KOÇ UNIVERSITY

MATH 102 - CALCULUS

Midterm II April 28, 2009

Duration of Exam: 90 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Surname, Name: ANSWERS

Signature: ______________________________

Section (Check One):

Section 1: Aybike Özer M-W (15:30) ___
Section 2: Burak Özbağcı M-W (14:00) ___
Section 3: E. Şule Yazıcı Tu-Th(11:00) ___
Section 4: E. Şule Yazıcı Tu-Th(14:00) ___
Section 5: Sinan Ünver M-W(11:00) ___

PROBLEM	POINTS	SCORE
1	35	
2	10	
3	10	
4	20	
5	25	
TOTAL	100	

Problem 1. (35 points) Calculate the following integrals

(a) $\int \frac{\cos x}{\sin^5 x}\, dx = \int \frac{du}{u^5} = \int u^{-5}\, du = -\frac{u^{-4}}{4} + C = -\frac{1}{4\sin^4 x} + C$

$u = \sin x$

$du = \cos x\, dx$

(b) $\int_{-1}^{2} |x^3|\, dx = \int_{-1}^{0} -x^3\, dx + \int_{0}^{2} x^3\, dx = \left[-\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^4}{4}\right]_{0}^{2} = \left[0 + \frac{1}{4}\right] + \left[\frac{16}{4} - 0\right]$

$= \frac{1}{4} + \frac{16}{4} = \frac{17}{4}$

(c) $\int \sqrt{x^3} + \frac{1}{\sqrt[3]{x^5}}\, dx = \int (x^{3/2} + x^{-5/3})\, dx = \frac{2}{3} x^{5/2} - \frac{3}{5} x^{-2/3} + C$

$= \frac{2}{3}\sqrt{x^5} - \frac{3}{5\sqrt[3]{x^2}} + C$

(d) $\int_{0}^{1} 3x^2\, dx = \left.\frac{3x^3}{3}\right]_{0}^{1} = 1 - 0 = 1$

(e) $\int_e^{e^2} \frac{(\ln x)^5}{x}\,dx = \int_1^2 u^5 . dx = \left.\frac{u^6}{6}\right]_1^2 = \frac{2^6}{6} - \frac{1}{6} = \frac{64}{6} - \frac{1}{6} = \frac{63}{6} = \frac{21}{2}$

$u = \ln x$ $\qquad \ln e = 1$

$du = \frac{dx}{u}$ $\qquad \ln e^2 = 2$

(f) $\int_0^1 x e^{x^2}\,dx = \int_0^1 e^u \frac{du}{2} = \frac{1}{2}\int_0^1 e^u du = \frac{1}{2}\left[e^u\right]_0^1 = \frac{e}{2} - \frac{e^0}{2} = \frac{e-1}{2}$

$u = x^2$ $\qquad 0^2 = 0$

$du = 2x\,dx$ $\qquad 1^2 = 1$

$\frac{du}{2} = x\,dx$

(g) $\int \frac{3x}{x^2+1}\,dx = \int \frac{3\,du}{2u} = \frac{3}{2}\int \frac{1}{u}\,du = \frac{3}{2}\ln u + C$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

Problem 2. (10 points) Find the critical points of the function $f(x) = \int_0^x (1 - e^{t^2-1})\,dt$

$f'(x) = \frac{d}{dx}\left(\int_0^x (1 - e^{t^2-1})\,dt\right) = (1 - e^{x^2-1}) - (1 - e^{0-1})$ by Fundamental Theorem of Calculus

$f'(x) = e^{-1} - e^{x^2-1} = \frac{1}{e} - \frac{e^{x^2}}{e} = \frac{1-e^{x^2}}{e}$

$f'(x) = 0 \Rightarrow e^{-1} = e^{x^2-1} \Rightarrow x^2 - 1 = -1 \Rightarrow x^2 = 0 \Rightarrow x = 0$

$f(0) = \int_0^0 (1 - e^{t^2-1})\,dt = 0$

$f'(x)$ exists everywhere. So the only critical point is $(0,0)$.

Problem 3. (10 pts) Calculate the following limit using the L'hospital rule

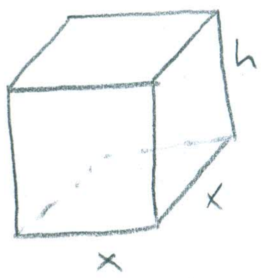
$$\lim_{x\to\infty} x\tan\frac{1}{x} =$$

$$\lim_{x\to\infty} x\tan\frac{1}{x} = \lim_{x\to\infty} \frac{\tan\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sec^2\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$$

$$\frac{0}{0}$$

$$= \lim_{x\to\infty} \sec^2\left(\frac{1}{x}\right) = \lim_{x\to\infty} \frac{1}{\cos^2\left(\frac{1}{x}\right)} = 1.$$

Problem 4. (20 pts) If $1200cm^2$ of material is available to make a box with a square base and an open top, find the largest possible volume of the box.



$$x^2 + 4xh = 1200 \Rightarrow h = \frac{1200 - x^2}{4x}$$

$$V = x^2 h$$

$$V(x) = x^2\left(\frac{1200 - x^2}{4x}\right) = 300x - \frac{x^3}{4}$$

$$V'(x) = 300 - \frac{3}{4}x^2 = 0 \Rightarrow x^2 = \frac{1200}{3} = 400 \Rightarrow x = 20$$

$$V''(x) = -\frac{3}{2}x \qquad V''(20) = -30 < 0$$ so $x = 20$ maximizes the volume.

$x = 20$ $\quad h = \frac{1200 - 400}{80} = 10$ $\quad V = (20)^2 10 = 4000\ cm^3.$

Problem 5. A continuous function f satisfies the properties given below.

(1) The domain of f is $\mathbb{R}$

(2) $f(-1) = 0$; $f(2) = 0$; $f(0) = 1$.

(3) $\lim_{x \to \infty} f(x) = -\infty$; $\lim_{x \to -\infty} f(x) = 1$

(4) $f'(x) > 0$ when $x \in (-1, 1)$; $f'(x) < 0$ when $x \in (-\infty, -1) \cup (1, \infty)$; and $f'(-1) = f'(1) = 0$

(5) $f''(x) > 0$ when $x \in (-\sqrt{3}, 0)$; $f''(x) < 0$ when $x \in (-\infty, -\sqrt{3}) \cup (0, \sqrt{3}) \cup (\sqrt{3}, \infty)$; $f''(-\sqrt{3}) = f''(0) = f''(\sqrt{3}) = 0$

(a) (5 points) Find the local maximum points and local minimum points of f.

	$-\infty$		-1		1		∞
$f'(x)$		$-$	0	$+$	0	$-$	

$f(-1) = 0$ is the local minimum point of f.

$f(1)$ is the local maximum point of f.

(b) (5 points) Find inflection points and the intervals where the graph of f is concave up or concave down.

	$-\infty$		$-\sqrt{3}$		0		$\sqrt{3}$		∞
$f''(x)$		$-$	0	$+$	0	$-$	0	$-$	

$(-\sqrt{3}, f(-\sqrt{3}))$ and $(0, f(0)) = (0,1)$ are inflection point.

f is concave up on $(-\sqrt{3}, 0)$, f is concave down on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3}) \cup (\sqrt{3}, \infty)$.

(c) (5 points) Find the asymptotes of f.

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = 1 \longrightarrow y = 1$ is the horizontal asymptote of f.

Since the domain of f is $\mathbb{R}$, there is no vertical asymptote of f.