Question 1. (30 Points)

Consider the function $f(x) = 2x^3 + cx^2 + 2x$.

a) For what values of c will this function have extrema (maxima and/or minima)?

b) Find the x-coordinates of the turning points (inflection points) in terms of c.

c) For what values of c will this function have saddle points? (A saddle point is where the first derivative is zero and where the concavity changes. The first derivative does not change sign at a saddle point.)

d) The derivative of $f(x)$ is $f'(x) = 6(x + \frac{c}{6})^2 - \frac{c^2}{6} + 2$. Sketch $f(x)$ for $c = 6$ and $c = 3$ using the expression for the derivative.

Question 2. (20 Points)

a) The tip of an isoceles triangle is at $(0, 0)$, and its other two points are at (a, b) and $(-a, b)$, which are on a semi-circle with center $(0, 0)$ and radius 2. What is the maximum possible area of this triangle? (a and b are positive numbers.)

b) The tangent to the graph of the function $f(x) = \log_2 x$ at $x = 8$ is $y = 0.1803x + 1.5573$. The differential of $f(x)$ at $x = 8$ is $df = Adx$. What is A? Write the approximate value of $log_2(8.01)$ in terms of A.

Question 3. Find the following limits: (20 Points)

a) $\lim_{x \to 0} (1 - 2x)^{\frac{1}{x}}$

b) $\lim_{x \to 0} \dfrac{\cos x}{\sin x} - \dfrac{1}{x}$

c) $\lim_{x \to 0} \dfrac{\cos mx - \cos nx}{x^2}$

d) $\lim_{x \to 1} \dfrac{x}{x - 1} - \dfrac{1}{\ln x}$

Question 4. Evaluate the following integrals (40 Points)

a) $\int_0^1 xe^{-x^2}\ dx$

b) $\int \frac{2^x}{2^x+3}\ dx$

c) $\int \frac{\cos x}{\sin^2 x}\ dx$

d) $\int \cos\sqrt{x}\ dx$

e) $\int (\arcsin x)^2\ dx$

f) $\int x^3 \ln x\ dx$