# Math 204. Homework 13

**Problems from W.E. Boyce, R.C. Diprima, D.B. Meade** :

*Section 10.6* p. 500, Problems : 19, 20.

*Section 10.7* p. 510, Problems : 6 8a,b), 9(a), 12, 22(a)

and the following problems:

**Problem 1.** Use the method of separation of variables to find the solution of the problem

$$\begin{cases} u_t(x,t) + bu = a^2 u_{xx}(x,t), \quad x \in (0,L), t > 0, \\ u(0,t) = 0, \quad u(L,t) = 0, t \geq 0, \\ u(x,0) = g(x), \quad x \in [0,L], \end{cases}$$

where $a > 0, b > 0$ are given numbers and $g(x)$ is a given twice continuously differentiable function on $[0,L]$.
Are the solutions of this problem tending to zero as $t \to +\infty$?

**Problem 2.** Use the method of separation of variables to find the solution of the problem

$$\begin{cases} u_{tt}(x,t) = u_{xx}(x,t), \quad x \in (0,1), t > 0, \\ u(0,t) = 0, \quad u(1,t) = 0, t \geq 0, \\ u(x,0) = x(1-x), \quad u_t(x,0) = 0, \quad x \in [0,1], \end{cases}$$

**Problem 3.** Solve the initial value problem

$$\begin{cases} u_{tt}(x,t) = 16u_{xx}(x,t), \quad x \in (-\infty,\infty), \ t > 0, \\ u(x,0 =) = \cos(x), \quad u_t(x,0) = xe^{-x}, t \geq 0. \end{cases}$$

**Problem 4.** Consider the initial boundary value problem

$$\begin{cases} u_{tt}(x,t) = u_{xx}(x,t) + f(x), \quad x \in (0,L), t > 0, \\ u(0,t) = A, \quad u(L,t) = B, t \geq 0, \\ u(x,0) = g(x), \ u_t(x,0) = h(x), \quad x \in [0,L], \end{cases}$$

where $A, B$ are given numbers , $f, g, h$ are given functions defined on $[0,L]$
Show that this problem may not have two different solutions.

*Hint:* Assume that the problem has another solution $v(x,t)$. The function $w(x,t) = u(x,t) - v(x,t)$ will satisfy

$$w_{tt}(x,t) = w_{xx}(x,t) \quad x \in (0,L), t > 0, \tag{1}$$

$$w(0,t) = 0, \quad w(L,t) = 0, t \geq 0, \tag{2}$$

$$w(x,0) = 0, \quad x \in [0,L] \tag{2}$$

Multiply (1) by $w_t(x,t)$, integrate the obtained realtion with respect to $x$ over the interval $(0, L)$ and use the conditions (2) and (3).