KOÇ UNIVERSITY

MATH 102

SECOND MIDTERM MAY 13, 2011

Duration of Exam: 75 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and no talking allowed.
- You must always explain your answers and SHOW YOUR WORK to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: ______________________

Student ID no: ______________________

Signature: ______________________

(Check One):

(Selda Küçükçifçi – MW 9:30-10:45) : ——
(Selda Küçükçifçi – MW 12:30-13:45) : ——
(Tolga Etgü – TTh 14:00-15:15) : ——
(Tolga Etgü – TTh 17:00-18:15) : ——

PROBLEM	1	2	3	4	5	TOTAL
POINTS	15	25	20	20	30	110
SCORE						

Problem 1 (15 pts) Let $f(x) = 2x^4 - 12x^2 + 5$.

(a) **(5 pts)** Find the critical points of f.

$f'(x) = 8x^3 - 24x = 8x(x^2-3) = 8x(x-\sqrt{3})(x+\sqrt{3})$

Critical pts are $0, \sqrt{3}, -\sqrt{3}$

(b) **(3 pts)** Find the intervals on which f is increasing or decreasing.

x		$-\sqrt{3}$		0		$\sqrt{3}$	
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	$\searrow$		$\nearrow$		$\searrow$		$\nearrow$

f is decreasing on $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$

f is increasing on $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$.

(c) **(3 pts)** Find the points where f attains its local maximum and local minimum values.

f has a local max at $x=0$ and, local min at $x=-\sqrt{3}$ & $x=\sqrt{3}$

(d) **(4 pts)** Find the absolute maximum and absolute minimum values of f on the interval $[-1, 2]$. 0 and $\sqrt{3}$ are the critical pts in $[-1,2]$.

$f(-1) = -5$

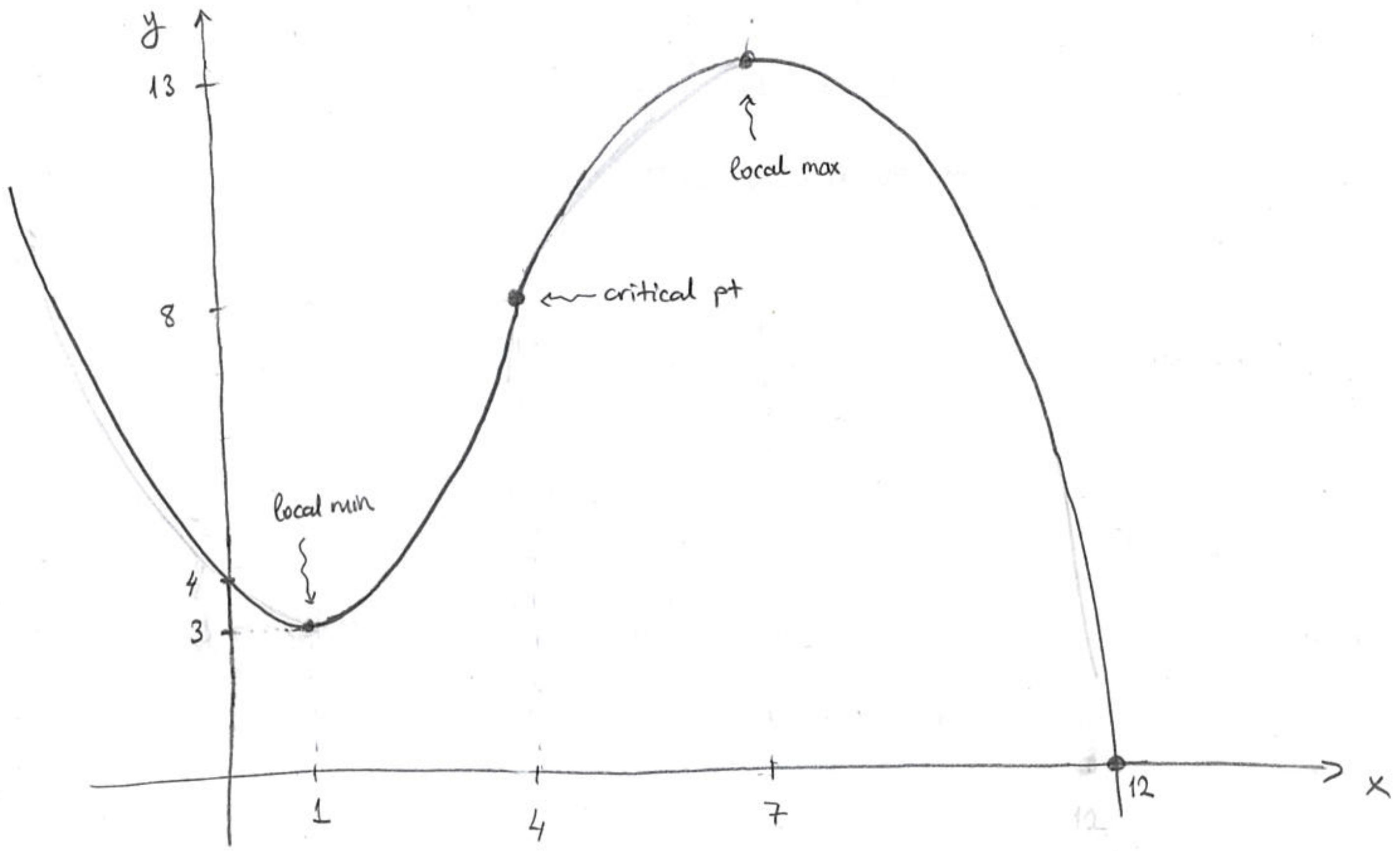
$f(2) = -11$

$f(0) = 5 \leftarrow$ absolute max.

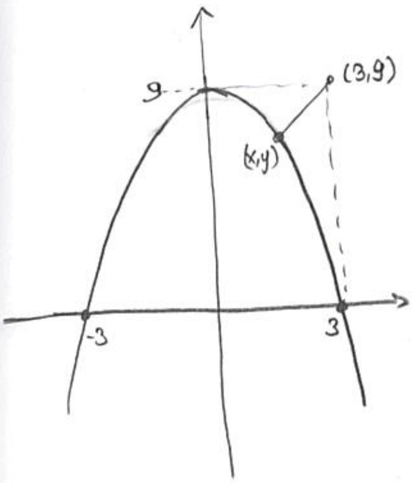
$f(\sqrt{3}) = -13 \leftarrow$ absolute min.

Problem 2 (25 pts) Sketch the graph of a twice-differentiable function $y = f(x)$ with the following properties, indicating the inflection points, local extrema (if they exist) and concavity on the graph.

- If $x < 1$, then $y' < 0$, $y'' > 0$.
- If $x = 1$, then $y = 3$.
- If $1 < x < 4$, then $y' > 0$, $y'' > 0$.
- If $x = 4$, then $y = 8$.
- If $4 < x < 7$, then $y' > 0$, $y'' < 0$.
- If $x = 7$, then $y = 13$.
- If $x > 7$, then $y' < 0$, $y'' < 0$.
- The x-intercept is 12 and the y-intercept is 4.



Problem 3 (20 pts) Find the point on the parabola $y = 9 - x^2$ that is closest to the point $(3, 9)$.



$$d^2 = (x-3)^2 + (y-9)^2$$

$$f(x) = d^2 = (x-3)^2 + (9-x^2-9)^2$$

$$f(x) = x^2 - 6x + 9 + x^4$$

Minimize $f(x) = x^4 + x^2 - 6x + 9$

$$f'(x) = 4x^3 + 2x - 6 = 0$$

$$(x-1)(4x^2+4x+6) = 0.$$

$x = 1$ is the only solution since $4x^2+4x+6=0$ has no root. $(\Delta = 16 - 4.4.6 < 0)$

		1	
$f'(x)$	$-$	0	$+$
$f(x)$	↘		↗

so $x = 1$ minimizes $f(x)$

$x = 1 \Rightarrow y = 8$

The closest pt on the parabola is $(1, 8)$.

Problem 4 (a) (10 pts) Evaluate the following limits.

(i) $\lim_{x\to 1} \frac{\ln x}{x^2-1}$ $\frac{0}{0}$ $\underset{\text{Use L'Hospital}}{=}$ $\lim_{x\to 1} \frac{1/x}{2x} = \lim_{x\to 1} \frac{1}{2x^2} = 1/2$

(ii) $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)$ $\infty . 0$

$= \lim_{x\to\infty} \frac{\sin(1/x)}{1/x}$ $\frac{0}{0}$ $\underset{\text{Use L'Hospital}}{=}$ $\lim_{x\to\infty} \frac{\cos(1/x) . \cancel{-1/x^2}}{\cancel{-1/x^2}} = \lim_{x\to\infty} \cos(1/x) = \cos 0 = 1$ //

(b) (10 pts) Evaluate $\frac{d}{dx}\left(\int_0^{x^2} \ln(1+t^2)dt\right)$

Since $\ln(1+t^2)$ is continuous on $(0,\infty)$ by Fundamental Theorem of Calculus

$\frac{d}{dx}\left(\int_0^{x^2} \ln(1+t^2)\,dt\right) = \ln(1+x^4) . 2x$

Problem 5 (a) Evaluate the integrals in (i) - (iii).

(i) **(7 pts)** $\int \left(\frac{4}{x^2} + \cos x + 2 \right) dx = \int (4x^{-2} + \cos x + 2)\, dx$

$= 4\frac{x^{-1}}{-1} + \sin x + 2x + C = -\frac{4}{x} + \sin x + 2x + C.$

(ii) **(8 pts)** $\int x^2 e^{x^3+1} dx = \frac{1}{3}\int e^u\, du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3+1} + C.$

$u = x^3+1$
$du = 3x^2\,dx$

(iii) **(8 pts)** $\int_0^2 \frac{x}{(1+x^2)^2}\, dx = \frac{1}{2}\int_1^5 \frac{du}{u^2} = \frac{1}{2}\left[\frac{u^{-1}}{-1}\right]_1^5 = \frac{1}{2}\left(-\frac{1}{5}+1\right) = \frac{4}{5}\cdot\frac{1}{2} = \frac{2}{5}$

$u = 1+x^2$
$du = 2x\,dx$

(b) **(7 pts)** Let $f(x)$ be an odd function and $\int_{-2}^{5} f(x)dx = 8$. Find $\int_2^5 f(x)dx$.

Since $f(x)$ is odd $\int_{-2}^{2} f(x)dx = 0$

$\int_{-2}^{5} f(x)\,dx = \int_{-2}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx \Rightarrow \int_{2}^{5} f(x)\,dx = 8$