Problem 1 a) (10 pts) Draw the graph of the function $f(x) = -x^2 + 6x - 8$. Find the intercepts and the vertex of the parabola.

$\frac{-b}{2a} = \frac{-6}{-2} = 3$

$f(3) = -9 + 18 - 8$

$= 1$

Vertex: $(3,1)$

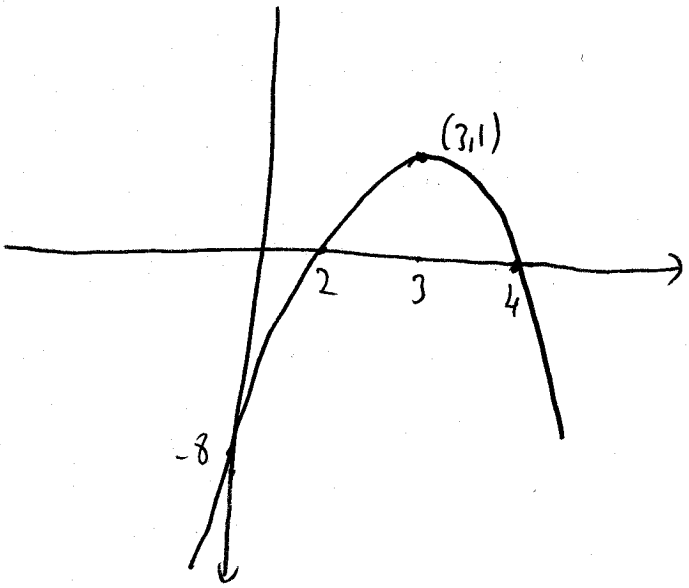
y-intercept: $f(0) = -8$

x-intercept: $-x^2 + 6x - 8 = 0$

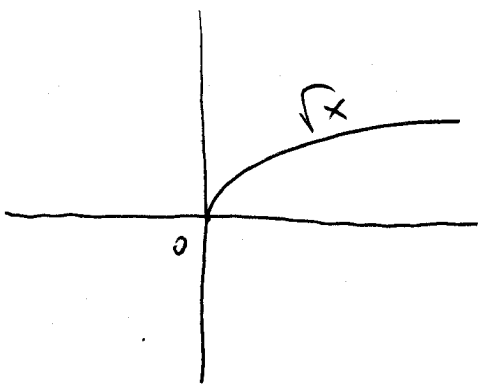
$-(x-2)(x-4) = 0$

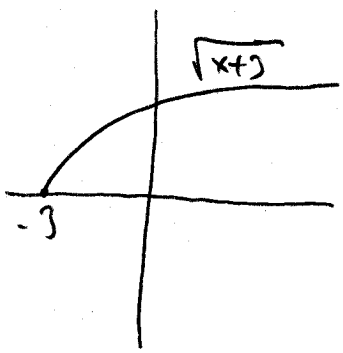
$x = 2$

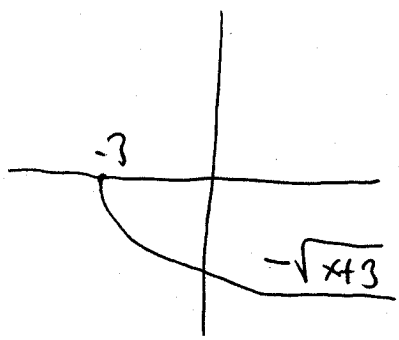
$x = 4$

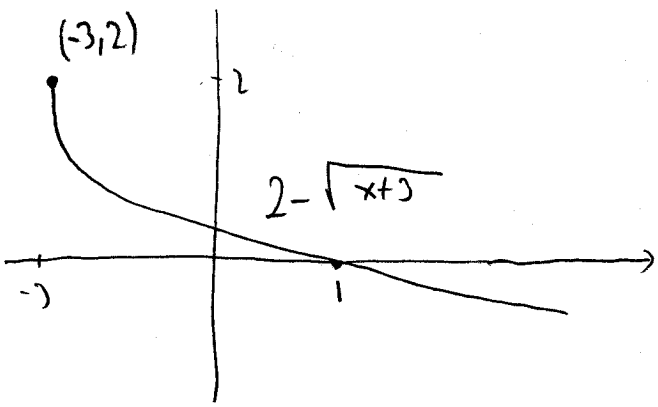


b) (10 pts) Draw the graph of the function $g(x) = 2 - \sqrt{x+3}$ by using the transformations. Find the domain and the range of the function.









Domain: $x+3 \geq 0 \Rightarrow x \geq -3 \Rightarrow \boxed{[-3, \infty)}$

Range: $f(x) \leq 2 \Rightarrow \boxed{(-\infty, 2]}$

Problem 2 (15 pts) Find the domains of the following functions.

a) $f(x)=\frac{1}{\sqrt{3-2x}}$

$3-2x \geq 0$ and $3-2x \neq 0 \Rightarrow 3-2x > 0$

$x < \frac{3}{2}$

$\boxed{(-\infty, \frac{3}{2})}$

b) $g(x) = \sqrt{\log_2 x} - 3$

$\log_2 x \geq 0 \Rightarrow x \geq 1 \Rightarrow \boxed{[1, \infty)}$

c) $h(x) = 3^{\frac{1}{x-2}}$

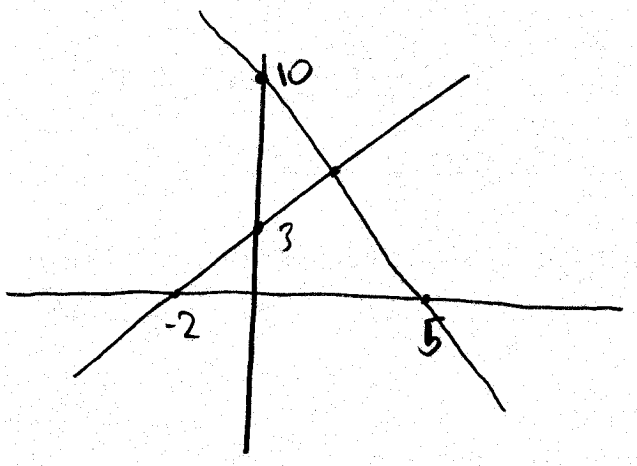
$x-2 \neq 0 \Rightarrow x \neq 2 \Rightarrow \boxed{\mathbb{R}-\{2\}}$ (or $(-\infty, 2) \cup (2, \infty)$)

Problem 3 a) (8 pts) Draw the following lines in the same coordinate plane.

$$2x + y = 10$$
$$3x - 2y = -6$$

$2x+y=10$
x-intercept 5
y-intercept 10

$3x-2y=-6$
x-int. -2
y-int. 3



b) (7 pts) Find the intersection point of the lines above (you may use any method you want).

$2x+y=10$
$3x-2y=-6$ $\Rightarrow$

$4x+2y=20$
$+\ 3x-2y=-6$

$7x=14$
$x=2$
$y=6$

$\boxed{(2,6)}$

c) (5 pts) Find the intersection point of the lines above by using the matrix method.

$$\begin{bmatrix} 2 & 1 & | & 10 \\ 3 & -2 & | & -6 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & -2 & | & -6 \\ 2 & 1 & | & 10 \end{bmatrix} \rightarrow \begin{matrix} \\ \times -1 \end{matrix}\begin{bmatrix} 1 & -3 & | & -16 \\ 2 & 1 & | & 10 \end{bmatrix} \times -2$$

$$\begin{bmatrix} 1 & -3 & | & -16 \\ 0 & 7 & | & 42 \end{bmatrix} \begin{matrix} \\ /7 \end{matrix} \rightarrow \begin{bmatrix} 1 & -3 & | & -16 \\ 0 & 1 & | & 6 \end{bmatrix} \times 3 \rightarrow \begin{bmatrix} 1 & 0 & | & 2 \\ 0 & 1 & | & 6 \end{bmatrix}$$

$x=2$
$y=6$

List of formulas

$I = P.r.t$ $\quad A = P.(1+r.t)$ $\quad A = P(1+\frac{r}{m})^{m.t}$ $\quad APY = (1+\frac{r}{m})^{m} - 1$

$FV = PMT\ \frac{(1+i)^n - 1}{i}$ $\quad PV = PMT\ \frac{1-(1+i)^{-n}}{i}$ $\quad i = \frac{r}{m}$ $\quad n = m.t$

Problem 4 a) (7 pts) How long will it take 10,000 TL to grow to 13,000 TL if it is invested at 12% simple interest.

$A = P.(1+rt)$

$13000 = 10000\ (1 + 0.12.t)$

$13000 = 10000 + 1200t$

$3000 = 1200t$

$t = \frac{3000}{1200} =$ **2.5 years**

b) (8 pts) How long will it take 10,000 TL to grow to 15,000 TL if it is invested at 12% compounded monthly.

$A = P.\left(1+\frac{r}{m}\right)^{m.t}$

$15000 = 10000\left(1+\frac{0.12}{12}\right)^{12t}$

$\frac{15000}{10000} = (1.01)^{12t}$

$1.5 = (1.01)^{12t}$

$\ln(1.5) = \ln(1.01)^{12t}$

$\ln(1.5) = 12t \cdot \ln(1.01)$

$12t = \frac{\ln(1.5)}{\ln(1.01)}$

$12t = 40.7$

$\Rightarrow$ **41 months**

or $\frac{40.7}{12}$ years

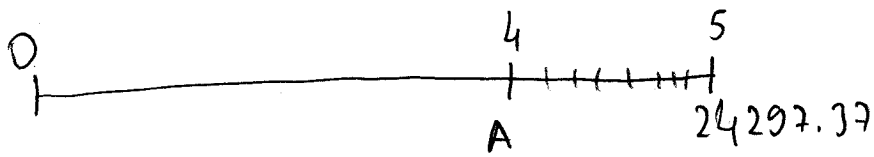
Problem 5 a) (8 pts) If \$1000 is deposited each quarter into an account paying 8% compounded quarterly for 5 years, how much money will be in the account at the end of 5 years?

$$FV = PMT\,\frac{(1+i)^n - 1}{i}$$

$$FV = 1000\,\frac{(1+0.02)^{20} - 1}{0.02}$$

$$= \boxed{24297.37}$$

b) (7 pts) How much interest is earned in the 5th year?



$$A = 1000\,\frac{(1.02)^{16} - 1}{0.02}$$

$$= 18\,639.29$$

$$24\,297.37 - 18\,639.29 = 5658.08$$ (Net change in 5th year)

Total Payments in 5th year

$$4 \times 1000 = 4000$$

$$5658.08 - 4000 = \boxed{1658.08}$$ (interest in the 5th year)

Problem 6 A family has a \$100,000, 20 year mortgage at 6% compounded monthly.

a) (10 pts) Find the monthly payment.

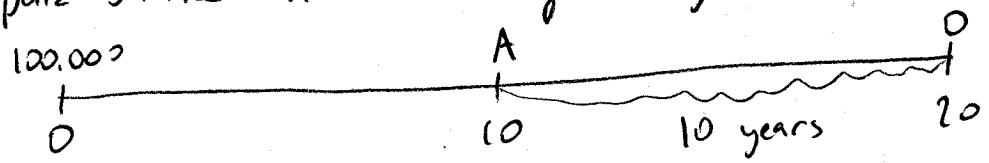
$$PV = PMT \frac{1-(1+i)^{-n}}{i}$$

$$100000 = PMT \frac{1-(1+0.005)^{-240}}{0.005}$$

$$PMT = \frac{100000 \cdot 0.005}{1-(1.005)^{-240}} = 716.43$$

b) (10 pts) If the family decides to add an extra \$100 to its mortgage payment each month starting 10 years later, how long will it take the family to pay off the mortgage.

Unpaid balance at the end of 10th year:



$$A = 716.43 \frac{1-(1.005)^{-120}}{0.005} = 64\,531.32 \quad (PV \text{ for } 10 \text{ years})$$

New Monthly Payments: 816.43

Time to pay off:

$$64\,531.32 = 816.43 \frac{1-(1.005)^{-n}}{0.005}$$

$$\frac{64\,531.32 \times 0.005}{816.43} = 1-(1.005)^{-n}$$

$$0.3952 = 1-(1.005)^{-n}$$

$$(1.005)^{-n} = 1-0.3952$$

$$\ln(1.005)^{-n} = \ln(0.6048)$$

$$-n = \frac{\ln(0.6048)}{\ln(1.005)}$$

$$n = 100.8$$

101 months or $\frac{100.8}{12}$ years