KOÇ UNIVERSITY

MATH 102

EXAM 2 December 2, 2019

Duration of Exam: 75 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and no talking allowed.
- You must always explain your answers and SHOW YOUR WORK to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: KEY

Student ID no:

Signature:

(Check One):

(Selda Küçükçifçi – TTh 8:30-9:45) : ___
(Selda Küçükçifçi – TTh 13:00-14:15) : ___
(E. Şule Yazıcı – TTh 16:00-17:15) : ___

PROBLEM	POINTS	SCORE
1	20	
2	30	
3	12	
4	12	
5	13	
6	13	
Bonus	10	
TOTAL	110	

Problem 1 (20 points) All 80 rooms in a motel will be rented each night if the manager charges \$40 per room. If he charges \$(40+x) per room, then $2x$ rooms will remain vacant (boş). If each rented room costs the manager \$10 per day and each unrented room costs \$2 per day, how much should the manager charge per room to maximize his daily profit?

80 rooms $40 / room (cost of rented room = $10 , cost of unrented room = $2)

80 - 2x $(40 + x)

$P(x) = R(x) - C(x)$

$= (80 - 2x)(40 + x) - (80 - 2x)\,10 - 2 \cdot 2x$

$= 3200 - \cancel{80x} + \cancel{80x} - 2x^2 - 800 + 20x - 4x$

$P(x) = -2x^2 + 16x + 2400 \longrightarrow \max$

$P'(x) = -4x + 16 = 0 \Rightarrow x = 4$

max : \$44 , $P''(x) = -4 < 0$

Problem 2 (30 points) Sketch the graph of $f(x) = \dfrac{x^2}{e^x}$. Find the domain, intercepts, asymptotes, intervals of increase and decrease, local extremum and determine the concavity.

$$\left(\text{Note: } f'(x) = \frac{x(2-x)}{e^x} \text{ and } f''(x) = \frac{x^2-4x+2}{e^x}\right)$$

Domain : $\mathbb{R}$

$$\lim_{x \to +\infty} \frac{x^2}{e^x} = \lim_{x \to \infty} \frac{2x}{e^x} = \lim_{x \to \infty} \frac{2}{e^x} = 0 \qquad \lim_{x \to -\infty} \frac{x^2}{e^x} = \infty$$

$y = 0$ is the horizontal asymptote

No vertical asymptote $\qquad (0, 0)$ intercept.

		0		$2-\sqrt{2}$		2		$2+\sqrt{2}$	
f'	$-$	0	$+$		$+$	0	$-$		$-$
f''	$+$		$+$	0	$-$		$-$	0	$+$
f	↘ ⌣		↗ ⌣		↗ ⌢		↘ ⌢		↘ ⌣

$f'(x) = 0 \Rightarrow x = 0, \; x = 2$

$$f''(x) = 0 \Rightarrow x = \frac{4 \mp \sqrt{16 - 4.1.2}}{2} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$$

$x = 2 - \sqrt{2}$

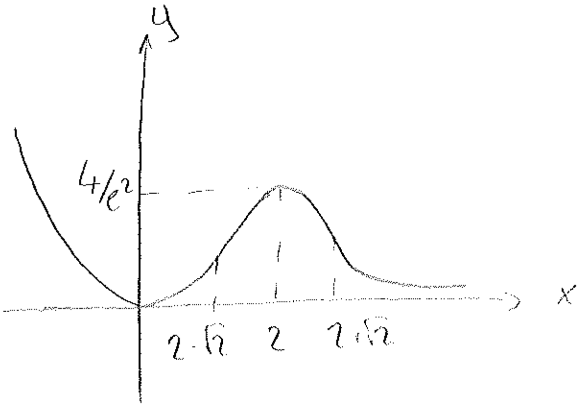
$x = 2 + \sqrt{2}$

local min : $(0, 0)$

local max : $\left(2, \dfrac{4}{e^2}\right)$

inf. points : $\left(2-\sqrt{2}, \; (2-\sqrt{2})^2 / e^{2-\sqrt{2}}\right)$

$\left(2+\sqrt{2}, \; (2+\sqrt{2})^2 / e^{2+\sqrt{2}}\right)$



Problem 3 (12 points) Calculate the following limit using the L'Hospital's rule.

$$\lim_{x\to 0}\frac{\ln(1+x)}{1-e^x} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{\frac{1}{x+1}}{-e^x} = \frac{1}{-1} = -1$$

Problem 4 (12 points) Let $f'(x) = \dfrac{(x+1)^2}{\sqrt{x}}$. If $f(0) = 5$, find $f(x)$.

$$f'(x) = \frac{x^2+2x+1}{\sqrt{x}} = x^{3/2} + 2x^{1/2} + x^{-1/2}$$

$$f(x) = \frac{2}{5}x^{5/2} + \frac{4}{3}x^{3/2} + 2x^{1/2} + C \quad , \quad f(0) = 5 \Downarrow C = 5$$

$$f(x) = \frac{2}{5}x^{5/2} + \frac{4}{3}x^{3/2} + 2x^{1/2} + 5$$

Problem 5 (13 points) Find y' if $\sin(xy) = \frac{1}{2}$ using implicit differentiation.

$$\cos(xy)\,(y + y'\cdot x) = 0$$

$$y\cos(xy) + xy'\cos(xy) = 0$$

$$y' = \frac{y\cdot\cancel{\cos(xy)}}{x\,\cancel{\cos(xy)}} \qquad y' = -\frac{y}{x}$$

Problem 6 (13 points) Find $f'(x)$ if $f(x) = (x^2 + 1)^x$.

$$y = (x^2+1)^x$$

$$\ln y = x \ln (x^2+1)$$

$$\frac{1}{y}\, y' = \ln (x^2+1) + x \cdot \frac{1}{x^2+1} \cdot 2x$$

$$y' = (x^2+1)^x \left(\ln (x^2+1) + \frac{2x^2}{x^2+1} \right)$$

BONUS (10 points) Evaluate $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \underbrace{\frac{1}{n}}_{\Delta x} \underbrace{\sqrt{\frac{i}{n}}}_{f(x_i)}$.

$$0 \quad \frac{1}{n} \quad \frac{2}{n} \quad \cdots \quad 1 = \frac{n}{n}$$

$$\int_0^1 \sqrt{x}\, dx = \left[\frac{x^{3/2}}{3/2} \right]_0^1 = \frac{2}{3}$$