MECH 303
Machine Design
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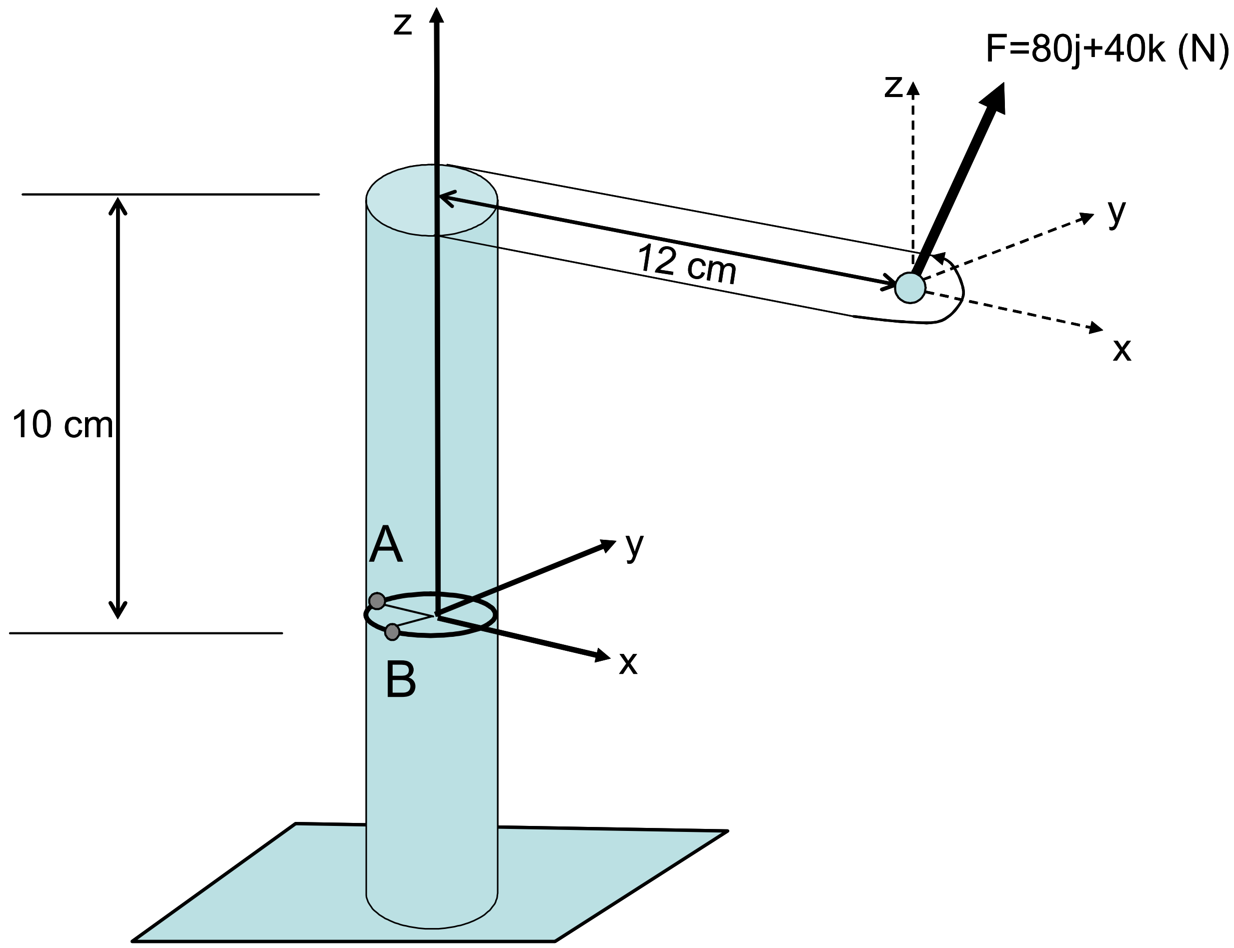
Example Problem: The steel pipe shown in the figure has an inner diameter of 2 cm and outer diameter of 2.15 cm. If it is subjected to the force F = 80j + 40 k (N), determine the principal stresses in the pipe at point A and B which are located on the surface of the pipe.



**Solution:**

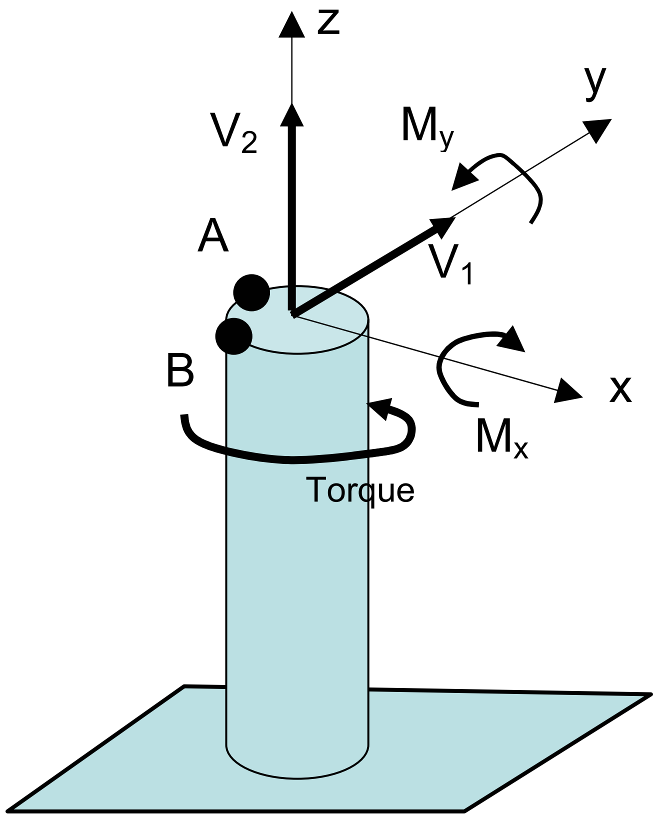
Hollow tube:

$$I = \frac{\pi}{4}\left[ (\frac{2.15.10^{-2}}{2})^4 - (\frac{2.0.10^{-2}}{2})^4 \right] = 2.63.10^{-9}\ m^4$$

$$J = \frac{\pi}{2}\left[ (\frac{2.15.10^{-2}}{2})^4 - (\frac{2.0.10^{-2}}{2})^4 \right] = 5.26.10^{-9}\ m^4$$

$$A = \pi\left[ (\frac{2.15.10^{-2}}{2})^2 - (\frac{2.0.10^{-2}}{2})^2 \right] = 4.88.10^{-5}\ m^2$$

**Free-Body Diagram**




$M_x$ = 80. 10.10$^{-2}$ = 8 Nm

$M_y$ = 40. 12.10$^{-2}$ = 4.8 Nm

T = 80. 12.10$^{-2}$ = 9.6 Nm

$V_1$ = 80 N

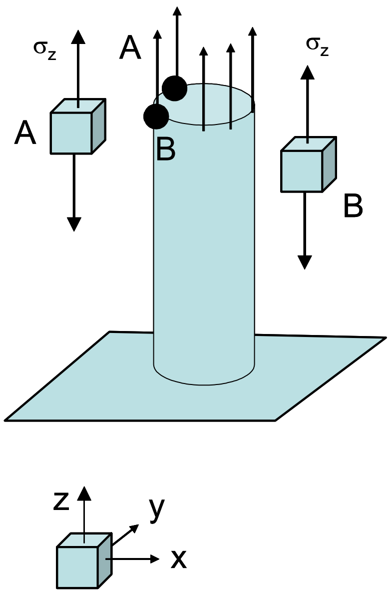
$V_2$ = 40 N

**Applied Stresses at Point A:**

Normal Stress (Axial Tension):

$$\sigma_z = \frac{V_2}{A} = \frac{40}{4.88.10^{-5}} = 8.2.10^5\ Pa$$ (along + z axis)

Normal Stresses (Axial Tension)

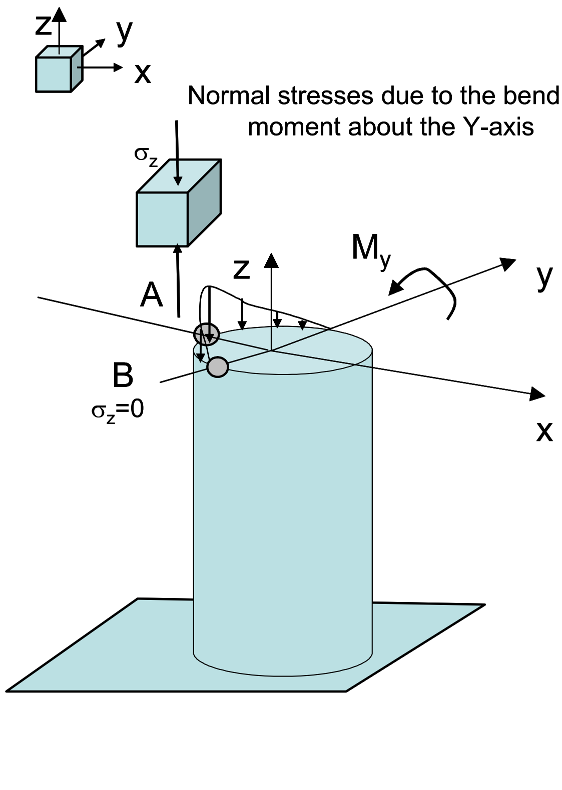



<u>Normal stress due to the bending moment about the x-axis</u> = 0

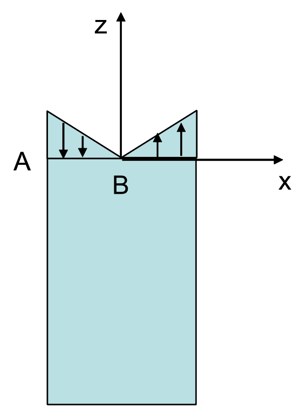
<u>Normal stress due to the bending moment about the y-axis</u>:

$$\sigma_z = \frac{M_y r_o}{I} = \frac{(4.8)*(\frac{2.15}{2}.10^{-2})}{2.63.10^{-9}} = 1.96.10^{7}\ Pa \quad \text{(along –z axis)}$$

Normal stresses due to the bending moment about the Y-axis



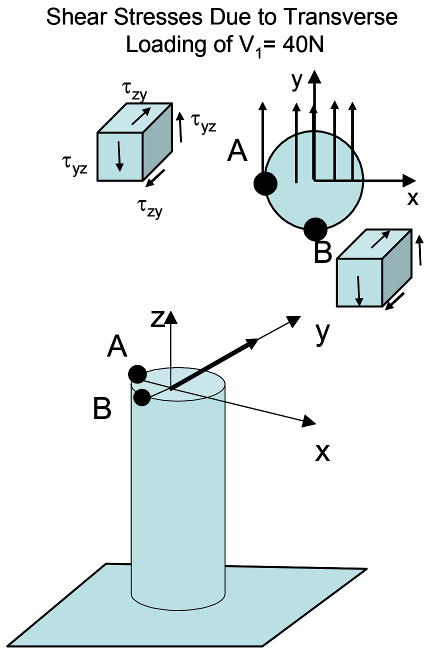

Normal stress distribution



Shear stress due to the bending moment about the x-axis

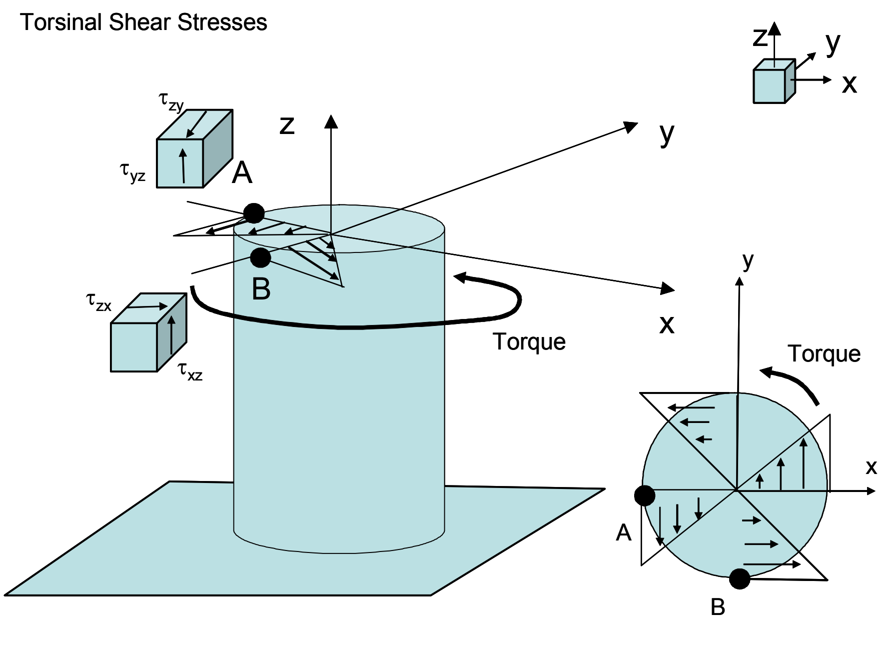
- Shear force (V1) is acting along the positive y-axis. Hence, the shear stress is on the XY plane and acting in the direction of positive y-axis.
- The shear stress is max along the moment axis. Hence, the shear stress is max at Point A and zero at Point B.
- Check the ratio of wall thickness to outer radius; (0.075/1.075 = 0.07 < 0.1)

$$\tau_{zy} = \frac{2V_1}{A} = \frac{2*80}{4.88.10^{-5}} = 32.8.10^{5}\ Pa \qquad \text{(along the + y axis)}$$




Torsional shear stress:

$$\tau_{zy} = \frac{Tr}{J} = \frac{9.6*(\frac{2.15*10^{-2}}{2})}{5.26.10^{-9}} = 1.96.10^{7}\ Pa \qquad \text{(along the –y axis; see the diagram)}$$



Total Normal Stress:

$$\sigma_z = 8.2 * 10^5 - 196 * 10^5 \approx -187 * 10^5\, Pa$$

Total Shear Stress:

$$\tau_{zy} = 32.8 * 10^5 - 196.10^5\ Pa \approx -163 * 10^5\, Pa$$

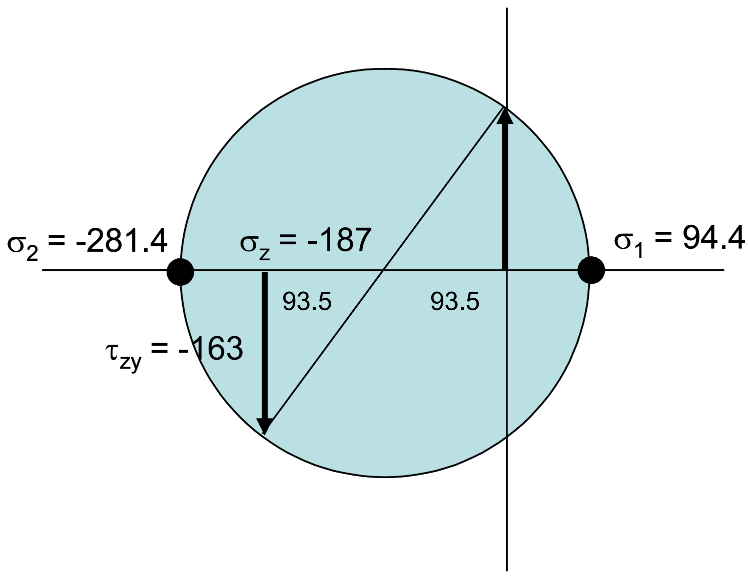
**Principal Stresses at Point A:**
The principal stresses can be computed as

$$\sigma_{1,2} = \frac{\sigma_y + \sigma_z}{2} \pm \sqrt{(\frac{\sigma_y - \sigma_z}{2})^2 + (\tau_{zy})^2}$$

$$\sigma_{1,2} = \frac{-187 * 10^5}{2} \pm \sqrt{(\frac{-187 * 10^5}{2})^2 + (-163 * 10^5)^2}$$

$$\sigma_1 = 94.4 * 10^5\, Pa$$

$$\sigma_2 = -281.4 * 10^5\, Pa$$

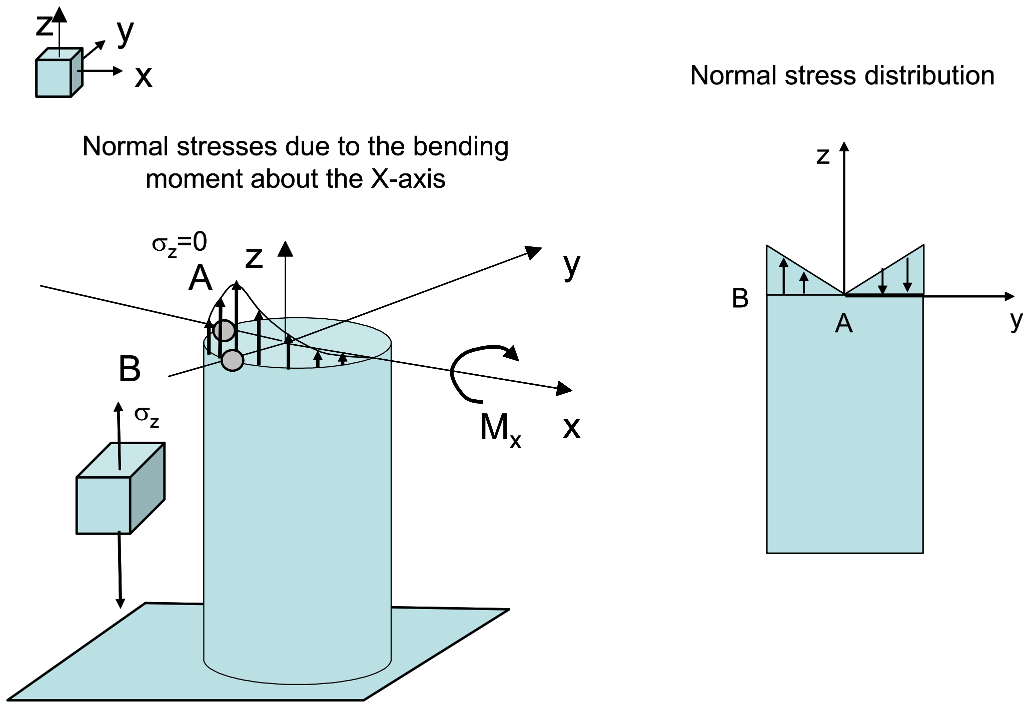



**Applied Stresses at Point B:**
Normal Stress (Axial Tension):

$$\sigma_z = \frac{V_2}{A} = \frac{40}{4.88.10^{-5}} = 8.2.10^5\ Pa \qquad \text{(along + z axis)}$$

Normal stress due to the bending moment about the x-axis:

$$\sigma_z = \frac{M_x r_o}{I} = \frac{(8) * (\frac{2.15}{2}.10^{-2})}{2.63.10^{-9}} = 3.3.10^7\ Pa \qquad \text{(along +z axis)}$$

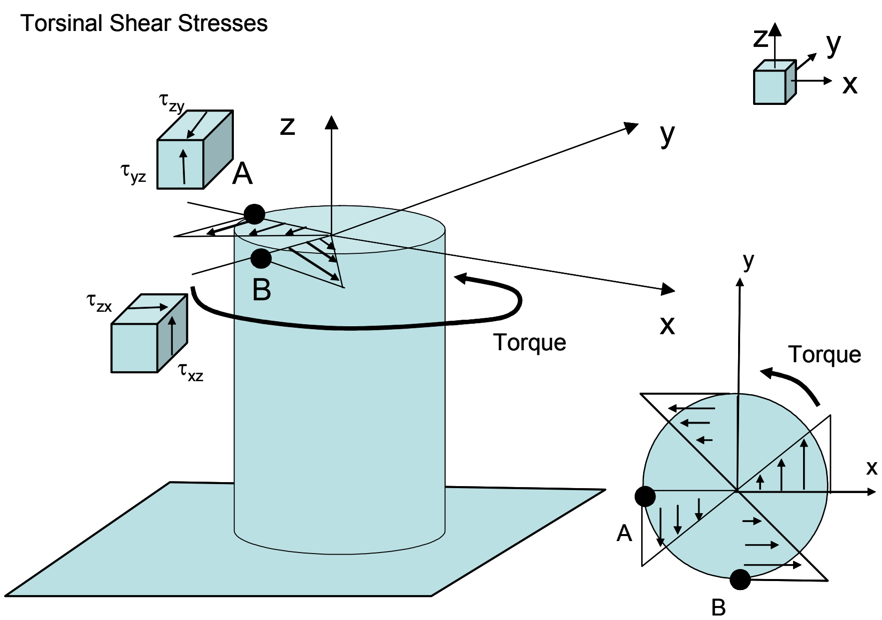



Normal stress due to the bending moment about the y-axis = 0

Shear stress due to the bending moment about the x-axis = 0

Torsional shear stress:

$$\tau_{zx} = \frac{Tr}{J} = \frac{9.6 * (\frac{2.15 * 10^{-2}}{2})}{5.26.10^{-9}} = 1.96.10^{7}\ Pa \qquad \text{(along the +x axis; see the diagram)}$$



Total Normal Stress:

$$\sigma_z = 8.2*10^5 + 330*10^5 \approx 338*10^5 \, Pa$$

Total Shear Stress:

$$\tau_{zx} = 196.10^5 \;\; Pa$$

**Principal Stresses at Point B:**
The principal stresses can be computed as

$$\sigma_{1,2} = \frac{\sigma_z + \sigma_x}{2} \pm \sqrt{(\frac{\sigma_z - \sigma_x}{2})^2 + (\tau_{zx})^2}$$

$$\sigma_{1,2} = \frac{338*10^5}{2} \pm \sqrt{(\frac{338*10^5}{2})^2 + (196*10^5)^2}$$

$$\sigma_1 = 427.8*10^5 \, Pa$$

$$\sigma_2 = -89.8*10^5 \, Pa$$