Question 1 (15 Points):

Find the following limits:

(a) $\lim_{h \to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h}$

(b) $\lim_{x \to 0} \frac{\tan(4x)}{\sin(5x)}$

(c) Let $f(x) = \frac{\tan(4x)}{\sin(5x)}$ for $-\pi/2 < x < \pi/2$, $x \neq 0$. How would you define $f(0)$ so that $f(x)$ is continuous?

Question 2 (15 Points):

(a) $y = f(x)$ is a one-to-one function, and the point (–1, 2) is on its graph. Let $f^{-1}(x)$ be the inverse function of $f(x)$, and $f'(x) = \frac{d}{dx} f(x)$ be the derivative of $f(x)$. The equation of the tangent to $y = f(x)$ at (–1, 2) is $y = 2x + b$. Find the following. Justify your answers.

(i) b

(ii) $f^{-1}(2)$

(iii) $f'(-1)$

(iv) $f^{-1}(f(-1))$

(v) $\frac{d}{dx} f^{-1}(x)\Big|_{x=2}$

(b) If $\sin(x) = -\frac{1}{2}$, then what are all possible values for $\tan(x)$?

Question 3 (15 Points):

Let $f'(x) = \dfrac{d}{dx} f(x)$ be the derivative of $f(x)$. Find

(a) $f'(x)$ for $f(x) = \sqrt[3]{\sin(x^2)}$

(b) The slope of the tangent at (1,–1) to the circle $x^2 + y^2 = 2$

(c) The function $f(x)$ is continuous in the interval (–5, 3). Find all local extrema of $f(x)$ in the interval (–5, 3) if $f'(1)$ does not exist and

x	(–5, –2)	–2	(–2, –1)	–1	(–1, 0)	0	(0, 1)	(1, 3)
$f'(x)$	–	0	+	0	+	0	–	+

Question 4 (10 Points):

(a) Find the $\frac{d}{dx}\int_{\sqrt{x}}^{3x} t^2 dt$ using the Fundamental Theorem of Calculus.

(b) Find $\frac{d}{dx}\int_{\sqrt{x}}^{3x} t^2 dt$ by first finding $\int_{\sqrt{x}}^{3x} t^2 dt$, and then taking the derivative of the result.

(c) Find $\int_{1}^{e} (2(\ln(x)+1))dx$ given that the derivative of $x^2\ln(x)$ is $2(\ln(x)+1)$.

Question 5 (20 Points):

(a) Evaluate

$$\int_{0,5}^{1} \frac{x^2+13}{x^2+1}\,dx$$

(b) Find the area between the curve $y = 2x\sqrt{x^2+1}$, $0 \le x \le \sqrt{3}$, and the x-axis

Question 6 (10 Points):

Determine whether the improper integral $\int_{0}^{\infty} e^{-x}\,dx$ is convergent or divergent. If the improper integral is convergent, evaluate it.

Question 7 (10 Points):

Determine whether the following sequence is convergent or divergent. If the sequence is convergent, find its limit.

(a) $a_n = \dfrac{(-1)^n n}{n+1}$

(b) $a_n = \dfrac{\ln(n+1)}{\sqrt{n}}$

Question 8 (10 Points):

For each of the following series, write the first 2 terms and determine whether the series is convergent or divergent. If the series converges, find its sum.

(a) $\sum_{n=1}^{\infty} (-1)^n$

(b) $\sum_{n=0}^{\infty} \frac{2^{2n}}{3^{n+1} 5^n}$