Math 204 Homework 13.

Problems from W.E. Boyce, R.C. Diprima, D.B. Meade

Section 10.4 , p. 485: Problems: 12, 19,28, 36,

Section 10.5 , p. 493: Problems: 7, 12, 20,

Section 10.6 , p. 500: Problems: 2, 6, 15,

Section 10.7 , p. 510: Problems: 2(a,b), 15,

and the following problems:

Consider the problem:

$$u_{tt}(x,t) = 4u_{xx}(x,t), \quad x \in (0,1), \; t > 0, \tag{1}$$

$$u(0,t) = u(1,t) = 0, \quad t > 0, \tag{2}$$

$$u(x,0) = f(x), \quad u_t(x,0) = g(x) \; t > 0, \tag{3}$$

where f, g are twice continuously differentiable functions on $[0,1]$.

Problem A. Show that if u is a solution of the problem (1)-(3), then

$$\int_0^1 \left[(u_t(x,t))^2 + (u_x(x,t))^2 \right] dx = \int_0^1 \left[(f'(x))^2 + (g(x))^2 \right] dx.$$

Problem B. Find the solution of the problem solution of the problem (1)-(3) for

$$f(x) = 4\sin(\pi x) + \sin(5\pi x), \quad g(x) = 0, \; x \in [0,1].$$