KOÇ UNIVERSITY

MATH 102 - CALCULUS

Final May 28, 2009

Duration of Exam: 120 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print** (use **CAPITAL LETTERS**) **and sign your name, and indicate your section below.**

Surname, Name: ______________________________

Signature: ______________________________

Section (Check One):

Section 1: Aybike Özer M-W (15:30) ___
Section 2: Burak Özbağcı M-W (14:00) ___
Section 3: E. Şule Yazıcı Tu-Th(11:00) ___
Section 4: E. Şule Yazıcı Tu-Th(14:00) ___
Section 5: Sinan Ünver M-W(11:00) ___

PROBLEM	POINTS	SCORE
1	25	
2	5	
3	10	
4	15	
5	10	
6	25	
7	10	
TOTAL	100	

Problem 1. (25 points) Calculate the following integrals

(a) $\int \frac{3x+1}{x^2+1}\,dx = \int \frac{3x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx = \frac{3}{2}\ln|x^2+1| + \arctan x + C$

$u = x^2+1$

$du = 2x\,dx$

$\frac{3}{2}\int \frac{1}{u}\,du = \frac{3}{2}\ln|x^2+1|$

$\int \frac{1}{x^2+1}\,dx = \arctan x$

(b) $\int_0^\infty \frac{2x}{(x^2+1)^2}\,dx = \lim_{t\to\infty} \int_0^t \frac{2x}{(x^2+1)^2}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\int_1^{t^2+1} \frac{1}{u^2}\,du = -\frac{1}{u}\Big|_1^{t^2+1} = -\frac{1}{t^2+1} + 1$

$= \lim_{t\to\infty}\left(1 - \cancel{\frac{1}{t^2+1}}^{\,0}\right) = 1$

(c) $\int \frac{x+3}{(x+1)(x+2)}\,dx = K$

$\frac{x+3}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} = \frac{Ax+2A+Bx+B}{(x+1)(x+2)} = \frac{x(A+B)+2A+B}{(x+1)(x+2)}$

$\Rightarrow \begin{array}{l} A+B=1 \Rightarrow B = 1-A \\ 2A+B = 3 \Rightarrow 2A+1-A = A+1 = 3 \\ \Rightarrow A = 2 \\ B = -1 \end{array}$

$K = \int \frac{2}{x+1}\,dx + \int \frac{-1}{x+2}\,dx = 2\ln|x+1| - \ln|x+2| + C$

$= 2\ln\frac{|x+1|}{|x+2|} + C.$

(d) $\int_0^{\frac{\pi}{2}} \underset{u}{x}\,\underset{dv}{\sin 2x\,dx} = -\frac{x\cdot\cos 2x}{2}\Big|_0^{\pi/2} - \int_0^{\pi/2} \frac{\cos 2x}{2}\,dx = \frac{\pi}{4} - \frac{\sin 2x}{4}\Big|_0^{\pi/2}$

$du = 1 \quad v = -\frac{\cos 2x}{2}$

$= \frac{\pi}{4} - (0-0) = \pi/4$

(e) $\int_e^{e^2} \frac{1}{x \ln x}\, dx = \int_1^2 \frac{1}{u}\, du = \ln|u|\Big|_1^2 = \ln(2) - \cancel{\ln(1)} = \ln(2)$

$u = \ln x$

$du = \frac{dx}{x}$

$\int_1^2 f(x)dx = F(2) - F(1)$

$F'(x) = f(x)$.

Problem 2. (5 points) Calculate $\frac{d}{dx}\int_1^2 te^{t^3}\, dt$

In the case that the limits of integral is constant i.e 1 and 2, since the integral $\int_1^2 te^{t^3}dt$ would be constant $\frac{d}{dx}\left(\int_1^2 te^{t^3}dt\right) = 0$. But if there is typo, the upper limit is $2x$ instead of 2 then we use Leibnitz Rule:

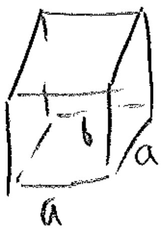
$\frac{d}{dx}\left(\int_1^{2x} \underbrace{te^{t^3}}_{f(t)} dt = f(2x)\cdot(2x)' = 2\, t\cdot e^{t^3}\Big|_{t=2x} = 4x\, e^{(2x)^3} = 4x e^{x^3} \cdot e^3 = 4e^3 x e^{x^3}\right.$

Problem 3. (10 pts) Calculate the following limit using the L'hospital rule

$$\lim_{x \to \pi/2} \frac{1 - \sin x}{1 + \cos 2x} \underset{L.H}{\overset{\frac{0}{0}}{=}} \lim_{x\to\pi/2} \frac{-\cos x}{-\sin 2x \cdot 2}$$

$$\underset{L.H}{\overset{\frac{0}{0}}{=}} \lim_{x\to\pi/2} \frac{\sin x}{-\cos 2x \cdot 4} = \frac{1}{4}$$

Problem 4. (15 pts) If $1200 cm^2$ of material is available to make a box with a square base and an open top, find the largest possible volume of the box.



$$a^2 + 4ab = 1200\,cm^2 \Rightarrow b = \frac{1200-a^2}{4a}$$

$$V = a^2 b = a^{\cancel{2}} \cdot \frac{1200-a^2}{4\cancel{a}}$$

$$V(a) = \frac{1200a - a^3}{4}$$

$$V'(a) = \frac{\overset{300}{\cancel{1200}}}{\cancel{4}} - 3a^2 = 0$$

$$a^2 = 100$$

$$a = 10$$

$$V = (10)^{\cancel{2}} \cdot \frac{1200-100}{4 \cdot \cancel{10}}$$

$$= \frac{11000}{4}$$

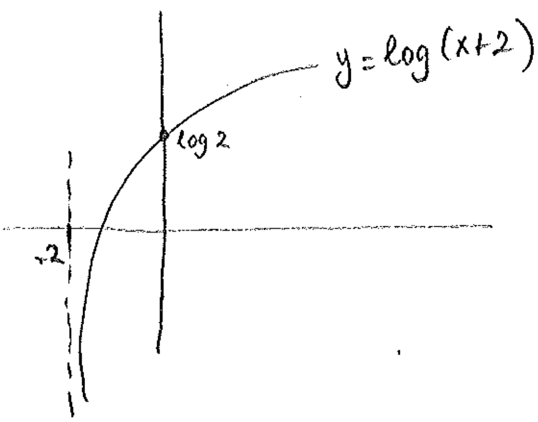
Problem 5. (10 pts) Find the domain and sketch the graph of the function $f(x) = \log(x+2)$.

log funct is defined on $(0, \infty)$

so $x+2 > 0 \Rightarrow x > -2$ so. domain$(f) = \{x \in \mathbb{R} : x > -2\}$

$= (-2, \infty)$

$x = 0$ $y = \log 2$



Problem 6.

(a) (10 points) Find the volume of the solid obtained by revolving the region bounded by the curve $y = \sqrt[4]{x}$ and the lines $y = 0$ and $x = 4$ about the x-axis.

(b) (15 points)Find c if the area of the region enclosed by $y = x^2 - c^2$ and $y = c^2 - x^2$ is $576\ cm^2$

Problem 7. (10 pts) Find the equation of the tangent line to the curve $\frac{x^2}{8}+\frac{y^2}{18}=1$ at the point $(-2,-3)$.

$$\frac{2x}{8}+\frac{2y}{18}y'=0$$

$$\frac{x}{4}+\frac{y}{9}y'=0 \overset{\text{at}(-2,-3)}{\Rightarrow} \frac{-2}{4}+\frac{-3}{9}y'=0 \Rightarrow -2-3y'=0$$

$$\Rightarrow 3y'=-2$$

$$\Rightarrow y'=\frac{-2}{3}$$

$$(y+3)=\frac{-2}{3}(x+2)$$

$$\boxed{3y=-2x-13}$$ Tangent line eqn.