KOÇ UNIVERSITY

MATH 102

EXAM 1 November 1, 2018

Duration of Exam: 75 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and no talking allowed.
- You must always explain your answers and SHOW YOUR WORK to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: ______________________

Student ID no: KEY ______________________

Signature: ______________________

(Check One):
(Selda Küçükçifçi – MW 8:30-19:45) : ____
(Selda Küçükçifçi – MW 11:30-12:45) : ____
(Hasan İnci – MW 16:00-17:15) : ____

PROBLEM	POINTS	SCORE
1	36	
2	36	
3	14	
4	14	
TOTAL	100	

Problem 1 Find the following limits, if they exist.

(a) (12 points) $\lim_{x\to\infty}\left(\frac{x^2+2x+2\sqrt{x}}{2x^2+\sqrt[3]{x}}\right)$ $= \lim_{X\to\infty} \dfrac{1+\frac{2}{X}+\frac{2}{X\sqrt{X}}}{2+\frac{1}{X^{5/3}}} = \frac{1}{2}.$

(b) (12 points) $\lim_{x\to 2^-}\left(\frac{|x-2|}{x^2-4}\right)$ $= \lim_{X\to 2^-} \dfrac{-(X-2)}{(X-2)(X+2)} = \lim_{X\to 2^-} \dfrac{-1}{X+2} = -\frac{1}{4}$

(c) (12 points) $\lim_{x\to 0}\left(\frac{5x-\sin(5x)}{2x+\sin(2x)}\right)$ $= \lim_{X\to 0} \dfrac{5-5\frac{\sin 5X}{5X}}{2+2\frac{\sin 2X}{2X}} = \frac{0}{4} = 0.$

since $\lim_{X\to 0}\frac{\sin 5X}{5X} = 1$ & $\lim_{X\to 0}\frac{\sin 2X}{2X} = 1.$

Problem 2 Find the derivative of the function f in (a) – (b). Simplify your answer, if it is possible.

(a) (12 points) $f(x) = e^{\cos^3(3x)}$

$$f'(x) = e^{\cos^3(3x)} \quad 3\cos^2(3x)(-\sin(3x)).3$$

$$= -9(\cos^2 3x)(\sin 3x)\, e^{\cos^3(3x)}$$

(b) (12 points) $f(x) = \dfrac{2x^2 - 3x}{(x-2)^2}$

$$f'(x) = \frac{(4x-3)(x-2)^2 - (2x^2-3x)2(x-2)}{(x-2)^4}$$

$$= \frac{(x-2)\left[\cancel{4x^2} - 8x - 3x + 6 - \cancel{4x^2} + 6x\right]}{(x-2)^4} = \frac{-5x+6}{(x-2)^3}$$

(c) (12 points) Express $\lim_{h\to 0} \left(\dfrac{\sin\sqrt{4+h} - \sin\sqrt{4}}{h} \right)$ as a derivative and evaluate it.

$$\lim_{h\to 0} \left(\frac{\sin\sqrt{4+h} - \sin\sqrt{4}}{h} \right) = f'(4)$$

where $f(x) = \sin\sqrt{x}$, $\quad f'(x) = (\cos\sqrt{x}) . \dfrac{1}{2\sqrt{x}}$

So $f'(4) = \dfrac{\cos 2}{4}$

Then $\lim_{h\to 0} \left(\dfrac{\sin\sqrt{4+h} - \sin\sqrt{4}}{h} \right) = \dfrac{\cos 2}{4}$.

Problem 3 (14 points) The function f is defined by $f(x) = \begin{cases} \cos x & x < 0 \\ mx + c & x \geq 0 \end{cases}$. Find the numbers m and c such that f is differentiable everywhere. Explain your work.

Since f is differentiable everywhere f must be continuous everywhere. So $\lim_{x \to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\lim_{x\to 0^-} \cos x = 1$ $\quad \lim_{x \to 0^+} (mx+c) = c = f(0)$.

So $c = 1$.

$f'(x) = \begin{cases} -\sin x & x < 0 \\ m & x > 0 \end{cases}$ $\quad f'(0) = \lim_{h\to 0^-} \frac{\cos(0+h)-1}{h} = \lim_{h \to 0^+} \frac{mh+1-1}{h}$

Since $f'(0) = 0 \Rightarrow m = 0$.

Problem 4 (14 points) Let $g(x) = 1 + \sqrt{x}$ and $(f \circ g)(x) = 3 + 2\sqrt{x} + x$. Find $f'(2)$.

$f(g(x)) = 3 + 2\sqrt{x} + x$

$(f\circ g)'(x) = f'(g(x))\, g'(x) = 2 \cdot \frac{1}{2\sqrt{x}} + 1 = \frac{1}{\sqrt{x}} + 1$.

$g(x) = 2 \Rightarrow 1 + \sqrt{x} = 2 \Rightarrow \sqrt{x} = 1 \Rightarrow x = 1$.

So $f'(2)\, g'(1) = 1 + 1 = 2$. But $g'(x) = \frac{1}{2\sqrt{x}}$ $\quad g'(1) = \frac{1}{2}$

So $f'(2) = \frac{2}{\frac{1}{2}} = 4$.