LECTURE 25

PRACTICAL ASPECTS OF LONG PATH FOLLOWING ALGORITHMS

Line Search

When choosing α_k one can use a backtracking line search.

ALGORITHM (Backtracking Line Search)

Given $(\Delta x_k, \Delta \pi_k, \Delta s_k)$, the Newton step

$\alpha_k = 1$

$(x_{k+1}, \pi_{k+1}, s_{k+1}) = (x_k, \pi_k, s_k) + \alpha_k(\Delta x_k, \Delta \pi_k, \Delta$

While $(x_{k+1}, \pi_{k+1}, s_{k+1}) \notin \mathcal{N}(\gamma)$

$\alpha_k = \alpha_k / 2$

[redacted] $(x_{k+1}, \pi_{k+1}, s_{k+1}) = (x_k, \pi_k, s_k) + \alpha_k(\Delta x_k, \Delta \pi_k, \Delta s$

end

REMARKS

* $(x_k, \pi_k, s_k) \in \mathbb{F}^{\circ}_{P,D} \Longrightarrow$

$(x_k, \pi_k, s_k) + \alpha_k(\Delta x_k, \Delta \pi_k, \Delta s_k) \in \mathbb{F}^{\circ}_{P,D}$

for all small $\alpha_k > 0$.

* $(x_k, \pi_k, s_k) \in \mathbb{F}^{\circ}_{P,D} \Longrightarrow$

$$(x_k + \alpha_k \Delta x_k)_j \, (s_k + \alpha_k \Delta x_k)_j \geq \gamma \mu$$

for all small $\alpha_k > 0$. (assuming $\gamma > 0$ is small enough)

Feasible Starting Points

Long path following algorithm requires an initial point

$$(x_0, \pi_0, s_0) \in \mathcal{N}(\gamma).$$

Typically a point satisfying

$$(x_0, \pi_0, s_0) \in \mathbb{F}^{\circ}_{P,D}$$

would be sufficient in practice assuming γ is small enough.

ALGORITHM (Finding a feasible point)

Choose an $(\bar{x}_0, \bar{\pi}_0, \bar{s}_0)$ such that $\bar{x}_0 > 0$ and $\bar{s}_0 > 0$.

$k = 0$

While $(\bar{x}_k, \bar{\pi}_k, \bar{s}_k) \notin \mathbb{F}^{\circ}_{P,D}$

(1) Solve

$$\begin{bmatrix} \cancel{0} & \cancel{A^T} & \cancel{I} \end{bmatrix}$$

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ \bar{S}_k & 0 & \bar{X}_k \end{bmatrix} \begin{bmatrix} \Delta x_k \\ \Delta \pi_k \\ \Delta s_k \end{bmatrix} = \begin{bmatrix} b - A\bar{x}_k \\ c - \bar{s}_k - A^T\bar{\pi}_k \\ -\bar{X}_k \bar{S}_k e + \sigma \mu_k \end{bmatrix}$$

for $(\Delta x_k, \Delta \pi_k, \Delta s_k)$ where

$$\bar{X}_k = \begin{bmatrix} (\bar{x}_k)_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & (\bar{x}_k)_n \end{bmatrix}, \quad \bar{S}_k = \begin{bmatrix} (\bar{s}_k)_1 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & (\bar{s}_k)_n \end{bmatrix}$$

and $\mu_k = \frac{1}{n} \sum_{j=1}^{k} \bar{x}_j \bar{s}_j$

(2) Apply a backtracking line search (starting with $\alpha_k = 1$) so that $(\bar{x}_k + \alpha_k \Delta x_k) > 0$ and $(\bar{s}_k + \alpha_k \Delta s_k) > 0$

$$(\bar{x}_{k+1}, \bar{\pi}_{k+1}, \bar{s}_{k+1}) = (\bar{x}_k, \bar{\pi}_k, \bar{s}_k) + \alpha_k (\Delta x_k, \Delta \pi_k, \Delta s_k)$$

(3) $k = k+1$

end (While)

$(x_0, \pi_0, s_0) = (\bar{x}_k, \bar{\pi}_k, \bar{s}_k)$