KOÇ UNIVERSITY

MATH 102 - CALCULUS

Midterm Exam May 6, 2005

Duration of Exam: 90 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below. GOOD LUCK!**

Surname, Name: ______________________

Student ID no: ______________________

Signature: ______________________

Section (Check One):

Section 1 (Vahap Erdoğdu) : ___
Section 2 (Burak Özbağcı- MW: 11:30-13:20): ___
Section 3 (Özgür Müstecaplıoğlu): ___
Section 4 (Tolga Etgü - MW: 9:30-11:20): ___
Section 5 (Tolga Etgü - MW: 12:30-14:20): ___
Section 6 (Burak Özbağcı- MW: 14:30-16:20) : ___

PROBLEM	1	2	3	4	TOTAL
POINTS	15	15	40	30	100
SCORE					

Name:

Problem 1

(1.a) **(5 pts)** Evaluate

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx.$$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} dx$

$$\int \cos u \cdot 2\, du = 2\int \cos u\, du = 2 \sin u + C = 2\sin\sqrt{x} + C.$$

constant.

(1.b) **(5 pts)** Find

$$\frac{d}{dx}\Big(\int_{\sqrt{2x}}^{e} e^{\sin(t^2)}\, dt\Big).$$

$$= -\frac{d}{dx}\left(\int_e^{\sqrt{2x}} e^{\sin(t^2)}\, dt\right) \underset{u=\sqrt{2x}}{=} \frac{d}{du}\left(\int_e^{u} e^{\sin t^2} dt\right)\cdot \frac{du}{dx} = e^{\sin u^2}\cdot \frac{1}{\sqrt{2x}}$$

By chain rule

by Fundamental thm of calculus.

$$= \frac{e^{\sin 2x}}{\sqrt{2x}}$$

(1.c) **(5 pts)** Which one is bigger:

$$\frac{2\ln 77 + 2\ln(\frac{1}{11})}{\ln 49} \quad \text{or} \quad \arcsin(\cos(\frac{\pi}{4}))?$$

Why?

$$\frac{2\ln 77 + 2\ln(\frac{1}{11})}{\ln 49} = \frac{2(\ln 7 + \ln 11) + 2(\ln 1 - \ln 11)}{2\ln 7} = \frac{2\ln 7}{2\ln 7} = 1 //$$

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \arcsin\left(\cos\frac{\pi}{4}\right) = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} //$$

$$1 > \frac{\pi}{4} \quad \text{so,} \quad \frac{2\ln 77 + 2\ln(\frac{1}{11})}{\ln 49} > \arcsin\left(\cos\frac{\pi}{4}\right)$$

Name:

Problem 2 Find the derivatives of the following functions:

(2.a) **(5 pts)** $y = \arctan\sqrt{x^2+1}$

$$\frac{dy}{dx} = \frac{1}{1+(x^2+1)} \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x.$$

(2.b) **(5 pts)** $y = 5^{-\cos(2x)}$

$$\ln y = -\cos(2x) . \ln 5$$

$$\frac{y'}{y} = 2\sin 2x . \ln 5 \Rightarrow y' = 5^{-\cos(2x)} . 2 . \ln 5 . \sin 2x$$

(2.c) **(5 pts)** $y = \ln(\ln(\ln x))$

$$\frac{dy}{dx} = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}.$$

Name:

Problem 3

(3.a) **(15 pts)** Find the arc-length of the curve $y = \ln(\sec x)$ for $0 \le x \le \frac{\pi}{4}$.

$$\int_0^{\pi/4} \sqrt{1 + ((\ln(\sec x))')^2}\, dx \qquad \ln(\sec x)' = \frac{1}{\sec x} \cdot (\sec x)'$$

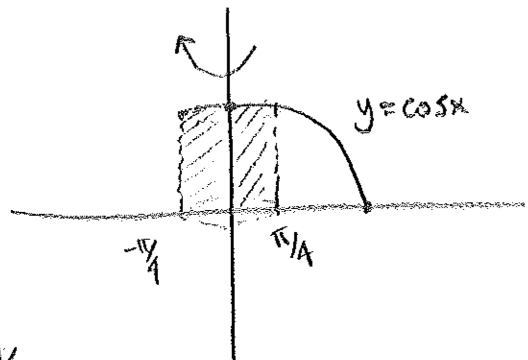
$$= \frac{\sec x \cdot \tan x}{\sec x} = \tan x$$

$$= \int_0^{\pi/4} \sqrt{\frac{1}{\cos^2 x}}\, dx = \int_0^{\pi/4} \frac{1}{\cos x}\, dx$$

$$= \int_0^{\pi/4} \sec x\, dx = \ln(\sec x + \tan x)\Big|_0^{\pi/4}$$

$$= \ln\left(\frac{1}{\sqrt{2}} + 1\right) - \underbrace{\cancel{\ln 1}}_{0}$$

(3.b) **(10 pts)** Find the volume of the solid obtained by revolving the region bounded by the curves $y = \cos x$, $y = 0$, $x = 0$ and $x = \frac{\pi}{4}$ about the x-axis.



$$\int_0^{\pi/4} 2\pi . x . \cos x\, dx = 2\pi \left(x . \sin x \Big|_0^{\pi/4} - \int_0^{\pi/4} \sin x\, dx \right)$$

$x = u$

$dx = du$

$\cos x\, dx = dv$

$\sin x = v$

$$= 2\pi \left(\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} + \cos x \Big|_0^{\pi/4} \right)$$

$$= 2\pi \left(\frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 \right)$$

Name:

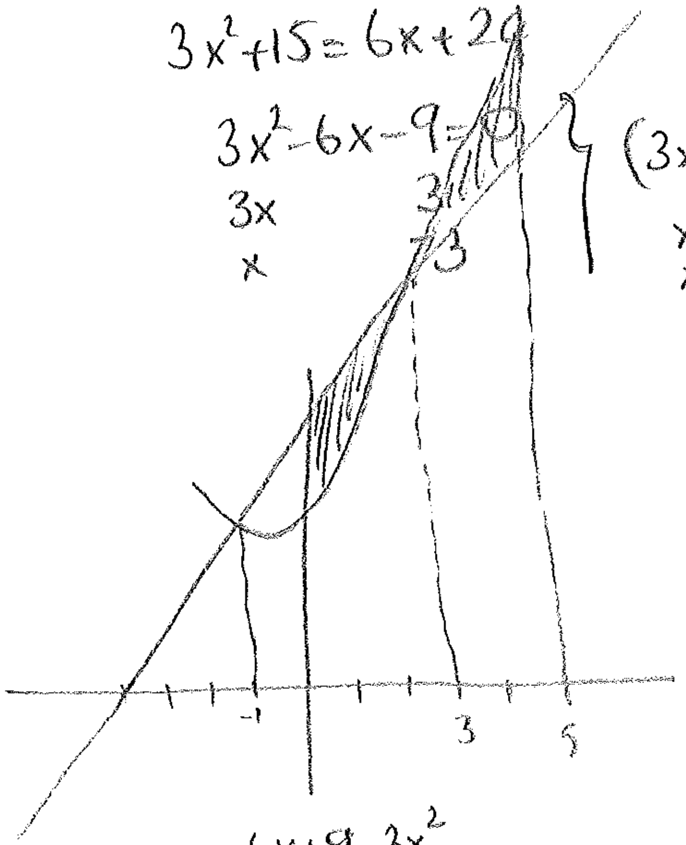
(3.c) **(15 pts)** Find the area of the region bounded by the curves $y = 3x^2 + 15$ and $y = 6x + 24$ for $0 \leq x \leq 5$.

$$3x^2+15 = 6x+24$$

$$3x^2-6x-9=0 \quad \left\{ (3x+3)(x-3)=0 \right.$$

$$\begin{array}{ll} 3x & 3 \\ x & -3 \end{array}$$

$x=3$
$x=-1$



$$\int_0^3 \overbrace{(6x+24)-(3x^2+15)}^{6x+9-3x^2}\,dx + \int_3^5 \overbrace{(3x^2+15)-(6x+24)}^{3x^2-6x-9}\,dx$$

$$= \frac{6x^2}{2}+9x-\frac{3x^3}{3}\Big|_0^3 + \frac{3x^3}{3}-\frac{6x^2}{2}-9x\Big|_3^5$$

$$= 27+\cancel{27}-\cancel{27}+5^3-3.5^2-9.5-\cancel{3^3}+\cancel{3.3^2}+9.3$$

$$= 54+125-75-45 = \underline{\underline{59}}$$

Name:

Problem 4

(4.a) **(10 pts)** Evaluate

$$\int_1^e x^3 \ln x \, dx.$$

By using Int. by Parts;

$\ln x = u$

$\frac{1}{x} dx = du$

$x^3 dx = dv$

$\frac{x^4}{4} = v$

$$u.v - \int v.du = \ln x . \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$$

$$= \ln x . \frac{x^4}{4} - \frac{x^4}{16}$$

(4.b) **(10 pts)** Evaluate

$$\int_0^1 \frac{dx}{(4-x^2)^{3/2}}$$

By using the method of substitution

$x = 2\cos\theta$

$dx = -2\sin\theta$

$x = 1 \Rightarrow \theta = \frac{\pi}{3}$

$x = 0 \Rightarrow \theta = -\frac{\pi}{2}$

$$\int_0^1 \frac{dx}{(4-x^2)^{3/2}} = \int_{-\pi/2}^{\pi/3} \frac{-2\sin\theta \, d\theta}{2^3 \sin^3\theta}$$

$$= -\int_{-\pi/2}^{\pi/3} \frac{d\theta}{4\sin^2\theta} = +\frac{1}{4} \cot\theta \Big|_{-\pi/2}^{\pi/3}$$

$$= \frac{1}{4}\left(\frac{1}{\sqrt{3}} - 0\right) = \frac{1}{4\sqrt{3}}$$

Name:

(4.c) **(10 pts)** Evaluate

$$\int \frac{4\,dx}{x^3+4x^2+4x}$$

$x(x^2+4x+4)$

$= x(x+2)^2$

$$\frac{4}{x^3+4x^2+4x} = \frac{4}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$(x+2)^2 \qquad x(x+2) \qquad (x)$

$$4 = A(x+2)^2 + Bx.(x+2) + C.x$$

$$4 = (A+B)x^2 + (4A+2B+C)x + 4A$$

$A+B=0 \Rightarrow A=-B$

$4A=4 \Rightarrow A=1 \Rightarrow B=-1$

$4A+2B+C=0 \Rightarrow 4-2+C=0 \Rightarrow C=-2.$

$$\int \frac{4}{x(x+2)^2} = \int \frac{1\,dx}{x} + \int \frac{-1}{x+2}dx + \int \frac{-2}{(x+2)^2}dx$$

$$= \ln|x| - \ln|x+2| + \frac{2}{x+2} + C$$

constant.

$\int -\frac{1}{x+2}dx = -\int \frac{1}{u}du = -\ln|x+2|+C$

$x+2=u$

$dx=du$

$\int \frac{-2}{(x+2)^2}dx = \int -2u^{-2}du = \frac{-2u^{-1}}{-1}+C$

$= \frac{2}{x+2}+C$

$x+2=u$

$dx=du$