EXERCISES 12.2

In Exercises 1–12, evaluate the indicated limit or explain why it does not exist.

1. $\lim_{(x,y)\to(2,-1)} xy + x^2$

2. $\lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2}$

3. $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{y}$

4. $\lim_{(x,y)\to(0,0)} \frac{x}{x^2+y^2}$

5. $\lim_{(x,y)\to(1,\pi)} \frac{\cos(xy)}{1-x-\cos y}$

6. $\lim_{(x,y)\to(0,1)} \frac{x^2(y-1)^2}{x^2+(y-1)^2}$

7. $\lim_{(x,y)\to(0,0)} \frac{y^3}{x^2+y^2}$

8. $\lim_{(x,y)\to(0,0)} \frac{\sin(x-y)}{\cos(x+y)}$

9. $\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x^2+y^2}$

10. $\lim_{(x,y)\to(1,2)} \frac{2x^2-xy}{4x^2-y^2}$

11. $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^4}$

12. $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{2x^4+y^4}$

13. How can the function

$$f(x,y) = \frac{x^2+y^2-x^3y^3}{x^2+y^2}, \qquad (x,y) \neq (0,0),$$

be defined at the origin so that it becomes continuous at all points of the xy-plane?

14. How can the function

$$f(x,y) = \frac{x^3-y^3}{x-y}, \qquad (x \neq y),$$

EXERCISES 12.3

In Exercises 1–10, find all the first partial derivatives of the function specified, and evaluate them at the given point.

1. $f(x, y) = x - y + 2, \quad (3, 2)$
2. $f(x, y) = xy + x^2, \quad (2, 0)$
3. $f(x, y, z) = x^3y^4z^5, \quad (0, -1, -1)$
4. $g(x, y, z) = \dfrac{xz}{y+z}, \quad (1, 1, 1)$
5. $z = \tan^{-1}\left(\dfrac{y}{x}\right), \quad (-1, 1)$
6. $w = \ln(1 + e^{xyz}), \quad (2, 0, -1)$
7. $f(x, y) = \sin(x\sqrt{y}), \quad \left(\dfrac{\pi}{3}, 4\right)$
8. $f(x, y) = \dfrac{1}{\sqrt{x^2+y^2}}, \quad (-3, 4)$
9. $w = x^{(y \ln z)}, \quad (e, 2, e)$
10. $g(x_1, x_2, x_3, x_4) = \dfrac{x_1 - x_2^2}{x_3 + x_4^2}, \quad (3, 1, -1, -2)$

In Exercises 11–12, calculate the first partial derivatives of the given functions at $(0, 0)$. You will have to use Definition 4.

11. $f(x, y) = \begin{cases} \dfrac{2x^3 - y^3}{x^2 + 3y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$
12. $f(x, y) = \begin{cases} \dfrac{x^2 - 2y^2}{x - y}, & \text{if } x \neq y \\ 0, & \text{if } x = y. \end{cases}$

In Exercises 13–22, find equations of the tangent plane and normal line to the graph of the given function at the point with specified values of x and y.

13. $f(x, y) = x^2 - y^2$ at $(-2, 1)$
14. $f(x, y) = \dfrac{x - y}{x + y}$ at $(1, 1)$
15. $f(x, y) = \cos(x/y)$ at $(\pi, 4)$
16. $f(x, y) = e^{xy}$ at $(2, 0)$
17. $f(x, y) = \dfrac{x}{x^2 + y^2}$ at $(1, 2)$
18. $f(x, y) = y\,e^{-x^2}$ at $(0, 1)$
19. $f(x, y) = \ln(x^2 + y^2)$ at $(1, -2)$
20. $f(x, y) = \dfrac{2xy}{x^2 + y^2}$ at $(0, 2)$
21. $f(x, y) = \tan^{-1}(y/x)$ at $(1, -1)$
22. $f(x, y) = \sqrt{1 + x^3y^2}$ at $(2, 1)$
23. Find the coordinates of all points on the surface with equation $z = x^4 - 4xy^3 + 6y^2 - 2$ where the surface has a horizontal tangent plane.
24. Find all horizontal planes that are tangent to the surface with equation $z = xye^{-(x^2+y^2)/2}$. At what points are they tangent?

In Exercises 25–31, show that the given function satisfies the given partial differential equation.

25. $z = x\,e^y, \quad x\dfrac{\partial z}{\partial x} = \dfrac{\partial z}{\partial y}$
26. $z = \dfrac{x + y}{x - y}, \quad x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0$
27. $z = \sqrt{x^2 + y^2}, \quad x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = z$
28. $w = x^2 + yz, \quad x\dfrac{\partial w}{\partial x} + y\dfrac{\partial w}{\partial y} + z\dfrac{\partial w}{\partial z} = 2w$
29. $w = \dfrac{1}{x^2 + y^2 + z^2}, \quad x\dfrac{\partial w}{\partial x} + y\dfrac{\partial w}{\partial y} + z\dfrac{\partial w}{\partial z} = -2w$
30. $z = f(x^2 + y^2)$, where f is any differentiable function of one variable,
$$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0.$$
31. $z = f(x^2 - y^2)$, where f is any differentiable function of one variable,
$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0.$$