

Question 1. Find the following limits: (32 Points)

a)  $\lim_{x \rightarrow 0} \sqrt{\frac{\tan 3x}{x}}$

b)  $\lim_{x \rightarrow -1} \frac{2x^2 - 2x - 4}{x^2 + 3x + 2}$

c)  $\lim_{x \rightarrow 0} \frac{3^x - 1}{x}$

i. Is it 1, more than 1, or less than 1? Justify your answer.

ii. Use your calculator to find the (approximate) answer.

d)  $\lim_{x \rightarrow a} \tan 3x = -\infty$ , what is  $a$ ?

e)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x}(x^2 + \frac{3}{x})}{\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} - x}$

f)  $\lim_{x \rightarrow \infty} \frac{\sin 2x}{x}$

g)  $\lim_{x \rightarrow \infty} (\sqrt{3x^2 + 8x + 6} - \sqrt{3x^2 + 3x + 1})$

Question 2. Let  $f(x)$  is given as follows: (20 Points)

$$F(x) = \begin{cases} mx - 2 & \text{if } x < -0.1 \\ (x - 1)^2 - b & \text{if } x \geq \frac{0.7}{\pi} \\ 3 \sin(2\pi x - 1.4) + 1 & \text{if } -0.1 \leq x \leq \frac{0.7}{\pi} \end{cases}$$

For which values of  $b$  and  $m$  is  $f$  continuous? Use your calculator to write the values up to two decimal places.

Question 3. Use the definition of derivative to calculate  $f'$ : (18 Points)

a)  $f(x) = \frac{1}{x^3}$

b)  $f(x) = \sqrt{x}$

Question 4. (30 Points)

a) Differentiate  $f(x) = (\ln(1 + e^x))^2$

b) Differentiate  $f(x) = x^x$

c) Differentiate  $f(x) = \sec\left(\frac{x^2 - 1}{x}\right)$

d) Find an equation of the tangent line to the curve  $y = 3 \cos(x/2)$  at the point  $(\pi, 0)$ .

e) Suppose  $f$  is a one-to-one differentiable function and  $f^{-1}$  is also differentiable. If  $f(4) = 5$  and  $f'(4) = \frac{2}{3}$ , find  $(f^{-1})'(5)$ .