Instructor: Emre Mengi

Math 304 (Spring 2010)

Study Guide for Weeks 13-14
Homework 9 concerns the topics listed below.

- Adams-Bashforth methods (Fausett - 12.3.1)
- Systems of ODEs and their numerical solution(Fausett - 13.1)
- Higher order differential equations and how to turn them into first order (Fausett - 13.1)
- Finite difference formulas and their errors (Fausett - 11.1.1-3)
- Numerical solution of Poisson equation via finite differences (Fausett - 15.3)

Homework 9 (due on June 11th, Friday by 3pm)

1. Derive the 3-step Adams-Bashforth formula for the numerical solution of ordinary differential equations

2. (Fausett - A.13.10 on page 558) A simple predator-prey relationship is described by the Lotka-Volterra model provided below, which is written in terms of

- a fox population $f(t)$ with birth rate b_f and death rate d_f, and
- a geese population $g(t)$ with birth rate b_g and death rate d_g

$$\begin{aligned} \frac{df}{dt} &= f(t)(b_f g(t) - d_f) \\ \frac{dg}{dt} &= g(t)(b_g - d_g f(t)) \end{aligned}$$

Find the populations on $[0, 2]$ using Euler's method with step-size $h = 1$ for the following parameter values and initial conditions

(a) $b_f = d_f = b_g = d_g = 1$, $f(0) = g(0) = 2$.

(b) $b_f = b_g = 1$, $d_f = d_g = 0.5$, $f(0) = 2$ and $g(0) = 10$.

3. Convert the second-order ODE

$$y'' + y + 2y^2 = 0 \;\; t > 0 \;\text{ and }\; y(0) = 2, \; y'(0) = 0$$

into a system first order ODEs. Then solve the resulting ODE on the interval $[0, 6]$ using Euler's method with the step-size $h = 2$.

4. Consider the following data for the function $y(t)$.

t	-1	0	1	2	3
$y(t)$	-2	0	2	16	54

(a) Approximate $y'(1)$ and $y'(2)$ using the forward difference, backward difference and central difference formulas. Choose the step size $h = 1$.

(b) Approximate $y''(1)$ and $y''(2)$ using the central difference formula. Choose the step size $h = 1$.

5. Consider the finite difference formula

$$D_m y(t_*) = \frac{y(t_* + \frac{3h}{4}) - y(t_* - \frac{h}{4})}{h}.$$

Show that

$$D_m y(t_*) - y'(t_*) = O(h)$$

and

$$D_m y(t_*) - y'(t_*) \neq O(h^2).$$

6. Solve the following Poisson equations using finite differences numerically with the step size $h = 0.25$. For the solution of resulting linear systems use Matlab.

(a)

$$u_{xx}(x, y) + u_{yy}(x, y) = 0$$

where $x, y \in [0, 1]$ with the boundary conditions

$$u(x, 0) = 0, \quad u(0, y) = 2y, \quad u(x, 1) = 2, \quad u(1, y) = 1 + y$$

(b)

$$u_{xx}(x, y) + u_{yy}(x, y) = 0$$

where $x, y \in [0, 1]$ with the boundary conditions

$$u(x, 0) = x^2 + 1, \quad u(0, y) = -y^2 + 1, \quad u(x, 1) = x^2, \quad u(1, y) = 2 - y^2$$