KOÇ UNIVERSITY

FALL 2013 FINAL EXAM

MATH 102 JANUARY 7, 2014

Duration of Exam: 135 minutes

INSTRUCTIONS:

- **No books, no notes, no questions, and talking allowed.**
- **You must always explain your answers and show your work to receive full credit.**
- **Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!**

SURNAME, Name: ________________________________

Student ID no: ________________________________

Signature: ________________________________

(Check One):

(Doğan Bilge – TTh 09:30-10:45) : ___
(Doğan Bilge – TTh 11:00-12:15) : ___
(Mehmet Sarıdereli – TTh 15:30-16:15) : ___

Problem	1	2	3	4	5	6	7	8	TOTAL
Points	10	20	10	15	15	20	15	10	115
Score									

Question 1. (10 Points)

Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$ and the x-axis about the x-axis.

Question 2. (20 Points)

Sketch the curve $y = \dfrac{x^2 - 4x}{x - 1}$.

Question 3. (10 Points)

A cylindrical tank with radius 5m is being filled with water at a rate of $3m^3/min$. How fast is the height of the water increasing?

Question 4. (15 Points)

a) Find $\dfrac{d}{dx}\displaystyle\int_{\sqrt{x}}^{3x} t^2 dt$ using the Fundamental theorem of Calculus.

b) Find $\dfrac{d}{dx}\displaystyle\int_{\sqrt{x}}^{3x} t^2 dt$ by first finding $\displaystyle\int_{\sqrt{x}}^{3x} t^2 dt$ and then taking the derivative of the result.

c) Find $\displaystyle\int_{1}^{e} (2x \ln x + x)dx$ given that the derivative of $x^2 \ln(x)$ is $2x \ln x + x$.

Question 5. (15 Points)

$y = f(x)$ is a one-to-one function, and the point $(1, 2)$ is on its graph. Let $f^{-1}(x)$ be the inverse function of $f(x)$, and $f'(x)$ be the derivative of $f(x)$. The equation of the tangent to $y = f(x)$ at $(-1, 2)$ is $y = 2x + b$. Find the following. Justify your answer.

i. b

ii. $f^{-1}(2)$

iii. $f'(-1)$

iv. $f^{-1}(f(-1))$

v. $\frac{d}{dx} f^{-1}(x)|_{x=2}$

Question 6. (20 Points)

Evaluate the following integrals:

a) $f(x) = \int \frac{x}{x^2 + 2x + 2} dx$ Hint: $x^2 + 2x + 2 = (x + 1)^2 + 1$

b) $f(x) = \int \sin(2x) \cos(2x) dx$

c) $f(x) = \int e^{5x} \cos(2x) dx$

d) $f(x) = \int \frac{3(x + 3)}{(x - 1)(x + 2)} dx$

Question 7. (15 Points)

The graph of $f(x) = x^3 + bx^2 + cx + d$ is increasing on the interval $x < -1$, decreasing on the interval $-1 < x < 3$ and increasing on the interval $x > 3$. The graph is concave down for $x < 1$ and concave up for $x > 1$. The inflection point is on the x-axis. Find the constants b, c and d.

Question 8. (10 Points)

A cone shaped drinking cup is made from a circular piece of paper of radius R by cutting out the sector BA and joining the edges OA and OB. Find the maximum capacity of such a cup.