KOÇ UNIVERSITY

MATH 102

FINAL EXAM January 2, 2018

Duration of Exam: 90 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and no talking allowed.
- You must always explain your answers and SHOW YOUR WORK to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: ____________________

Student ID no: KEY

Signature: ____________________

(Check One):

(Selda Küçükçifçi – MW 8:30-9:45) : ____
(Selda Küçükçifçi – MW 11:30-12:45) : ____
(Hasan İnci – MW 16:00-17:15) : ____

PROBLEM	POINTS	SCORE
1	20	
2	12	
3	16	
4	40	
5	15	
TOTAL	**103**	

Problem 1 (10 points) (a) Determine $\lim_{x \to 0} \frac{|4x-1| - |4x+1|}{x}$.

$$\lim_{x \to 0} \frac{|4x-1| - |4x+1|}{x} = \lim_{x \to 0} \frac{-4x+1-4x-1}{x} = \lim_{x \to 0} \frac{-8x}{x} = -8$$

(b) (10 points) Determine $\lim_{x \to 0+} \frac{\int_0^x e^{t^3}\, dt}{x}$. $\frac{0}{0}$.

$$= \lim_{x \to 0^+} \frac{\frac{d}{dx} \int_0^x e^{t^3} dt}{1} = \lim_{x \to 0^+} \frac{e^{x^3}}{1} = 1.$$

Problem 2 (12 points) Find the absolute maximum and absolute minimum values of

$$f(x) = xe^{x/2} \text{ on } [-1, 1].$$

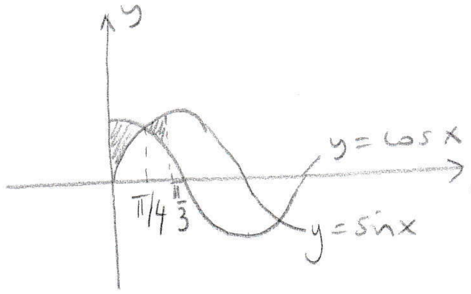
$f'(x) = e^{x/2} + xe^{x/2} \cdot \frac{1}{2} = e^{x/2}\left(1 + \frac{x}{2}\right)$

$f'(x) = 0 \Rightarrow \frac{x}{2} = -1 \Rightarrow x = -2 \notin [-1,1]$.

$f(-1) = -e^{-1/2} = -\frac{1}{\sqrt{e}}$ which is the absolute minimum.

$f(1) = e^{1/2} = \sqrt{e}$ which is the absolute maximum.

Problem 3 (16 points) Determine the area of the region bounded by $\cos x$, $y = \sin x$, $x = 0$ and $x = \frac{\pi}{3}$.



$\int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/3} (\sin x - \cos x)\,dx$

$= \Big[\sin x + \cos x\Big]_0^{\pi/4} + \Big[-\cos x - \sin x\Big]_{\pi/4}^{\pi/3}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0+1) + \left[-\frac{1}{2} - \frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\right]$

$= \frac{4\sqrt{2}}{2} - \frac{3}{2} - \frac{\sqrt{3}}{2} = \frac{4\sqrt{2} - 3 - \sqrt{3}}{2}$.

Problem 4 Evaluate the following integrals.

(a) (12 points) $\int \sin^2 x \sin(2x)\, dx$ (HINT: $\sin(2x) = 2\sin x \cos x$)

$$= \int \sin^2 x \, 2\sin x \cos x \, dx = 2\int \sin^3 x \cos x \, dx$$

$u = \sin x$
$du = \cos x \, dx$

$$= 2\int u^3 \, du = 2\frac{u^4}{4} + C = \frac{1}{2}(\sin x)^4 + C$$

(b) (12 points) $\int_1^e x^3 \ln x \, dx$

$u = \ln x$ $\quad v = \frac{x^4}{4}$

$du = \frac{1}{x}dx$ $\quad dv = x^3 dx$

$$\int_1^e x^3 \ln x \, dx = \left[\frac{x^4}{4}\ln x\right]_1^e - \frac{1}{4}\int_1^e x^3 \, dx = \left[\frac{x^4}{4}\ln x\right]_1^e - \frac{1}{4}\left[\frac{x^4}{4}\right]_1^e$$

$$= \frac{e^4}{4} - \frac{1}{16}(e^4 - 1) = \frac{3e^4 + 1}{16}$$

(c) (16 points) $\displaystyle\int \frac{5x}{(x+2)(x^2+1)}\,dx$

$$\frac{5x}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$5x = A(x^2+1) + (x+2)(Bx+C)$$

$x=-2 \Rightarrow -10 = 5A \Rightarrow A = -2.$

$$5x = -2x^2-2+Bx^2+2Bx+Cx+2C = \underbrace{(-2+B)}_{0}x^2 + \underbrace{(2B+C)}_{5}x + \underbrace{(2C-2)}_{0}$$

$B=2 \qquad C=1.$

$$\int \frac{5x}{(x+2)(x^2+1)}\,dx = -2\int \frac{dx}{x+2} + \int \frac{2x\,dx}{x^2+1} + \int \frac{dx}{x^2+1}$$

$$= -2\ln|x+2| + \ln(x^2+1) + \arctan x + C.$$

Problem 5 (15 points) Determine whether the following integral is convergent or divergent. Evaluate it if it is convergent.

$$\int_e^{e^2} \frac{dx}{x(1-lnx)}$$

$u = 1-\ln x$

$du = -\frac{1}{x}\,dx$

$$-\int_0^{-1} \frac{du}{u} = \int_{-1}^{0} \frac{du}{u} = \lim_{t\to 0^-} \int_{-1}^{t} \frac{du}{u} = \lim_{t\to 0^-} \Big[\ln|u|\Big]_{-1}^{t}$$

$$= \lim_{t\to 0^-} (\ln|t| - 0) = -\infty.$$

So this integral is divergent.