KOÇ UNIVERSITY

MATH 102 - CALCULUS

Midterm II May 5, 2006

Duration of Exam: 90 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Surname, Name: ____________________

Signature: ____________________

Section (Check One):

Section 1: S. Küçükçifci ___
Section 2: T. Albu (9:30) ___
Section 3: E.Ş. Yazici (15:30) ___
Section 4: T. Albu (12:30) ___
Section 5: E.Ş. Yazici (11:00) ___

PROBLEM	POINTS	SCORE
1	40	
2	10	
3	10	
4	10	
5	10	
6	10	
TOTAL	100	

Problem 1. Evaluate the following indefinite and definite integrals:

(a) (10 pts) $\int \frac{x^3}{\sqrt{x^4+4}}\,dx =$

$u = x^4+4$

$du = 4x^3dx$

$\Rightarrow \int \frac{x^3}{\sqrt{x^4+4}}\,dx = \frac{1}{4}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{4}\int u^{-1/2}du = \frac{\sqrt{u}}{2}$

$= \frac{\sqrt{x^4+4}}{2}$

(b) (10 pts) $\int_0^1 \sqrt{2x}\,(\sqrt{x}+\sqrt{2})\,dx$

$\int_0^1 \sqrt{2x}(\sqrt{x}+\sqrt{2})dx = \int_0^1 (x\sqrt{2} + 2\sqrt{x})\,dx = \frac{\sqrt{2}}{2}x^2 + \frac{4}{3}x\sqrt{x}\Big|_0^1 = \frac{3\sqrt{2}+8}{6}$

(c) (10 pts) $\int \sin x \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\Rightarrow \int \sin x \cos x\,dx = \int u\,du = \frac{u^2}{2}$

$= \frac{\sin^2 x}{2}$

(d) (5 pts) $\int_1^{-1} x^2(x^3+1)^4 dx$

$u = x^3+1$

$du = 3x^2dx$

$x=-1 \Rightarrow u=0$

$x=1 \Rightarrow u=2$

$\int_1^{-1} x^2(x^3+1)^4dx = \frac{1}{3}\int_2^0 u^4du$

$= \frac{u^5}{15}\Big|_2^0 = \frac{32}{15}$

Problem 2. Calculate the following limit or show that it does not exist:

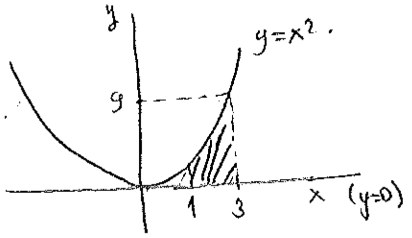
$$\lim_{x\to 0} \frac{1-\cos 3x}{1-\cos 5x} = \frac{0}{0}$$ Use L'Hospitals Rule.

$$\lim_{x\to 0} \frac{\sin 3x \cdot 3}{\sin 5x \cdot 5} = \lim_{x\to 0} \frac{\overbrace{\frac{\sin 3x}{3x}}^{1} \cdot 3\cancel{x} \cdot 3}{\underbrace{\frac{\sin 5x}{5x}}_{1} \cdot 5\cancel{x} \cdot 5} = \frac{9}{25}$$

since $\lim_{x\to 0} \frac{\sin x}{x} = 1$

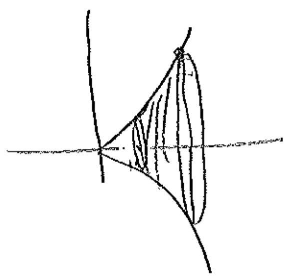
Problem 3.

(a) (5 pts) Find the area between the curve $y = x^2$ and the x-axis between 1 to 3.



$$\int_1^3 (x^2-0)\,dx = \int_1^3 x^2\,dx = \left.\frac{x^3}{3}\right|_1^3 = 9 - \frac{1}{3} = \frac{26}{3}$$

(b) (5 pts) Find the volume of the solid formed by revolving the area you obtain above (problem (3-a)) about the x-axis.



$$\int_1^3 2\pi (x^2)^2\,dx = 2\pi \left.\frac{x^5}{5}\right|_1^3 = 2\pi\left(\frac{243}{5} - \frac{1}{5}\right)$$

$$= 2\pi \cdot \frac{242}{5}$$

Problem 4. Find the absolute extremum of the function $f(x) = \dfrac{2x}{(x+2)^2}$ on [-1,3].

$$f'(x) = \frac{2(x+2)^2 - 2x \cdot 2(x+2)}{(x+2)^4} = \frac{2(x+2) - 4x}{(x+2)^3} = \frac{4-2x}{(x+2)^3}.$$

critical pts of $f'(x)$ are $x=2$ & $x=-2$.

we don't need since $-2 \notin [-1,3]$.

$f(2) = \frac{1}{4} \rightarrow$ absolute max.

$f(-1) = -2 \rightarrow$ absolute min.

$f(3) = \frac{6}{25}$

Problem 5. The revenue of a manufacture's product is given by the function

$$R(q) = 20q - \frac{q^2}{4}$$

where q is the number of units. At What production level will there be a maximum revenue? What is the maximum revenue?

$R'(q) = 20 - \frac{q}{2}$ $\Rightarrow$ critical pts of R is $q=40$

$R(40) = 20 \cdot 40 - \frac{40^2}{4} = 800 - 400 = 400$

Problem 6. A function $f(x)$ satisfies the properties given below.

1-)Domain of f : $\mathbb{R}$

2-)$f(0) = 1$; $f(1) = 0$; $f(-1) = 0$; $f(2) = 1$.

3-) $\lim_{x\to\infty} f(x) = 2$; $\lim_{x\to-\infty} f(x) = -1$

4-)$f'(1) = 0$ and $f'(0)$ is undefined

5-)The sign table of $f'(x)$ is as follows

	$-\infty$		0		1		∞
f'	|	$+++$	||	$---$	|	$+++$	|
		$\nearrow$		$\searrow$		$\nearrow$	

6-) The sign table of $f''(x)$ is as follows

	$-\infty$		0		1		2		∞
f''	|	$+++$	||	$++$	||	$+$	|	$---$	|

a-) State the local maximum points, local minimum points, inflection points and the intervals where the graph is concave up or concave down.

local max at $x=0$

local min at $x=1$

inflection pts at $x=2$

Concave up on $(-\infty, 2)$

Concave down on $(2, \infty)$

b-) Sketch the graph of a function which satisfies the properties given.