KOÇ UNIVERSITY

MATH 106 - CALCULUS 1

Midterm II May 4, 2015

Duration of Exam: 75 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and no talking allowed. You must always **explain your answers** and **show your work** to receive **full credit.** Use the back of these pages if necessary. **Print** (use **CAPITAL LETTERS**) **and sign your name, and indicate your section below.**

Name: ____________________

Surname: KEY

Signature: ____________________

Section (Check One):

Section 1: Selda Küçükçifçi M-W (8:30) ___
Section 2: Ayberk Zeytin T-Th(10:00) ___

PROBLEM	POINTS	SCORE
1	36	
2	18	
3	36	
4	15	
TOTAL	105	

1. (36 points) Compute the following limits.

(a) $\lim_{x \to 0+} \underbrace{(e^x + 2x)^{3/x}}_{y}$

$y = (e^x + 2x)^{3/x} \Rightarrow \ln y = \frac{3}{x} \ln(e^x + 2x)$

$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{3 \ln(e^x + 2x)}{x} \quad \frac{0}{0} \quad = \lim_{x \to 0^+} \frac{3 \frac{1}{e^x + 2x} \cdot (e^x + 2)}{1} = 3 \cdot \frac{1}{1} \cdot 3 = 9$

$\lim_{x \to 0^+} \ln y = 9 \Rightarrow \lim_{x \to 0^+} y = e^9$.

(b) $\lim_{x \to 0+} \frac{\int_{x+1}^{1-x^2} t^{106} \ln(t)\, dt}{\ln(x+1)} \quad \frac{0}{0} \quad = \lim_{x \to 0^+} \frac{(1-x^2)^{106} \ln(1-x^2)(-2x) - (x+1)^{106} \ln(x+1)}{\frac{1}{x+1}}$

$= \frac{0 - 0}{1} = 0$.

(c) $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n} \left(\frac{2i}{n} + 1\right)^5 = \int_0^1 (2x+1)^5 \, dx$

$= \left[\frac{(2x+1)^6}{2 \times 6}\right]_0^1 = \frac{3^6}{12} - \frac{1}{12} = \frac{182}{3}$

2. (18 points) Find the absolute maximum and minimum values of the function

$$f(x) = x^2e^{-x^2}.$$

$$f'(x) = 2xe^{-x^2} + x^2e^{-x^2}(-2x)$$

$$= 2xe^{-x^2}(1-x^2)$$

$$= 2xe^{-x^2}(1-x)(1+x)$$

$$f'(x) = 0 \Rightarrow x=0,\ x=1,\ x=-1.$$

		-1		0		1	
f'	$+$		$-$		$+$		$-$
f	$\nearrow$		$\searrow$		$\nearrow$		$\searrow$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x^2}{e^{x^2}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to-\infty} \frac{2x}{e^{x^2}\,2x} = \lim_{x\to-\infty} \frac{1}{e^{x^2}} = 0$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to+\infty} \frac{x^2}{e^{x^2}} = 0.$$

$f(-1) = e^{-1}$

$f(1) = e^{-1}$

$f(0) = 0$

So absolute maximum value is $\frac{1}{e}$ and obtained at $(-1, e^{-1})$ & $(1, e^{-1})$.

Absolute minimum value is 0 and obtained at $(0,0)$.

3. Determine the following integrals in (a)-(b).

(a) (12 points) $\int_0^4 x^2 e^{2x}\,dx$

$u = x^2 \qquad dv = e^{2x}\,dx$

$du = 2x\,dx \qquad v = \frac{1}{2}e^{2x}$

$$\int_0^4 x^2 e^{2x}\,dx = \left[\frac{x^2}{2}e^{2x}\right]_0^4 - \int_0^4 x e^{2x}\,dx$$

$u = x \qquad dv = e^{2x}\,dx$

$du = dx \qquad v = \frac{1}{2}e^{2x}$

$$\int_0^4 x^2 e^{2x}\,dx = \left[\frac{x^2}{2}e^{2x}\right]_0^4 - \left[\frac{x}{2}e^{2x}\right]_0^4 + \frac{1}{2}\int_0^4 e^{2x}\,dx$$

$$= 8e^8 - 2e^8 + \frac{1}{2}\left[\frac{1}{2}e^{2x}\right]_0^4$$

$$= 6e^8 + \frac{1}{4}(e^8 - 1) = \frac{25}{4}e^8 - \frac{1}{4}.$$

(b) (12 points) $\int \frac{-3\,dx}{x^3 - 3x^2}$

$$\frac{-3}{x^3 - 3x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$$

$$-3 = Ax(x-3) + B(x-3) + Cx^2$$

$x = 0 \Rightarrow -3 = -3B \Rightarrow B = 1$

$x = 3 \Rightarrow -3 = C\,9 \Rightarrow C = -1/3$

$\Rightarrow A = 1/3.$

$$\int \frac{-3\,dx}{x^3 - 3x^2} = \frac{1}{3}\int \frac{dx}{x} + \int \frac{dx}{x^2} - \frac{1}{3}\int \frac{dx}{x-3}$$

$$= \frac{1}{3}\ln|x| - \frac{1}{x} - \frac{1}{3}\ln|x-3| + C.$$

(c) (12 points) Determine whether the following improper integral $\int_e^\infty \frac{dx}{x(\ln x)^2}$ is convergent or divergent.

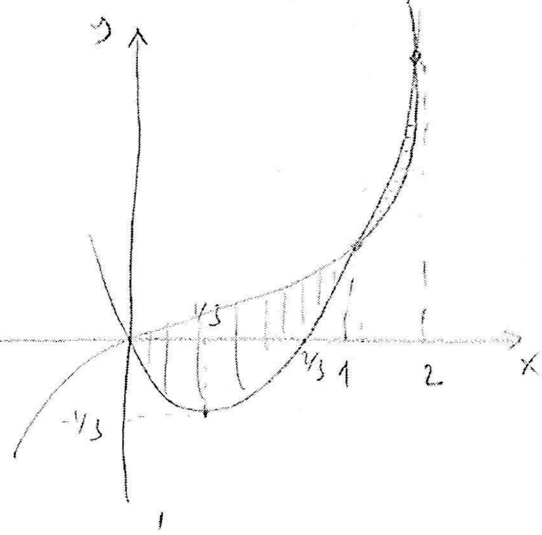
$$\int_e^\infty \frac{dx}{x(\ln x)^2} = \lim_{t\to\infty} \int_e^t \frac{dx}{x(\ln x)^2} = \lim_{t\to\infty} \int_1^{\ln t} \frac{du}{u^2}$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \lim_{t\to\infty} \left[\frac{u^{-1}}{-1}\right]_1^{\ln t} = \lim_{t\to\infty} \left(\frac{-1}{\ln t} + 1\right) = 1$$

4. (15 points) Find the area of the finite region between $y = x^3$ and $y = 3x^2 - 2x$.



$x^3 = 3x^2 - 2x$

$x(x^2 - 3x + 2) = 0$

$x(x-1)(x-2) = 0$

$x = 0 \quad x = 1 \quad x = 2$

$$\int_0^1 (x^3 - 3x^2 + 2x)\,dx + \int_1^2 (3x^2 - 2x - x^3)\,dx$$

$$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 + \left[x^3 - x^2 - \frac{x^4}{4}\right]_1^2$$

$$= \frac{1}{4} - 1 + 1 + 8 - 4 - 4 - 1 + 1 + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$