KOÇ UNIVERSITY

MATH 102

SECOND MIDTERM April 26, 2014

Duration of Exam: 75 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and no talking allowed.
- You must always explain your answers and SHOW YOUR WORK to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: ______________________

Student ID no: ______________________

Signature: ______________________

(Check One):

(Şule Yazıcı – MWF 11:00-12:15) : ——
(Şule Yazıcı – MWF 12:30-13:45) : ——
(Şule Yazıcı – MWF 15:30-16:45) : ——
(Fatih Demirkale – TuTh 12:30-13:45) : ——
(Fatih Demirkale – TuTh 14:00-15:15) : ——

PROBLEM	1	2	3	4	5	6	TOTAL
POINTS	10	10	35	20	10	15	100
SCORE							

Problem 1 (10 points) Estimate $\ln(0.97)$ by linear approximation.

We will use a linear approximation for $f(x)=\ln x$ at $a=1$.

$$L(x)=f(a)+f'(a)(x-a)$$

Here, $f(a)=\ln 1=0$ and $f'(x)=\frac{1}{x}$, so $f'(a)=\frac{1}{1}=1$.

Thus, $L(x)=0+1(x-1)=x-1$.

Hence, $L(0.97)=0.97-1=-0.03$.

Therefore, $\ln(0.97)\approx -0.03$.

Problem 2 (10 pts) Find $f'(x)$ if $f(x)=\cos^2(\sin 2x)$.

$$\begin{aligned} f'(x) &= 2\cos(\sin 2x)\cdot(\cos(\sin 2x))' \\ &= 2\cos(\sin 2x)\cdot(-\sin(\sin 2x))\cdot(\sin 2x)' \\ &= -2\cos(\sin 2x)\cdot\sin(\sin 2x)\cdot\cos 2x\cdot(2x)' \\ &= -4\cos(\sin 2x)\cdot\sin(\sin 2x)\cdot\cos 2x \end{aligned}$$

Problem 3 (35 pts) Consider the function $f(x) = \dfrac{2x^2-8}{x^2-16}$.

Given that $f'(x) = \dfrac{-48x}{(x^2-16)^2}$ and $f''(x) = \dfrac{48(3x^2+16)}{(x^2-16)^3}$.

(a) (2 pts) Find the domain of $f(x)$.

$\mathbb{R} \setminus \{-4, 4\}$.

(b) (2 pts) Find the x and y intercepts of the graph of f if they exist.

x-intercepts : $y=0 \Rightarrow 2x^2-8=0 \Rightarrow x=\pm 2$. So $(\pm 2, 0)$.

y-intercept : $x=0 \Rightarrow y=\frac{-8}{-16}=\frac{1}{2}$. So $(0, 1/2)$.

(c) (3 pts) Find the horizontal and vertical asymptotes of the graph of f if they exist.

$\lim_{x\to -4^-} f(x) = +\infty$ and $\lim_{x\to 4^+} f(x) = +\infty$. So $x=\pm 4$ are vertical asymptotes.

$\lim_{x\to \pm\infty} f(x) = 2$. So $y=2$ is a horizontal asymptote.

(d) (9 pts) Find the intervals on which the function f is increasing or decreasing and; determine the local extreme values of f.

If $f'(x)=0$, then $x=0$.

		0	
$f'(x)$	+		−
$f(x)$	↗		↘

f is increasing on $(-\infty, 0)$.

f is decreasing on $(0, \infty)$.

local max at $(0, 1/2)$.

No local min.

(e) (9 pts) Determine the intervals where the graph of the function f is concave up and concave down, and find the inflection points, if they exist.

$f''(x) \neq 0$ for all x, and $f''(x)$ is undefined for $x = \pm 4$.

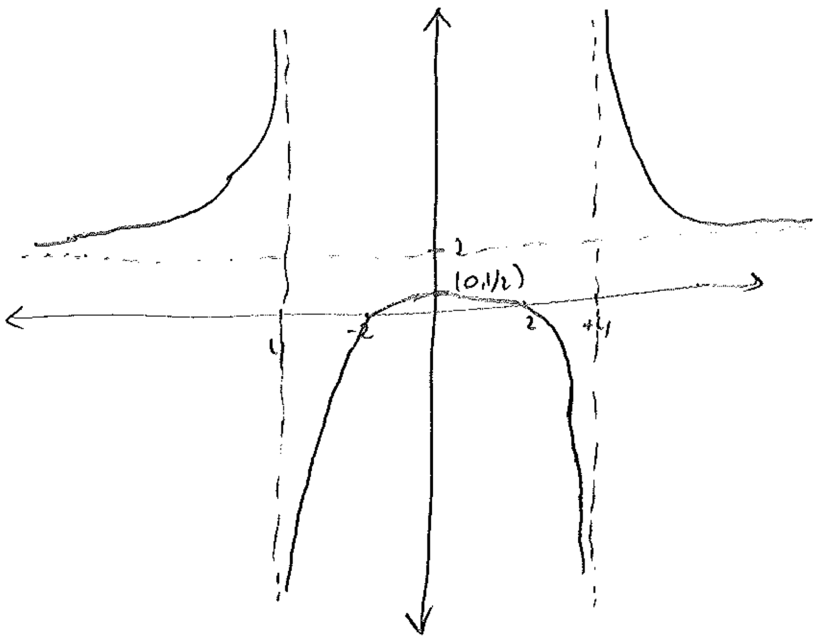
		-4		4	
$f''(x)$	$+$		$-$		$+$
$f(x)$	$\cup$		$\cap$		$\cup$

f is concave up on $(-\infty, -4) \cup (4, \infty)$.

f is concave down on $(-4, 4)$.

No inflection points.

(f) (10 pts) Sketch the graph of the function f.



Problem 4 (20 pts) Find the points on the curve $y = \sqrt{x}$ that are closest and farthest away from the point $(2,0)$ when $0 \le x \le 3$.

$d = \sqrt{(x-2)^2 + (y-0)^2}$ where $y = \sqrt{x}$.

So, $d = f(x) = \sqrt{(x-2)^2 + x} = \sqrt{x^2 - 3x + 4}$.

We will find the abs. max. and abs. min of $f(x)$ when $0 \le x \le 3$.

Alternatively, we can find the abs. max. and abs. min of $d^2 = x^2 - 3x + 4$ when $0 \le x \le 3$.

If $f'(x) = \dfrac{2x-3}{2\sqrt{x^2-3x+4}} = 0$, then $x = 3/2$. Note that, $x^2 - 3x + 4 > 0$ for all x.

So, there are no numbers which make $f'(x)$ undefined.

Here, $x = 3/2$ is the only critical number.

Next, $f(0) = \sqrt{4} = 2$.

$f(3/2) = \sqrt{\frac{9}{4} - \frac{9}{2} + 4} = \sqrt{\frac{7}{4}}$

$f(3) = \sqrt{9 - 9 + 4} = 2$.

By closed interval method, $(3/2, \sqrt{3/2})$ is the closest point and $(0,0)$ and $(3, \sqrt{3})$ are the farthest away from $(2,0)$.

Problem 5 (10 pts) Find $\lim_{x\to\infty}(1-\frac{3}{x})^x$.

$y=(1-\frac{3}{x})^x$. So $\ln y = x\cdot\ln(1-\frac{3}{x})$.

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty} x\cdot\ln(1-\tfrac{3}{x}) = \lim_{x\to\infty}\frac{\ln(1-\frac{3}{x})}{\frac{1}{x}}$$

L'Hospitals Type $\frac{0}{0}$

$$= \lim_{x\to\infty}\frac{\frac{1}{1-\frac{3}{x}}\cdot\frac{-1}{x^2}\cdot(-3)}{\frac{-1}{x^2}}$$

$$= \lim_{x\to\infty}\frac{-3}{1-\frac{3}{x}} = -3.$$

$\lim_{x\to\infty}\ln y = -3 \Rightarrow \lim_{x\to\infty} y = e^{-3}$.

Problem 6 (15 pts) Find dy/dx for the point $(3,1)$ if $5y^2+\ln y = x^2y-4$. Use implicit differentiation.

$10y\cdot y' + \frac{1}{y}\cdot y' = 2xy + y'\cdot x^2 - 0$.

$\Rightarrow y'(10y+\frac{1}{y}-x^2) = 2xy$

$\Rightarrow \frac{dy}{dx} = \frac{2xy}{10y+\frac{1}{y}-x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{6}{10+\frac{1}{1}-9} = 3$ at $(3,1)$.