KOÇ UNIVERSITY

FALL 2017 MATH102

Final Exam December 30, 2017

Duration of the exam: 75 minutes

Instructions: Calculators are not allowed. No books, no notes, no talking allowed. Explain your answers to get full credit. You can use the back of these pages.

Name, Surname: _____

Signature: _____

Section: ☐ 1 (Mo & We 14.30-15.45) ☐ 2 (Mo & We 16.00-17.15)

Problem	**Points**	**Score**
1	20	
2	25	
3	25	
4	30	
Total	**100**	

Problem 1

a) Find the derivative of the function (5 points)

$$f(x) = \sqrt{1 + xe^{-2x}}$$

b) Use implicit differentiation to find an equation of the tangent line to the curve

$$y^2(y^2 - 4) = x^2(x^2 - 5)$$

at the point $(0, -2)$. (10 points)

c) Use logarithmic differentiation to find the derivative of (5 points)

$$y = (\sqrt{x})^x$$

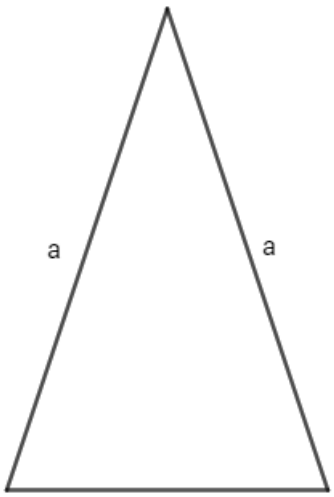
Problem 2

a) Find the absolute maximum and absolute minimum values of

$$f(x) = \frac{x}{x^2 - x + 1}$$

on the interval $[0, 3]$. (10 points)

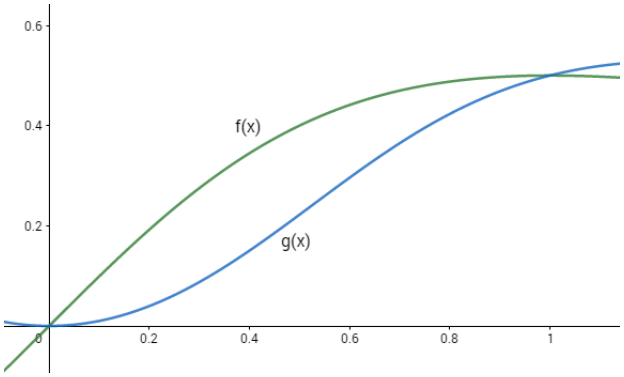
b) If the two equal sides of an isosceles triangle have length a, find the length of the third side that maximizes the area of the triangle. *(Hint: Pythagoras)*(15 points)



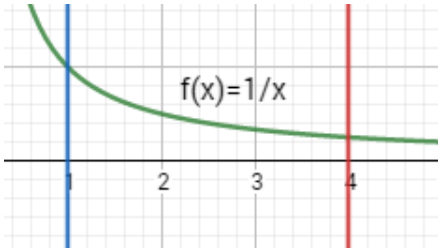
Problem 3

a) Find the area enclosed by $f(x)$ and $g(x)$ between $x = 0$ and $x = 1$. *(Hint: Look at the integrals separately and apply the Substitution rule)* (15 points)

$$f(x) = \frac{x}{1+x^2}, \quad g(x) = \frac{x^2}{1+x^3}$$



b) Find the volume of the solid obtained by rotating the region enclosed by $x = 1$, $x = 4$, $y = 0$ and $f(x) = \frac{1}{x}$ about the x-axis. (10 points)



Problem 4

a) Evaluate the integral *(Hint: Integration by parts)* (10 points)

$$\int_1^2 \frac{(\ln(x))^2}{x^3}\, dx$$

b) Evaluate the integral *(Hint: Partial fractions)* (10 points)

$$\int_0^1 \frac{x-4}{x^2-5x+6}\, dx$$

c) Is the improper integral (10 points)

$$\int_0^\infty \frac{1}{x^2+3x+2}\, dx$$

convergent or divergent? If it is convergent, what is its value? *(Hint: Partial fractions)*