KOÇ UNIVERSITY
MATH 102 - CALCULUS
Final exam   May 31, 2006
Duration of Exam: 150 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below.**

Surname, Name: ——————————————————

Signature: ——————————————————

Section (Check One):

Section 1: S. Küçükçifci	——
Section 2: T. Albu (9:30)	——
Section 3: E.Ş. Yazici (15:30)	——
Section 4: T. Albu (12:30)	——
Section 5: E.Ş. Yazici (11:00)	——

PROBLEM	POINTS	SCORE
1	15	
2	20	
3	10	
4	20	
5	15	
6	8	
7	12	
TOTAL	**100**	

Problem 1. Calculate the following limits or show that they do not exist.

(a) (5 pts) $\lim\limits_{x\to 1} \dfrac{(x-1)\sqrt{2x}}{|3x-3|}$

(b) (4 pts) $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{\cos x}$

(c) (6 pts) For which values of a, the function

$$f(x) = \begin{cases} x^3 - 2x & \text{for} & x \leqslant a \\ x^2 & \text{for} & x > a \end{cases}$$

is continuous at $x = a$?

Problem 2. Find the derivative of the following function f in (a)-(c). Simplify your answers.

(a) (3 pts) $f(x) = e^{x^2+2} + \ln\left(\dfrac{x+1}{x^2+2}\right)$

(b) (6 pts) $f(x) = (\sin x)^{\cos x}$

(c) (6 pts) $f(x) = \sqrt[3]{\dfrac{1+x^3}{1-x^3}}$

(d) (5 pts) Find the equation of the tangent line at the point $P(1, e)$ to the curve defined by the equation $y = e^{1/x}$.

Problem 3. Consider the function

$$f(x) = \frac{e^x}{x + 2}$$

(a) (6 pts) Find the horizontal and vertical asymptotes of the graph of f if they exist.

(b) (2 pts) Find the intervals on which the function f is increasing and decreasing.

(c) (2 pts) Determine the local extreme values of the function f.

Problem 4. Calculate the following integrals.

(a) (5 pts) $\displaystyle \int \frac{1}{x^2} \sin \frac{1}{x} \, dx$

(b) (8 pts) $\displaystyle \int \frac{3x^2 + 4x + 4}{x^3 + x} \, dx$

(c) (7 pts) $\displaystyle \int e^x \sin x \, dx$

Problem 5. (a) (9 pts) The region between the curve $y = \sin x$, $0 \le x \le \frac{\pi}{4}$, and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

(b) (6 pts) Find the length of the curve $y = 2x^{3/2} + 1$ from $x = 0$ to $x = 1$.

Problem 6. Find the limit of the following sequences (a_n) and determine whether (a_n) converges or diverges.

(a) (4 pts) $a_n = \dfrac{(-1)^n \sin(2n + 3)}{3n + 4}$

(b) (4 pts) $a_n = \dfrac{3^n}{n^3}$

Problem 7. Determine whether the following series converges or diverges. If the series converges, find its sum.

(a) (4 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^4 - 2n + 1}{5n^4 - 2n^2 + n}$

(b) (4 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$

(a) (4 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$