz continuously differentiable? Justify your answer.

(c) (10 points) Assume a sequence $\{x_k\}$ generated by Newton's method for $F(x)$ converges to the root $x_* = (0, 1)$. Does the sequence $\{x_k\}$ converge to $(0, 1)$ q-linearly, q-superlinearly or q-quadratically? Explain. (Note: Don't try to derive the order of convergence. Rely on a theorem discussed in class.)

Question 3. (30 points) Given the functions

$$\begin{aligned} f(x) &= x_1^2 + 4x_1x_2 + 4x_2^2 + 2x_1 - 2x_2 + 1 \\ g(x) &= x_1^2 + x_2^2 + 4x_1 - 2x_2 + 3 \end{aligned}$$

from $\mathbf{R}^2$ to $\mathbf{R}$. For each of the functions indicate whether the function has a local minimizer or not. If the function has a local minimizer, determine a local minimizer and argue why it must be a local minimizer. If the function does not have any local minimizer, provide a formal argument why the function cannot have a local minimizer.